Model Order Reduction of Discrete Time Interval System Using Improved Bilinear Routh Approximation

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Abstract--This paper deals with a new method for order reduction of higher order discrete time interval systems into lower order discrete time interval. Model Order Reduction is done by using conventional method, Improved Bilinear Routh Approximation method and Kharitonov's theorem. By this method, the obtained lower order model retains the properties of the higher order system like stability, settling time and attains impulse energy. The proposed method is illustrated with the help of typical numerical example considered from the literature.

***Index Terms*-- Discrete systems, Improved Bilinear Routh Approximation, Interval system, Kharitonov's theorem.**

I.INTRODUCTION

Original system is generally a higher order system in which analysis is tedious and costlier due to its complexity. By implementing Model Order Reduction method, complexity of system can be reduced. So that, design and analysis of control system becomes easier. Model Order Reduction system is a branch of control systems which deals with the reduction of complexity and preserving the input-output behavior of the system [1]. These methods are mainly classified as 1.Frequency domain and 2.Time domain order reduction methods. These are again classified as 1.Fixed and 2.Interval systems. The frequency domain order reduction methods uses transfer function model, whereas the time domain order reduction methods uses state space model. Several methods are available in the literature for the order reduction of linear continuous systems in time domain as well as frequency domain. The reduced order model obtained in the frequency domain gives better matching of the impulse response with its higher order system[2].

Some of the most popularly used frequency domain discrete order reduction methods are Routh Approximation (RA) [1] method, Bilinear Routh Approximation (BRA) [2] method. BRA method is extension of RA method to discrete time systems. It inherits the properties of RA method such as continuous attention due to its stability preserving and time moments matching properties. Thus the reduced model obtained still possesses the stability preserving, time moments matching. In literature, it has been shown that BRA method and it's modified method i.e., IBRAM is improved by letting the impulse response energy of the original system also be conserved in the reduced model. Many systems the coefficients are constants but uncertain within a finite range. Such systems are classified as interval systems. Some methods like [4] and [5] are used for reduction techniques and some methods [3] are used for fixed systems and extended to interval systems by using kharitonov's theorem to increase its effectiveness. Kharitonov's theorem converts the interval higher order system into fixed higher order system and by applying MOR method, it can be converted to fixed lower order system and rearranged to form an interval lower order system.

2. PROBLEM FORMULATION

Consider a higher order discrete time interval by the transfer function.

$$\mathrm{G_n(z)} = \frac{\mathrm{N(z)}}{\mathrm{D(z)}} = \frac{[\mathrm{B_0^-\ B_0^+}] + [\mathrm{B_1^-\ B_1^+}]\mathrm{z} + \ldots\ldots\ldots + [\mathrm{B_{n-1}^-\ B_{n-1}^+}]\mathrm{z^{n-1}}}{[\mathrm{A_0^- A_0^+}] + [\mathrm{A_1^- A_1^+}]\mathrm{z} + \ldots\ldots\ldots. + [\mathrm{A_n^- A_n^+}]\mathrm{z^n}}$$

Where $[A_i^-\ A_i^+]$ represents the denominator coefficients of $G_n(z)$ for i=0,1,2,……n with $A_i^-\ and\ A_i^+$ represents the lower and upper bounds of the interval $[A_i^-\ A_i^+]$ respectively and $[B_i^-\ B_i^+]$ represents the numerator coefficients of $G_n(z)$ for i=0,1,2,……n with $B_i^-\ and\ B_i^+$ represents the lower and upper bounds of the interval $[B_i^-\ B_i^+]$ respectively. The above system is proposed to obtain the reduced order interval model such as

$$G_r(z) = \frac{n(z)}{d(z)} = \frac{[b_0^-\ b_0^+] + [b_1^-\ b_1^+]z + \ldots\ldots\ldots + [b_{r-1}^-\ b_{r-1}^+]z^{r-1}}{[a_0^- a_0^+] + [a_1^- a_1^+]z + \ldots\ldots\ldots. + [a_r^- a_r^+]z^r}$$

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Where $[a_i^- \; a_i^+]$ represents the denominator coefficients of $G_r(z)$ for i=0,1,2,……n with a_i^- *and* a_i^+ represents the lower and upper bounds of the interval $[a_i^- \; a_i^+]$ respectively and $[b_i^- \; b_i^+]$ represents the numerator coefficients of $G_r(z)$ for i=0,1,2,……n with b_i^- *and* b_i^+ represents the lower and upper bounds of the interval $[b_i^- \; b_i^+]$ respectively.

3. PROPOSED METHOD

3.1 Reduction procedure

Consider the family of real interval transfer functions as:

$$G_n(z) = \frac{N(z)}{D(z)} = \frac{[B_0^- \; B_0^+] + [B_1^- \; B_1^+]z + \ldots\ldots\ldots + [B_{n-1}^- \; B_{n-1}^+]z^{n-1}}{[A_0^- A_0^+] + [A_1^- A_1^+]z + \ldots\ldots\ldots + [A_n^- A_n^+]z^n} \quad (1)$$

The four fixed kharitonov's transfer functions associated with $G_n(z)$ (1) are given as:

$$G^1(z) = \frac{N^1(z)}{D^1(z)} = \frac{B_0^- + B_1^- z + B_2^+ z^2 + \ldots\ldots\ldots + B_{n-1}^- z^{n-1}}{A_0^- + A_1^- z + A_2^+ z^2 + \ldots\ldots\ldots + A_n^- z^n}$$

$$= \frac{B_{10}^- + B_{11}^- z + B_{12}^+ z^2 + \ldots\ldots\ldots + B_{1n-1}^- z^{n-1}}{A_{10}^- + A_{11}^- z + A_{12}^+ z^2 + \ldots\ldots\ldots + A_{1n}^- z^n} \quad (2)$$

$$G^2(z) = \frac{N^2(z)}{D^2(z)} = \frac{B_0^- + B_1^+ z + B_2^+ z^2 + \ldots\ldots\ldots + B_{n-1}^- z^{n-1}}{A_0^- + A_1^+ z + A_2^+ z^2 + \ldots\ldots\ldots + A_n^- z^n}$$

$$= \frac{B_{20}^- + B_{21}^+ z + B_{22}^+ z^2 + \ldots\ldots\ldots + B_{2n-1}^- z^{n-1}}{A_{20}^- + A_{21}^+ z + A_{22}^+ z^2 + \ldots\ldots\ldots + A_{2n}^- z^n} \quad (3)$$

$$G^3(z) = \frac{N^3(z)}{D^3(z)} = \frac{B_0^+ + B_1^- z + B_2^- z^2 + \ldots\ldots\ldots + B_{n-1}^+ z^{n-1}}{A_0^+ + A_1^- z + A_2^- z^2 + \ldots\ldots\ldots + A_n^+ z^n}$$

$$= \frac{B_{30}^+ + B_{31}^- z + B_{32}^- z^2 + \ldots\ldots\ldots + B_{3n-1}^+ z^{n-1}}{A_{30}^+ + A_{31}^- z + A_{32}^- z^2 + \ldots\ldots\ldots + A_{3n}^+ z^n} \quad (4)$$

$$G^4(z) = \frac{N^4(z)}{D^4(z)} = \frac{B_0^+ + B_1^+ z + B_2^- z^2 + \ldots\ldots\ldots + B_{n-1}^+ z^{n-1}}{A_0^+ + A_1^+ z + A_2^- z^2 + \ldots\ldots\ldots + A_n^+ z^n}$$

$$= \frac{B_{40}^+ + B_{41}^+ z + B_{42}^- z^2 + \ldots\ldots\ldots + B_{4n-1}^+ z^{n-1}}{A_{40}^+ + A_{41}^+ z + A_{42}^- z^2 + \ldots\ldots\ldots + A_{4n}^+ z^n} \quad (5)$$

The above Kharitonov's transfer functions are, in general represented as:

$$G_n^I(z) = \frac{N_n^I(z)}{D_n^I(z)} = \sum_{J=0}^{n-1} B_{IJ} z^J \Big/ \sum_{J=0}^{n-1} A_{1J} z^J$$

Where I=1,2,3,4

Determination of the denominator and numerator coefficients of lower order model for first Kharitonov transfer function by Improved Bilinear Routh Approximation method using [2] [5]

IBRAM method

$$G^1(z) = \frac{N^1(z)}{D^1(z)}$$
$$= \frac{B_{10}^- + B_{11}^- z + B_{12}^+ z^2 + \ldots\ldots\ldots + B_{1n-1}^- z^{n-1}}{A_{10}^- + A_{11}^- z + A_{12}^+ z^2 + \ldots\ldots\ldots + A_{1n}^- z^n}$$

Let the reduced order model for first kharitonov transfer function be:

$$G_r^1(z) = \frac{N_r^1(z)}{D_r^1(z)}$$
$$= \frac{b_{10}^- + b_{11}^- z + b_{12}^+ z^2 + \ldots\ldots\ldots + b_{1r-1}^- z^{r-1}}{a_{10}^- + a_{11}^- z + a_{12}^+ z^2 + \ldots\ldots\ldots + a_{1r}^- z^r}$$

Let numerator represented by B(z) and denominator represented by A(z) and the reduced method be represented by $B_r(z)$ *and* $A_r(z)$ respectively. Let

$$C_1(z) = \frac{A(z) + z^n A(z^{-1})}{2} \quad (6)$$

and

$$D_1(z) = \frac{A(z) - z^n A(z^{-1})}{2} \quad (7)$$

let the α and β parameters are computed by

$$C_{i+1}(z) = \frac{D_i(z)}{z-1} \quad (8)$$

Where

$$\alpha_i = \frac{C_i(1)}{2C_{i+1}(1)} \quad (9)$$

And

$$D_{i+1}(z) = \frac{C_i(z) - \alpha_i(z+1)C_{i+1}(z)}{z-1} \quad (10)$$

$$\beta_i = \frac{B_i(1)}{2C_{i+1}(1)} \quad (11)$$

$$B_{i+1}(z) = \frac{B_i(z) - \beta_i(z+1)C_{i+1}(z)}{z-1} \quad (12)$$

With $B_i(z) = (z+1)B(z)$. (13)

G(z) is stable if and only if a_i>0 for all 1≤ i≤ n.

For reducing the order to k^{th} term, then set α_i =0 for all k+1≤ i≤ n where 'n' is the order of higher order system and k is the order of reduced order system.

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Therefore $G_k(z) = \frac{B_k(z)}{A_k(z)}$ (14)

Compute the above recursive equations for $1 \leq m \leq k$.

$$B_m(z) = \alpha_m(z+1)B_{m-1}(z) + (z-1)^2 B_{m-2}(z) + \beta_m(z-1)^{m-1} \quad (15)$$

$$A_m(z) = \alpha_m(z+1)A_{m-1}(z) + (z-1)^2 A_{m-2}(z) \quad (16)$$

Where $B_{-1}(z) = B_0(z) = 0, A_{-1}(z) = \frac{1}{z-1}$ and $A_0(z) = 1$.

Since the first k(α,β) parameters of G_n(z) are preserved in $G_k(z)$, $G_k(z)$ is stable whenever G_n(z) is stable and $G_k(z)$ fits the first k time moments of G_n(z) then if we expand G_n(z) and $G_k(z)$ in taylor series about z=1 as:

$$G_n(z) = t_0 + t_1(z-1) + \frac{t_2}{2!}(z-1)^2 + \ldots\ldots$$

$$G_k(z) = l_0 + l_1(z-1) + \frac{l_2}{2!}(z-1)^2 + \ldots\ldots\ldots$$

And $t_i = l_i$ for all i=0,1,2,3,……k-1

Which represents the two responses represents the same time moments.

The impulse response energies of $G_n(z)$ and $G_k(z)$ are represented respectively as:

$$I = \frac{1}{2\pi i}\oint G_n(z)G_n(z^{-1})\frac{1}{z}dz = \frac{1}{4}\sum_{i=1}^{n}\frac{\beta_i^2}{\alpha_i} \quad (17)$$

$$I_k = \frac{1}{2\pi i}\oint G_k(z)G_k(z^{-1})\frac{1}{z}dz = \frac{1}{4}\sum_{i=1}^{k}\frac{\beta_i^2}{\alpha_i} \quad (18)$$

Where $\oint denotes$ the integral along the unit circle in the complex plane.It was also shown in [] that for $0 \leq m \leq n-1$,the time moments t_m of G(z) is of the form:

$$t_m = f(\alpha_1, \alpha_2, \ldots\ldots \alpha_m, \beta_1, \beta_2, \ldots\ldots \beta_m) + g(\alpha_1, \alpha_2, \ldots\ldots \alpha_m)\frac{\beta_{m+1}}{\alpha_{m+1}}$$

With $t_0 = \frac{\beta_1}{2\alpha}$ i.e., the first k time moments $t_0, t_1, \ldots\ldots t_{k-1}$ are not changed as long as the ratio $\frac{\beta_k}{\alpha_k}$ is kept the same with (α_i, β_i), i=1,2,3,…..k-1 fixed. Hence we can replace the last parameters $\alpha_{k\text{ and }}, \beta_k,$ in eq.(14) by $\widehat{\alpha_k}\, and\, \widehat{\beta_k}$ in such a way that $\frac{\beta_k}{\alpha_k} = \frac{\widehat{\beta_k}}{\widehat{\alpha_k}}$ (19) and $I_k = I$, (20) and compute the new model $\widehat{G_k}$(z) from eq.(15)& (16) using (α_i, β_i),i=1,2,….k-1 and $\widehat{\alpha_k}, \widehat{\beta_k}$. Then $\widehat{G_k}$(z) preserves the impulse response energy as well as the stability and the first k time moments of the original system.

The above procedure is repeated for remaining Kharitonov's transfer function after obtaining four Kharitonov reduced order transfer function, the reduced order interval transfer function is obtained by using below equation

$$G_k(z) = \frac{\sum_{j=0}^{k-1}[\min(b_{Ij}), \max(b_{Ij})]z^j}{\sum_{j=0}^{k}[\min(a_{Ij}), \max(a_{Ij})]z^j}$$

4. Numerical Example

Consider a higher order interval system

$$G(z) = \frac{[2.3\ 2.55]z^4 + [2.45\ 2.65]z^3 + [3.25\ 3.35]z^2 + [2.5\ 2.65]z + [1.8\ 2.2]}{[8.3\ 8.35]z^5 + [4.6\ 4.8]z^4 + [2.4\ 2.5]z^3 + [2\ 2.2]z^2 + [1.5\ 1.8]z + [2.1\ 2.15]}$$

This higher order interval system can be represented as four Kharitonov higher order transfer functions. They are given as:

$$G_5^1(z) = \frac{2.3z^4 + 2.65z^3 + 3.35z^2 + 2.5z + 1.8}{8.3z^5 + 4.6z^4 + 2.5z^3 + 2.2z^2 + 1.5z + 2.1}$$

$$G_5^2(z) = \frac{2.3z^4 + 2.45z^3 + 3.35z^2 + 2.65z + 1.8}{8.35z^5 + 4.6z^4 + 2.4z^3 + 2.2z^2 + 1.8z + 2.1}$$

$$G_5^3(z) = \frac{2.55z^4 + 2.45z^3 + 3.25z^2 + 2.65z + 2.2}{8.35z^5 + 4.8z^4 + 2.4z^3 + 2z^2 + 1.8z + 2.15}$$

$$G_5^4(z) = \frac{2.55z^4 + 2.65z^3 + 3.25z^2 + 2.5z + 2.2}{8.3z^5 + 4.8z^4 + 2.5z^3 + 2z^2 + 1.5z + 2.15}$$

From the first Kharitonov transfer function,

$$G_5^1(z) = \frac{2.3z^4 + 2.65z^3 + 3.35z^2 + 2.5z + 1.8}{8.3z^5 + 4.6z^4 + 2.5z^3 + 2.2z^2 + 1.5z + 2.1}$$

By using formulae (6) to (13) we get alpha and beta values

From that we calculate the reduced system by using (15) to (18).

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The reduced transfer function is

$$G_2^1(\mathrm{z}) = \frac{3.97201\mathrm{z} + 2.76933}{9.25966\mathrm{z}^2 + 3.66684\mathrm{z} - 1.59282}$$

Similarly, the other reduced transfer functions are represented as

$$G_2^2(\mathrm{z}) = \frac{7.5343\mathrm{z} + 5.8285}{17.3175\mathrm{z}^2 + 9.4192\mathrm{z} - 3.8979}$$

$$G_2^3(\mathrm{z}) = \frac{29.461\mathrm{z} + 25.2808}{65.6688\mathrm{z}^2 + 42.9248\mathrm{z} - 18.7442}$$

$$G_2^4(\mathrm{z}) = \frac{13.6354\mathrm{z} + 10.51087}{29.6461\mathrm{z}^2 + 17.5105\mathrm{z} - 8.1356}$$

By rearranging the above reduced models to represent an interval model as

$$G_k(\mathrm{z}) = \frac{[3.97201\ 29.461]\mathrm{z} + [2.76933\ 25.2808]}{[9.25966\ 65.6688]\mathrm{z}^2 + [3.66684\ 42.9248]\mathrm{z} + [-18.7442\ -1.59282]}$$

Thus, we can represent the above four reduced order step responses respectively as:

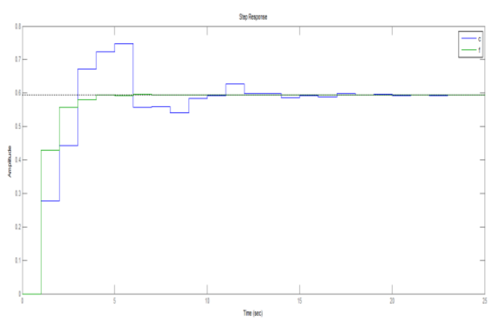

Fig1: step response of first transfer function

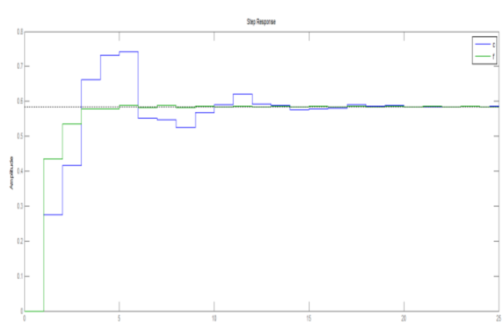

Fig2: step response of second transfer function

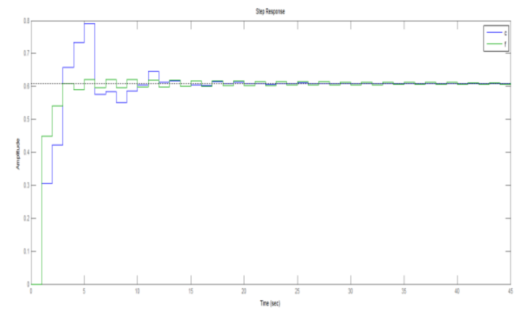

Fig3: step response of third transfer function

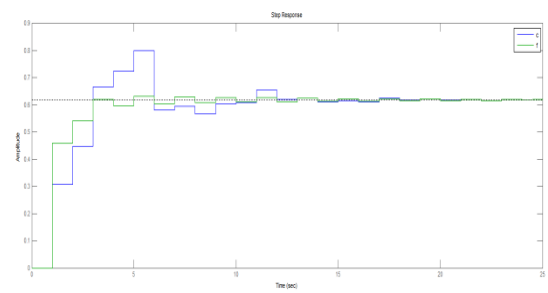

Fig4: step response of fourth transfer function

5.Conclusion

An Improved Bilinear Routh Approximation method (IBRAM)[3] for interval system has been proposed in this paper. The reduced model is obtained by using kharitonov theorem of discrete time interval systems and IBRAM technique. It preserves the stability, time-moments and impulse response energy of all kharitonov polynomials of the original system in their reduced models.

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