**S- Prime Meet Matrices on posets**

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Abstract - We consider S-prime meet matrices as an abstract generalization of S-prime greatest common divisor (GCD) matrices. We also found determinant and inverse and discuss the some of the most important properties of S-prime GCD matrices are presented in terms of Error! Reference source not found. meet me meet matrices.


I. INTRODUCTION

Let $S = \{x_1, x_2, x_3, \ldots, x_n\}$ be a set of n positive integers with $x_1 < x_2 < x_3 < \ldots < x_n$ and let $f : P \rightarrow \mathbb{C}$ be a complex valued function on $\mathbb{Z}_+$(i.e., arithmetical function). Let $(x_i, x_j)$ denotes the greatest common divisor (GCD) of $x_i$ and $x_j$ and define the nxn matrices $(S)_{ij} = ((S)_{ij}) = f(x_i, x_j).$ We refer to (S) as the GCD matrix on S with respect to f. The set S is said to be factor closed if it contains every positive divisor of each $x_i \in S$ clearly a factor closed set is always GCD – closed further converse does not hold.

In 1876, the concept of classical Smith determinant with entries on $\mathbb{Z}_+$ was introduced by H.J.S. Smith [12],

$$\prod_{i=1}^{n} \Phi(x_i)$$

where $\Phi$ is the Euler’s totient function. The GCD matrix with respect to $f$ is,

$$\begin{bmatrix}
    f(x_1, x_1) & f(x_1, x_2) & \ldots & f(x_1, x_n) \\
    f(x_2, x_1) & f(x_2, x_2) & \ldots & f(x_2, x_n) \\
    \vdots & \vdots & \ddots & \vdots \\
    f(x_n, x_1) & f(x_n, x_2) & \ldots & f(x_n, x_n)
\end{bmatrix}$$

$$\det \left[ f(x_i, x_j) \right] = \prod_{k=1}^{n} (f \ast \mu) (x_k)$$

In this paper describes an abstract generalization of S-prime GCD matrices, namely S-prime meet matrices on posets.

Previously results in this direction were obtained in [1, 2, 3, 5, 9, 10, 11]. The purpose of the paper is to express sum of the most important properties of S-prime GCD matrices on a factor closed sets in the language of S-prime meet matrices, more precisely to set a structure theorem for S-prime meet matrices then derive explicit expression and found further determinant and inverse of S-prime meet matrices.

2. Structure of S-prime Meet Matrices on posets

2.1 Definition

Let $(p, \leq) = (\mathbb{Z}^+ , |)$ be a finite poset. We call P be a meet - semi lattice if for any $x, y \in P$ there exist a unique $z \in P$ such that (i) $z \leq x$ and $z \leq y$ and (ii) if $w \leq x$ and $w \leq y$ for some $w \in P$, then $w \leq z$.

In such a case z is called the meet of x and y is denoted by $x \wedge y$. For each $x \in P$, the principal order ideal $\downarrow x$ is defined by $\downarrow x = \{ y \in P | y \leq x \}$ p.246, [8]

2.2 Definition

Let S be a subset of subset of P .we call S be a lower - closed if for every $x, y \in P$ and $x \in S$ and $y \in S$ we have $y \in S$.

2.3 Definition

Let S be a subset of P then S is said to be meet-closed if for every $x, y \in S$ we have $x \wedge y \in S$.

In this case S itself is a meet lattices. It is clear that a lower –closed subset of a meet semi- lattice is always meet-closed but not conversely. The concept “lower–closed” and “Meet -closed” are generalization of “factor -closed” and “GCD-closed” [6,7] respectively.
In what follows, let P always denote a finite meet lattice, S a poset that can be embedded in a Meet-semi lattice and \( \mathcal{F} \) the unique minimal meet semi-lattice containing S.

**2.4 Definition**

Let \( x \) and \( y \) be two elements the poset P and \( \mu \) is the mobius function of the poset \( (S, \prec) \) then

\[
\mu(x, y) = \begin{cases} 
0 & \text{if } x \neq y \\
1 & \text{if } x = y \\
-\sum_{z \in S} \mu(x, z) & \text{otherwise}
\end{cases}
\]

**2.5 Definition**

Let \((P, \prec, \wedge, \vee)\) be a S-prime meet-semi lattice, let \( S = \{x_1, x_2, x_3, \ldots, x_n\} \) be a subset of P such that \( x_i \prec x_j \Rightarrow i < j \) and let \( f \) be a complex valued function on P. Then \( n \times n \) matrix \((s)_{ij} = ((s)_{ij})_{ij} = (f_{ij})\) where \( f_{ij} = 4(x_i \wedge x_j) + 1 \) is called the S-prime Meet Matrix on S with respect to \( f \).

**3. Generalized Totient functions**

**3.1 Definition**

let \( S = \{x_1, x_2, x_3, \ldots, x_n\} \) be a subset of P, and let \( f \) be a function on P with complex values. Then the function \( g_{s,f} \) on S is defined inductively by

\[
g_{s,f}(x_j) = f(x_j) - \sum_{x_i \leq x_j} g_{s,f}(x_i)
\]

Where \( x_i \leq x_j \) means that \( x_i \leq x_j \), \( x_i \neq x_j \) or \( f(x_j) = \sum_{x_i \leq x_j} g_{s,f}(x_i) \) (p.2,[2])

**3.2 Remark**

If S is a factor-closed set of positive integer ordered by divisibility and \( f(x) = x \) for all x, then \( g_{s,f} = \Phi \) Euler’s totient function. Thus \( g_{s,f} \) in definition 3.1 is a generalization of Euler’s totient function

**3.3 Theorem**

Let \( S = \{x_1, x_2, x_3, \ldots, x_n\} \) be S-prime Meet-closed. Without loss of generality we may assume that \( i < j \) whenever \( x_i \prec x_j \) then

\[
g_{s,f}(x_j) = \sum_{x_i \leq x_j} \sum_{z \in S} f(w)\mu(w, z)
\]

Where \( \mu \) is the mobius function of P.

**Proof:**

By using the definition 3.1

\[
f(x_j) = \sum_{x_i \leq x_j} g_{s,f}(x_i) = \sum_{x_i \leq x_j} \sum_{z \in S} f(w)\mu(w, z)
\]

We write,

\[
f(x) = \sum_{z \in S} g(z) \text{ or } g(x) = \sum_{z \in S} f(z)\mu(z, x)
\]

for all \( x \in P \)

It has to be prove that,

\[
\sum_{z \in S} g(z) = \sum_{x_i \leq x_j} \sum_{z \in S} f(w)\mu(w, z)
\]

Now consider the sum of R.H.S of equation (1)

Let \( x_i \leq x \) and \( z \leq x_i \Rightarrow z \leq x_i \)

Thus every \( z \) occurring on the right side of equation (1) occurs on the left side of equation (1).

Conversely, consider the sum on the left side of equation (1).

Suppose that \( z \leq x_i \) we have \( z \leq x_i \) by minimality of i, we have \( r = i \) or \( x_i = x_i \) therefore \( x_i \leq x_i \) means \( x_i \leq x_i \) thus every \( z \) occurring on the side of equation (1).

This completes the proof.

**3.4 Theorem**

If S is lower closed subset of p.

Then \( g_{s,f}(x_j) = \sum_{x_i \leq x_j} f(x_i)\mu(x_i, x_j) \)

**Proof:**

Already we know that the result,

\[
g_{s,f}(x_j) = \sum_{z \in S, w \leq z} f(w)\mu(w, z)
\]

It reduces we get the proof of theorem Then S is lower closed.

**3.5 Example**

Let \( S = \{x_1, x_2, \ldots, x_n\} \) be a chain with \( x_1 \prec x_2 \prec \ldots \prec x_n \) Then \( g_{s,f}(x_1) = f(f(x_1)), g_{s,f}(x_2) = f(f(x_2)) - f(x_1) \)

In general \( g_{s,f}(x_j) = f(x_j) - f(x_{j-1}) \) where \( j = 2, 3, 4, \ldots, n \).

**3.6 Example**

Let \( S = \{x_1, x_2, \ldots, x_n\} \) be an incomparable set and let \( S = \{x_0, x_1, x_2, \ldots, x_n\} \). Then, \( g_{s,f}(x_0) = f(x_0) \), \( g_{s,f}(x_i) = f(x_i) - f(x_0) \), and \( g_{s,f}(x_2) = f(x_2) - f(x_0) \).

In general \( g_{s,f}(x_j) = f(x_j) - f(x_0) \) for \( j = 1, 2, 3, \ldots, n \)

**3.7 Theorem** (STRUCTURE THEOREM)

Let \( S = \{x_1, x_2, \ldots, x_n\} \) and \( T = \{y_1, y_2, \ldots, y_m\} \) be any two subsets of P. Define the incidence matrix whose entry is 1 if \( y_j \leq x_i \), or \( j \)-entry is 1 if \( y_j \leq x_i \), and zero otherwise namely that is,

\[
E(S, T) = (\epsilon_{i,j})_{n \times m}
\]

where \( \epsilon_{i,j} = \begin{cases} 
1 & \text{if } y_j \leq x_i \\
0 & \text{otherwise}
\end{cases} \)

**Example:**

1. We consider \( S = \{5, 9, 13\}, T = \{9, 17, 21\} \) are the S-prime number subsets.
Then the incidence matrix of \((S, T)\) is \(E(S, T)\)
\[
= (e_{i,j}) = \begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 0 \\
1 & 1 & 0 \\
\end{bmatrix}
\]

2. We consider \(S=\{5,7,8,4\}\) and \(T=\{2,6,3,7\}\) are the subsets of \(P\). Then the incidence matrix of \((S, T)\) is
\[
E(S, T) = (e_{i,j}) = \begin{bmatrix}
1 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 0 & 1 & 0 \\
\end{bmatrix}
\]

3.8 Definition
If \(T=\{y_1, y_2, \ldots, y_m\}\) be a \(S\)-prime Meet-closed subset of \(P\) containing \(S\) \((m \geq n)\). Let \(D=\{d_1, d_2, \ldots, d_m\}\) be any subset of \(P\) containing the elements \(4(x_i \land x_j) + 1; \quad i,j=1,2,3,\ldots,n\). Let the elements of \(D\) be arranged so that,
\[
d_i \leq d_j \Rightarrow i \leq j
\]

3.9 Theorem
If \(T=\{y_1, y_2, \ldots, y_m\}\) be a \(S\)-prime Meet closed subset of \(P\) containing \(S=\{x_1, x_2, \ldots, x_m\}\) \((m \geq n)\) then \((s)_f = E \land E^T = AA^T\)

Where, \(E=E(S,T) ; \quad \Lambda = \text{diag}(g_{f,s}(y_1), g_{f,s}(y_2), \ldots, g_{f,s}(y_m))\)

and \(A = E \Lambda^{\frac{1}{2}}\).

**Proof:**
Now we consider the Example
\[
S=\{2, 3\}, \quad T=\{1,2,3,\}
\]
Then by using definition(2.3),
\[
(s)_f = \left[ f(4(x_{i} \land x_{j}) + 1) \right] = \begin{bmatrix}
f(4(2 \land 2) + 1) & f(4(2 \land 2) + 1) \\
f(4(3 \land 2) + 1) & f(4(3 \land 3) + 1) \\
f(4(2 + 1) & f(4(2 + 1) \\
f(4(3 + 1) & f(4(3 + 1)
\end{bmatrix}
\]

\[
E=E(S,T) = \begin{bmatrix}
1 & 1 & 0 \\
1 & 1 & 1
\end{bmatrix}
\]

\[
\Lambda = \text{diag}(g_{f,s}(d_1), g_{f,s}(d_2), g_{f,s}(d_3))
\]
\[
g_{s,f}(x_1) = f(x_1), \quad g_{s,f}(x_2) = f(x_2) - f(x_1), \quad g_{s,f}(x_3) = f(x_3) - f(x_1)
\]
\[
g(1) = f(1) = f(9), \quad g(2) = f(2) - f(1) = f(9) = f(3), \quad g(3) = f(3) - f(2) = f(13) = f(9)
\]

\[
\Lambda = \text{diag}(g_{s,f}(d_1), g_{s,f}(d_2), g_{s,f}(d_3))
\]

\[
E \land E^T = \begin{bmatrix}
1 & 1 & 0 \\
1 & 1 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
f(9) & 0 & 0 & 1 & 1 \\
0 & f(9) - f(3) & 0 & 1 & 1 \\
0 & 0 & f(13) - f(9) & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
f(3) & f(9) - f(3) & 0 \\
f(3) & f(9) - f(3) & f(13) - f(9)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
f(9) & f(9) \\
f(9) & f(13)
\end{bmatrix} = (S)_f
\]

Now \(A = E \Lambda^{\frac{1}{2}}\)
\[
\Lambda^T = (E \Lambda^{\frac{1}{2}})^T = (\Lambda^T)^T E^T
\]

\[
AA^T = E \Lambda E^T = \begin{bmatrix}
f(9) & f(9) \\
f(9) & f(13)
\end{bmatrix}
\]

\(S_h = AA^T = E \Lambda E^T\)

Let \(S=\{1,2,3\}\) Then
\[
(S)_h = \begin{bmatrix}
f(5) & f(5) & f(5) \\
f(5) & f(9) & f(5) \\
f(5) & f(5) & f(13)
\end{bmatrix}
\]

\(\text{diag}(S)_h = f(5)f(13)-f(5)^2- f(5)f(13)f(9)^2 - f(5)^3f(9)^2 + f(5)^3f(9)f(13)) \]

**Proof:**
Now we consider the example,
\(S = \{1, 2\} \quad \text{and} \quad T = \{1,2,3\}\). Then
\[
(S)_f = \left[ f(4(x_{i} \land x_{j}) + 1) \right] = \begin{bmatrix}
f(4(1 \land 1) + 1) & f(4(1 \land 2) + 1) \\
0 & f(4(2 \land 1) + 1) \\
0 & f(4(2 \land 2) + 1)
\end{bmatrix}
\]

\[
\Lambda = \text{diag}(g_{s,f}(d_1), g_{s,f}(d_2), g_{s,f}(d_3))
\]

\[
E=E(S,T) = \begin{bmatrix}
1 & 0 & 0 \\
1 & 0 & 0
\end{bmatrix}
\]

Since \(\Lambda = \text{diag}(g_{s,f}(d_1), g_{s,f}(d_2), g_{s,f}(d_3))\)

and by \(g_{s,f}(x_i) = f(x_i) - f(x_{i-1})\)

where \(j=2,3,4,\ldots,n\).
\[ g_{x,f}(x_i) = f(x_i); \quad g_{x,f}(x_2) = f(x_2) - f(x_1); \]
\[ g_{x,f}(x_3) = f(x_3) - f(x_2). \]

\[ g(1) = f(5), \quad g(2) = f(9) - f(5), \]
\[ g(3) = g(13) - g(9). \]

\[ E \Lambda^T = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \]
\[ = \begin{bmatrix} f(5) & 0 & 0 \\ 0 & f(9) - f(5) & 0 \\ 0 & 0 & f(13) - f(9) \end{bmatrix} \]
\[ = \begin{bmatrix} f(9) & f(5) \\ f(9) & f(13) \end{bmatrix} \]

Now \( A = E \Lambda^2 \)
\[ = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \]
\[ = \begin{bmatrix} f(5) & 0 & 0 \\ 0 & f(9) - f(5) & 0 \\ 0 & 0 & f(13) - f(9) \end{bmatrix} \]
\[ = \begin{bmatrix} f(5) & f(5) \\ f(5) & f(13) \end{bmatrix} \]

3.10 Theorem

If \( S \) is a s-prime meet -closed. Then \( \text{det}(S) = \prod_{i=1}^{n} g_{x,f}(x_i) \)

Proof:

The theorem is proved and verified with a suitable example. Consider the set \( S=\{1, 2, 3\} \)

Then \[ \left( S \right) = \begin{bmatrix} f(4) & f(4) & f(4) \\ f(4) & f(4) & f(4) \\ f(4) & f(4) & f(4) \end{bmatrix} \]

\[ \left( S \right)_f = \begin{bmatrix} f(5) & f(5) & f(5) \\ f(5) & f(5) & f(5) \\ f(5) & f(5) & f(5) \end{bmatrix} \]

\[ \left( S \right)_r = \begin{bmatrix} f(5)[f(9)f(13)-f(5)^2] - f(5)[f(13)-f(5)^2] \\ f(5)[f(5)^2 - f(5)f(9)] \end{bmatrix} \]

From the equation (1) and (2), to obtain
\[ \text{det}(S) = \prod_{i=1}^{n} (g(x_i)) \]

\[ \text{Hence the theorem is proved.} \]

3.11 Corollary

If \( S=\{x_1, x_2, x_3, ..., x_n\} \) is a chain with \( x_1 < x_2 < x_3 < ... < x_n \). Then
\[ \text{det}(S) = \prod_{i=1}^{n} (f(x_i) - f(x_{i-1})) \]

Proof:

By using theorem,

If \( S \) is a S-prime meet closed then
\[ \text{det}(S) = \prod_{i=1}^{n} (g_{x,f}(x_i)) \]

We have \( f(5) \neq f(9) \neq f(13) \)
\[ \text{det}(S) = g(1)g(2)g(3) \]

3.12 Theorem

Let \( T = \{y_1, y_2, y_3, ..., y_m\} \) be a S-prime Meet -closed subset of \( P \) containing \( S=\{x_1, x_2, x_3, ..., x_n\} \). Then,
\[ \text{det}(S) = \sum_{k=1}^{m} \text{det}(E(k_1, k_2, ..., k_n)^2 g_{T,f}(y_{k_1}, ..., g_{T,f}(y_{k_2}, ..., g_{T,f}(y_{k_n})}) \]

Where, \( E = E(S,T) \)

Proof:

\[ (S)_E = E \Lambda E^T, \quad \text{also det}(E) = \text{det}(E^T), \quad \text{by using known theorem.} \]

Now we consider the example ,
\[ S = \{2,3\} \quad \text{and} \quad T = \{1,2,3\} \]. Then,
\[ (S)_f = \begin{bmatrix} f(4) & f(4) & f(4) \\ f(4) & f(4) & f(4) \\ f(4) & f(4) & f(4) \end{bmatrix} \]

The incident matrix of \( S \& T \) is,
\[ E = E(S,T) = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \]
E \Lambda E^T =
\begin{bmatrix}
1 & 1 & 0 \\
1 & 1 & 1 \\
0 & f(9) - f(5) & 0 \\
0 & 0 & f(13) - f(5)
\end{bmatrix}
\begin{bmatrix}
1 & 1 \\
1 & 1 \\
1 & 1 \\
0 & 0
\end{bmatrix}
= \begin{bmatrix}
f(9) & f(5) \\
0 & f(13) - f(5)
\end{bmatrix}
(S_i)_{E \Lambda E^T} = E \\
\det(E) \Rightarrow E = \begin{bmatrix}
1 & 1 & 0 \\
1 & 1 & 1 \\
0 & 0 & 1
\end{bmatrix} = 0
\det(E^T) = \begin{bmatrix}
1 & 1 \\
1 & 1 \\
0 & 0
\end{bmatrix}
\det(E) = \det(E^T)
\det(S_i) = \sum_{1 \leq k_1, \ldots, k_n \leq m} \det(E(k_1, k_2, \ldots, k_n)^2 g_{T, f}(y(k_1), g_{T, f}(y(k_2), \ldots, g_{T, f}(y(k_n)]
Hence proved.

4. Determinant and inverse S-prime Meet Matrix on posets

4.1 Definition
Let the element of S be arranged so that $x_i \leq x_j \Rightarrow i < j$.
Let $d = \{d_1, d_2, d_3, \ldots, d_n\}$ be any subset of P continuous on element $x_i \wedge x_j, i = 1, 2, 3, \ldots, n$. Let the elements of D be arranged so that $d_i \leq d_j \Rightarrow i < j$. The arithmetical function $g_{D, f}$ on D derived by
$$g_{D, f}(d_i) = \sum_{d_i \leq d_j} f(d_j) \mu(d_i, d_j)$$
where $\mu(D)$ is the moebius function of the posets (D, $\leq$).

4.2 Theorem
Let $S = \{x_1, x_2, x_3, \ldots, x_n\}$ be a subset of P with $D = \{d_1, d_2, d_3, \ldots, d_m\}$. Let g be an arithmetical function then
$(S_i)_{E \Lambda E^T} = E \Delta (g(d_1), g(d_2), g(d_3), \ldots, g(d_m))$ $E^T$ where E is $(S, D)$
Proof:
Similar to a proof of 3.9

4.3 Theorem
Let S, D, F, and g be as in the theorem 4.2 then where
$$(S_i)_{E \Lambda E^T} = \sum_{1 \leq k_1, \ldots, k_n \leq m} \det[E(k_1, k_2, \ldots, k_n)^2 g(d_1), g(d_2), g(d_3), \ldots, g(d_m)]$$
Where $k_1, k_2, k_3, \ldots, k_n$ is the sub-matrix of E, E is $(S, D)$ consist of the $k_1, k_2, \ldots, k_n$ columns of E. Further if g is a function with positive value then $\det(M) \geq g(x_i)g(x_j)g(x_k)\ldots g(S_m)$ and the equality holds iff S is meet-closed .
Proof:
Since $(S_i)_{E \Lambda E^T} = E(g(d_i))E^T$ and $\det E = \det E^T$ so the proof of the theorem is obvious.

4.4 Theorem
If S is a lower closed subset of P then
$$\det(S) = \prod_{i=1}^{n} (g(d_i))$$
$g(d_i) = \sum_{d_i \leq d_j} (4d_i + 1)\mu(d_j, d_i)$

Proof:
By using theorem 4.2 and definition 4.1 to get the proof.

4.5 Theorem
$B = (b_{ij})$ where
$$b_{ij} = \frac{(-1)^{i+j}}{\det(S)_{f}} \sum_{1 \leq k_1, \ldots, k_n \leq m} \det E(S_{j})_{(k_1, k_2, \ldots, k_{n-1})}$$
$E \Delta (g(d_1), g(d_2), \ldots, g(d_n))$
Proof:
By using the theorem 4.2, 4.3.

4.6 Theorem
If $(S)_{f} = (f_{ij})$ is invertible then the inverse of $(S)_{f}$ in the n$x$n matrix $B = (b_{ij})$
where $b_{ij} = \frac{\alpha_{ij}}{\det(s)_{f}}$, Where $\alpha_{ij}$ in the co-factor of the $ij^{th}$ entry of $(S)_{f} = (f_{ij})$.

Proof:
It is a general method used to prove.

4.7 Theorem
Suppose that S is meet-closed . If $(S)_{f}$ is invertible then the inverse of $(S)_{f}$ is $B = (b_{ij})$
Where $b_{ij} = \sum_{d_j \leq d_i} \mu(d_j, d_i)\mu(d_j, d_i)$

Proof:
It is similar to the proof of theorem.

4.8 Example
Let $S = \{1, 2\}$ Then by definition (2.5)
$$(S)_{M} = \begin{bmatrix} 4(x_1 \land x_2) + 1 \end{bmatrix}$$
$$(S)_{f} \begin{bmatrix} (4(1 \land 1) + 1) & (4(1 \land 2) + 1) \\
(4(2 \land 1) + 1) & (4(2 \land 2) + 1)
\end{bmatrix}$$
$$= \begin{bmatrix} 4(1) + 1 & 4(1) + 1 \\
4(1) + 1 & 4(2) + 1
\end{bmatrix}$$
$$= \begin{bmatrix} 5 & 5 \\
5 & 9
\end{bmatrix}$$
And by definition (4.4) \( \det (s)_M = \prod_{i=1}^{n} (g(d_i)) \)

\[
g(d_i) = \sum_{d_i \leq 1} (4d_i + 1) \mu(d_i, d_i)
\]

\[
g(1) = \sum_{d_i \leq 1} (4d_i + 1) \mu(d_i, 1) = (4(1) + 1) \mu(1,1) = 5
\]

\[
g(2) = \sum_{d_i \leq 2} (4d_i + 1) \mu(d_i, 2)
\]

\[
= (4(1) + 1) \mu(1,2) + (4(2) + 1) \mu(2,2)
\]

\[
= 5(-1) + 9(1) = 4
\]

\[
\det (S)_M = g(1)g(2) = (5)(4) = 20
\]

And by using definition (4.7),

\[
b_{11} = \sum_{1 \leq d_j \leq d_k} \frac{\mu(1,d_j)\mu(1,d_k)}{g(d_k)}
\]

\[
b_{11} = \sum_{1 \leq d_j \leq 1/d_k} \frac{\mu(1,d_j)^2}{g(d_k)}
\]

\[
b_{11} = \frac{1}{5} + \frac{1}{4} \frac{9}{20}
\]

\[
b_{12} = \sum_{1 \leq d_j \leq 2/d_k} \frac{\mu(1,d_j)\mu(2,d_k)}{g(d_k)}
\]

\[
b_{12} = \frac{\mu(1,2)\mu(2,2)}{g(2)}
\]

\[
b_{12} = \frac{(1)(-1)}{4}
\]

\[
b_{12} = -\frac{1}{4}
\]

\[
b_{21} = \sum_{2 \leq d_j \leq 1/d_k} \frac{\mu(2,d_j)\mu(1,d_k)}{g(d_k)}
\]

\[
b_{21} = \frac{\mu(2,1)\mu(2,2)}{g(2)}
\]

\[
b_{21} = -\frac{1}{4}
\]

\[
b_{22} = \sum_{2 \leq d_j \leq 2/d_k} \frac{\mu(2,d_j)\mu(2,d_k)}{g(d_k)}
\]

\[
b_{22} = \frac{1}{4}
\]

\[
(S)_M^{-1} = \begin{bmatrix}
9 & -1 \\
20 & 4
\end{bmatrix}
\]

REFERENCES


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