Comparative Study of Beam by Flexibility Method & Slope Deflection Method

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Abstract: *The design process of structural planning and design required not only imagination and conceptual thinking but also sound knowledge of science of structural engineering besides the knowledge of practical aspec, such as recent design codes bye laws, backed up by ample experience and judgement for designing of beam, it is necessary to know the moments they are subjected to. For this purpose we use static method for analysis of beam.*

Keywords- *Beam, Flexibility method and Slope Deflection Method.*

I. INTRODUCTION

Structural analysis deals with study and determination of forces in various components of a structure subjected to loads. As the structural system as a whole and the loads acting on it may be of complex nature certain simplifying assumptions with regard to the quality of material, geometry of the members, nature and distribution of loads and the extent of connectivity at the joints and the supports are always made to make the analysis simpler.

For the design of beam, flexibility method and Slope deflection method of analysis are mainly used, which allows the engineer to analyse beam easily and design it economically. The research is concluded by evaluating a selection of beam, with practical dimensions in order to substantiate the conclusions as stated below.

Beam: Beam is a structural element that is capable of withstanding load primarily loads by resisting against bending. Beams are traditionally description of building or civil engineering structural elements, but smaller structure such as truck or automobile frame, machine frames and other mechanical or structural systems contain beam structure that are designed and analyzed in a similar fashion.

II. METHODS OF ANALYSIS

- Slope and Deflection Method
- Flexibility or Force Method
- Strain Energy Method
- Moment Distribution Method

2.1 Flexibility Method:

Since twentieth century, indeterminate structures are being widely used for its obvious merits. It may be recalled that, in the case of indeterminate structures either the reactions or the internal forces cannot be determined from equations of statics alone. In such structures, the number of reactions or the number of internal forces exceeds the number of static equilibrium equations. In addition to equilibrium equations, compatibility equations are used to evaluate the unknown reactions and internal forces in statically indeterminate structure. In the analysis of indeterminate structure it is necessary to satisfy the equilibrium equations (implying that the structure is in equilibrium) compatibility equations (requirement if for assuring the continuity of the structure without any breaks) and force displacement equations (the way in which displacement are related to forces). We have two distinct method of analysis for statically indeterminate structure depending upon how the above equations are satisfied:

1. Force method of analysis (also known as flexibility method of analysis, method of consistent deformation, flexibility matrix method)
2. Displacement method of analysis (also known as stiffness matrix method). In the displacement method of analysis, the primary unknowns are the displacements. In this method, first force - displacement relations are computed and subsequently equations are written satisfying the equilibrium conditions of the structure. After determining the unknown displacements, the other forces are calculated satisfying the compatibility conditions and force displacement relations. The displacement-based method is amenable to computer programming and hence the method is being widely used in the modern day structural analysis.

2.2 Slope Deflection Method:

This method was first devised by Heinrich Mandrels and Otto Mohr to study the secondary stresses in trusses and was further developed by G.A. Maney extend its application to analyze indeterminate beams and framed structures. The basic assumption of this method is to consider the

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deformations caused only by bending moments. It's assumed that the effects of shear force or axial force deformations are negligible in indeterminate beams or frames.

By forming slope deflection equations and applying joint and shear equilibrium conditions, the rotation angles (or the slope angles) are calculated. Substituting them back in to the slope deflection equations, member end moments are readily determined.

III. METHODOLOGY ADOPTED

This paper presents the analysis of Beam, which is the most common in practice by using two most common methods via flexibility method & slope deflection method. The moment of inertia of both spans is I.

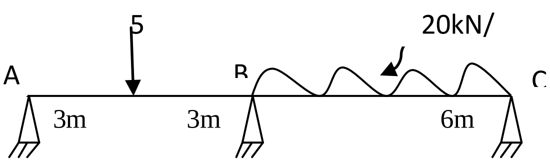

Fig.1. Beam considered for the analysis

3.1. Application of Flexibility Method for the Analysis of Beam:

3.1.1 Degree of Kinematic Indeterminacy: DOKI is equal to the number of unrestrained degrees of freedom used in beam. It is the number of unknowns to be solved in the stiffness method. DOKI = 3n – s

Where, n is the no. of nodes in the beam is equal to 3 and The total number of restraints for the beam is denoted here as s are 6.

Therefore DOKI = (3x3) - 6 = 3

3.1.2 Formulation of Members Stiffness Matrix:

$$SM = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{-12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & \frac{-6EI}{L^2} & \frac{2EI}{L} \\ \frac{-12EI}{L^3} & \frac{-6EI}{L^2} & \frac{+12EI}{L^3} & \frac{-6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & \frac{-6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

$$SM_{AB} = \begin{bmatrix} 0.056 & 0.167 & -0.05\,6 & 0.167 \\ 0.167 & 0.67 & -0.167 & 0.33 \\ -0.056 & -0.167 & 0.056 & -0.167 \\ 0.167 & 0.33 & -0.167 & 0.67 \end{bmatrix}$$

$$SM_{BC} = \begin{bmatrix} 0.187 & 0.375 & -0.187 & 0.375 \\ 0.375 & 1 & -0.375 & 0.5 \\ -0.187 & -0.375 & 0.187 & -0.375 \\ 0.37 & 0.5 & -0.375 & 1 \end{bmatrix}$$

3.1.3 Stiffness Matrix: In the given problem of the beam, the degree of kinematic indeterminacy is equal to 3. Therefore the stiffness matrix should be of 3x3.

$$[\boldsymbol{K}] \text{ X } [\boldsymbol{DF}] = [\boldsymbol{AFC}]$$

Where,

$$\mathbf{K} = \begin{bmatrix} \boldsymbol{K_{11}} & \boldsymbol{K_{12}} & \boldsymbol{K_{13}} \\ \boldsymbol{K_{21}} & \boldsymbol{K_{22}} & \boldsymbol{K_{23}} \\ \boldsymbol{K_{31}} & \boldsymbol{K_{32}} & \boldsymbol{K_{33}} \end{bmatrix}$$

$$K = \begin{bmatrix} 0.67 & 0.33 & 0 \\ 0.33 & 1.67 & 0.5 \\ 0 & 0.5 & 1 \end{bmatrix}$$

3.2.3 Formulation of Load Matrix or Global Load Matrix:

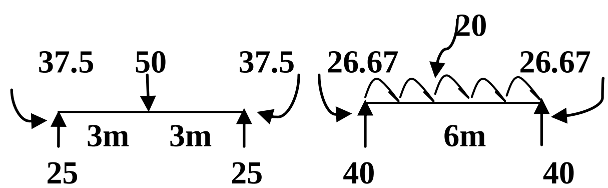

Fig: 3 Figure shows fixed moments and forces

Action due to moment loading:

$$AML_1 = \begin{bmatrix} 25 \\ 37.5 \\ 25 \\ -37.5 \end{bmatrix}, \quad AML_2 = \begin{bmatrix} 40 \\ 26.67 \\ 40 \\ -26.67 \end{bmatrix}$$

Action due to free co-ordinates:

$$AFC = \begin{bmatrix} -37.5 \\ 10.83 \\ 26.67 \end{bmatrix}$$

$$[\boldsymbol{K}] \text{ X } [\boldsymbol{DF}] = [\boldsymbol{AFC}]$$

$$EI \text{ x} \begin{bmatrix} 0.67 & 0.33 & 0 \\ 0.33 & 1.67 & 0.5 \\ 0 & 0.5 & 1 \end{bmatrix} \text{ x} \begin{bmatrix} DF1 \\ DF2 \\ DF3 \end{bmatrix} = \begin{bmatrix} -37.5 \\ 10.83 \\ 26.67 \end{bmatrix}$$

By solving equation we get,

$DF_1 = \frac{-62.22}{EI}$, $DF_2 = \frac{12.69}{EI}$, $DF_3 = \frac{20.32}{EI}$

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Final Member Forces:

For span AB,

$AM = [AML] + \{[SM] \times [DF]\}$

$$AM_{AB} = \begin{bmatrix} 16.728 \\ 0 \\ 33.27 \\ -49.53 \end{bmatrix}$$

$$AM_{BC} = \begin{bmatrix} 52.38 \\ 49.52 \\ 27.62 \\ 0 \end{bmatrix}$$

3.1.4 Final End Moments:

The final end moments have been calculated,

$M_{AB} = 0$

$M_{BA} = +49.5$ kNm

$M_{BC} = -49.5$ kNm

$M_{CB} = 0$

3.2 Application of Slope Deflection Method for the Analysis of Portal Frame:

3.2.1 Fixed End Moments:

The fixed end moments of each member are,

$FEM_{AB} = -37.5$ kNm

$FEM_{BA} = +37.5$ kNm

$FEM_{BC} = -26.67$ kNm

$FEM_{CB} = +26.67$ kNm

3.2.2 Formulation of Members Slope Deflection Equation:

In the given problem, only joint B rotates. Hence, in this problem we have three unknown displacements to be evaluated. The ends A and C are hinge. Hence, $\theta_A = \theta_c = 0$

$M_{AB} = FEM_{AB} + \frac{2EI}{L}(2\theta_A + \theta_B)$----------------------(i)

$M_{BA} = FEM_{BA} + \frac{2EI}{L}(2\theta_B + \theta_A)$-----------------------(ii)

$M_{BC} = FEM_{BC} + \frac{2EI}{L}(2\theta_B + \theta_C)$-----------------------(iii)

$M_{CB} = FEM_{CB} + \frac{2EI}{L}(2\theta_C + \theta_B)$-----------------------(iv)

3.2.3 Applying Equilibrium Condition:

In the slope deflection method we have to calculate unknown joints, rotation and displacements.

Now, consider the summation of moment at A,

$M_{AB} = 0$

$0.67EI\theta_A + 0.33EI\theta_B = 37.5$..................................... (A)

Now, consider the summation of moment at B,

$M_{BA} + M_{BC} = 0$

$0.33EI\,\theta_A + 1.67\,EI\,\theta_B + 0.5EI\theta_C = -10.83$………..(B)

Now, consider the summation of moment at C,

$M_{CB} = 0$

$0.5EI\,\theta_B + EI\,\theta_C = -26.67$……………………...(C)

On solving the equation (A),(B) & (C) we get,

$\theta_A = \frac{62.22}{EI}$, $\theta_B = \frac{-12.7}{EI}$, $\theta_C = \frac{-20.32}{EI}$

3.2.4 Final End Moments:

Substituting the values of θ_A, θ_B and θ_C in the slope deflection equations we get the final end moments.

$M_{AB} = 0$

$M_{BA} = +49.5$ kNm

$M_{BC} = -49.5$ kNm

$M_{CB} = 0$ kNm

3.2.5 BENDING MOMENT DIAGRAM

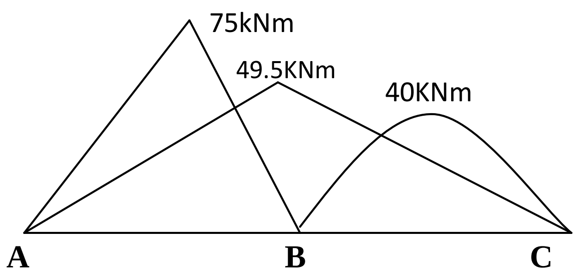

Fig:2 Final Bending Moment

IV. INVESTIGATIVE ANALYSIS

After the analysis is completed, the result of the end moments of the considered beam has been compared and an investigative analysis is done.

Table: 1 End Moments of structure

Moment at	End Moments (kNm)		% Variations
	Flexibility Method	Slope Deflection Method	
M_{AB}	0	0	0%
M_{BA}	+ 49.5	+49.53	0.03%
M_{BC}	- 49.5	-49.52	0.02%
M_{CB}	0	0	0%

V. CONCLUSIONS

The End Moments calculated by the application of flexibility method, for the analysis of consider beam, mostly matches with those calculated by the application of Slope Deflection method. The slope deflection method is more preferable than flexibility method because as its calculations are easy. In Slope deflection method we directly know unknown joints, displacement and rotations of any structure (Beam). The flexibility method is quite difficult as compared to slope deflection method as it based on matrices.

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