SOME EXTENSION OF 1-NEAR MEAN CORDIAL LABELING OF GRAPHS

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ABSTRACT : Let G = (V, E) be a simple graph. A bijective function f: V(G) → {0, 1,..., p − 1}is said to be a 1-Near Mean Cordial Labeling if for each edge uv, the induced map

$$f^*(uv) = \begin{cases} 0 & if \quad \frac{f(u)+f(v)}{2} \text{ is an integer} \\ 1 & otherwise \end{cases}$$

Satisfies the condition $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ is the number of edges with 0 label and $e_f(1)$ is the number of edges with 1 label. G is said to be a 1-Near Mean Cordial Graph if it has a 1- Near Mean Cordial Labeling. In this paper ,we proved that path, cycle, complete bipartite, star, fan, crown, comb and $D_2(P_n)$ are 1- Near Mean Cordial Graphs.

Keywords : 1-Near Mean Cordial Labeling, 1-Near Mean Cordial Graph.
AMS Subject Classification (2010): 05C78

1 INTRODUCTION

All graphs considered here are finite, simple and undirected. Gallian [2] has given a dynamic survey of labeling. For graph theoretic terminologies and notations we follow Harary [3]. The concept of mean cordial labeling was introduced by Raja Ponraj, Muthirulan Sivakumar and Murugesan Sundaram in the year 2012 [1,4,5,7]. Let f be a function from V(G) to {0,1,2}. For each edge uv of G, assign the label $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called a mean cordial labeling of G if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, i, j ∈ {0,1,2} where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with x (x=0,1,2) respectively. A graph with a mean cordial labeling is called Mean Graph. K. Palani, J. Rejila Jeya Surya [6] introduced a new concept called 1-Near Mean Cordial Labeling and investigated some standard graphs.

2 PRELIMINARIES

We define the concept of 1-Near Mean Cordial Labeling as follows,

Let G = (V, E) be a simple graph. A bijective function f: V → {0, 1,..., p − 1} said to be a 1-Near Mean Cordial Labeling if for each edge uv, the induced map

$$f^*(uv) = \begin{cases} 0 & if \quad \frac{f(u)+f(v)}{2} \text{ is an integer} \\ 1 & otherwise \end{cases}$$

Satisfies the condition $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ is the number of edges with 0 label and $e_f(1)$ is the number of edges with 1 label. G is said to be a 1-Near Mean Cordial Graph if it has a 1- Near Mean Cordial Labeling.

Definition 2.1. If all the vertices in a walk are distinct, then it is called a path and a path of length k is denoted by P_{k+1}.

ISSN: 2278 – 7798

Definition 2.2. A cycle is a circuit in which no vertex except the first (which is also the last) appears more than once.
Alternatively, a cycle can be defined as a closed path.

Definition 2.3. A graph G is called a complete bipartite graph $K_{m,n}$ with bipartition $V(G) = V_1 \cup V_2$ where $V_1 = \{x_1, x_2, ..., x_m\}$ and $V_2 = \{y_1, y_2, ..., y_n\}$ and all vertices in V_1 are adjacent to all vertices in V_2 but no vertices in V_1 and V_2.

Definition 2.4. The graph $K_{1,r}$, $r \geq 1$ is called a star at the vertex has degree r is called center.

Definition 2.5. The join $G_1 + G_2$ of G_1 and G_2 consists of $G_1 \cup G_2$ and all lines joining V_1 with V_2 as vertex set $V(G_1) \cup V(G_2)$ and edge set $E[G_1 \cup G_2] = E(G_1) \cup E(G_2) \cup [uv : u \in V(G_1)$ and $v \in V(G_2)]$. The graph $P_n + K_1$ is called a fan.

Definition 2.6. The crown $(C_n \odot K_1)$ is obtained by joining a pendant edge to each vertex of C_n.

Definition 2.7. The corona $(G_1 \odot G_2)$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has p_1 points) and p_1 copies of G_2 and then joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2. The graph $P_n \odot K_1$ is called a comb.

Definition 2.8. $D_2(P_n)$ is the graph of two copies of path graph P_n consisting of vertices u_i and v_i for $1 \leq i \leq n$. Hence $D_2(P_n)$ consists of V(G)=2n and E(G)=4(n-1).

3 MAIN RESULTS

Theorem 3.1. A path P_n is a 1-Near Mean Cordial Graph .
Proof. Let G=(V, E) be a simple graph.
Let G be P_n.
Let $V(G) = \{u_i : 1 \leq i \leq n\}$ and
$E(G) = \{(u_i u_{i+1}) : 1 \leq i \leq n - 1\}$
Case:(i) when n is even
Define $f : V(G) \to \{0, 1, ..., p - 1\}$ by
$f(u_{2i+1}) = i, \quad 0 \leq i \leq \frac{n}{2} - 1$
$f(u_{2i}) = n - i, \quad 1 \leq i \leq \frac{n}{2}$
The induced edge labeling are,

$$f^*(u_i u_{i+1}) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n-1$$

Here $e_f(0) = \frac{n}{2} - 1$, $e_f(1) = \frac{n}{2}$
Case:(ii) when n is odd
Define $f : V(G) \to \{0, 1, ..., p - 1\}$ by
$f(u_{2i+1}) = i, \quad 0 \leq i \leq \frac{n-1}{2}$
$f(u_{2i}) = n - i, \quad 1 \leq i \leq \frac{n-1}{2}$
The induced edge labeling are

$$f^*(u_i u_{i+1}) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n-1$$

Here $e_f(0) = e_f(1) = \frac{n-1}{2}$
Hence the graph satisfies the condition
$|e_f(0) - e_f(1)| \leq 1$
Therefore, a path P_n is a 1-near mean cordial graph.

Illustration 1. The 1-near mean cordial graph of P_8 and P_7 are shown in the figure 1(a) and 1(b)

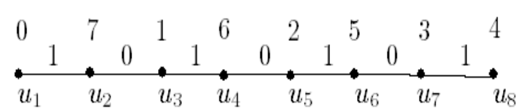

Figure 1(a): P_8

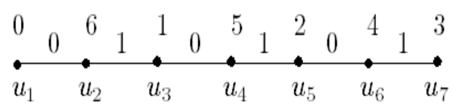

Figure 1(b): P_7

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Theorem 3.2. A cycle C_n is a 1-Near Mean Cordial Graph for $n \not\equiv 2 \mod 4$, $n \geq 3$.

Proof. Let G = (V, E) be a simple graph.

Let G be C_n

Let $V(G) = \{u_i : 0 \leq i \leq n-1\}$ and

$E(G)=\{[(u_iu_{i+1}) : 1 \leq i \leq n-2] \cup (u_{n-1}u_0)\}$

Case(i): when $n \equiv 0 \mod 4$

Define $f : V(G) \to \{0, 1, ..., p-1\}$ by

$f(u_{2i-1}) = n - i, \quad 1 \leq i \leq \frac{n}{2}$

$f(u_{2i}) = i, \quad 0 \leq i \leq \frac{n}{2} - 1$

The induced edge labeling are

$f^*(u_iu_{i+1}) = \begin{cases} 0 & i \equiv 1 \mod 2 \\ 1 & i \equiv 0 \mod 2 \end{cases} \quad 0 \leq i \leq n-2$

$f^*(u_{n-1}u_0) = 0$

Here $e_f(0) = e_f(1) = n$

Hence the graph satisfies the condition $|e_f(0) - e_f(1)| \leq 1$

Therefore, the graph, C_n, is a 1-near mean cordial graph.

For example,The 1-near mean cordial graph of C_8 is shown in the figure 2(a)

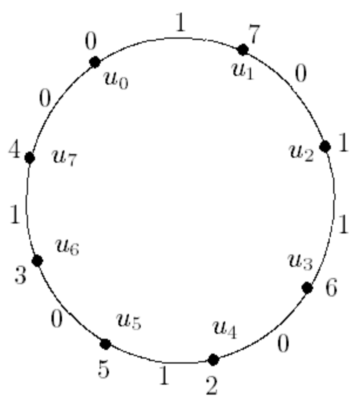

Figure 2(a): C_8

Case(ii): when $n \equiv 1 \mod 4$

Define $f : V(G) \to \{0, 1, ..., p-1\}$ by

$f(u_{2i-1}) = n - i, \quad 1 \leq i \leq \frac{n-1}{2}$

$f(u_{2i}) = i, \quad 0 \leq i \leq \frac{n-1}{2}$

The induced edge labeling are

$f^*(u_iu_{i+1}) = \begin{cases} 0 & i \equiv 0 \mod 2 \\ 1 & i \equiv 1 \mod 2 \end{cases} \quad 0 \leq i \leq n-2$

$f^*(u_{n-1}u_0) = 0$

Here, $e_f(0) = \frac{n+1}{2}$, $\quad e_f(1) = \frac{n-1}{2}$

Hence the graph satisfies the condition $|e_f(0) - e_f(1)| \leq 1$

Therefore, the graph, C_n, is a 1-near mean cordial graph.

For example, the 1-near mean cordial graph of C_5, is shown in the figure 2(b)

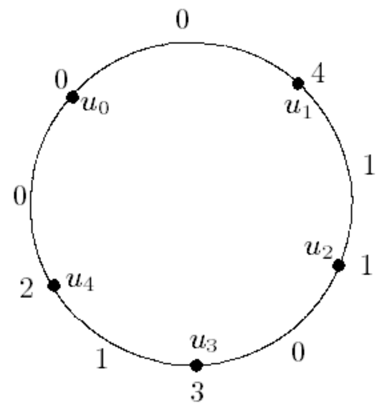

Figure 2(b): C_5

Case(iii): when $n \equiv 3 \mod 4$

Define $f : V(G) \to \{0, 1, ..., p-1\}$ by

$f(u_{2i-1}) = n - i, \quad 1 \leq i \leq \frac{n-1}{2}$

$f(u_{2i}) = i, \quad 0 \leq i \leq \frac{n-1}{2}$

The induced edge labeling are

$f^*(u_iu_{i+1}) = \begin{cases} 0 & i \equiv 0 \mod 2 \\ 1 & i \equiv 1 \mod 2 \end{cases} \quad 0 \leq i \leq n-2$

$f^*(u_{n-1}u_0) = 1$

Here, $e_f(0) = \frac{n-1}{2}$, $\quad e_f(1) = \frac{n+1}{2}$

Hence the graph satisfies the condition $|e_f(0) - e_f(1)| \leq 1$

Therefore, the graph, C_n, is a 1-near mean cordial graph.

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For example, the 1-near mean cordial graph of C_7 is shown in the figure 2(c),

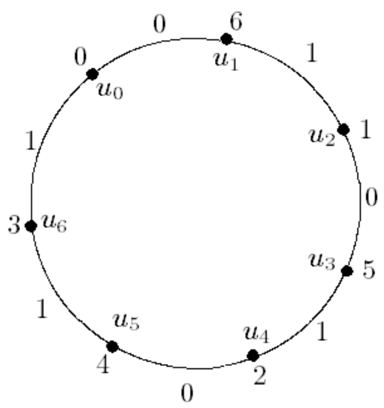

Figure 2(c): C_7

Theorem 3.3. The complete bipartite graph, $K_{m,n}$ is a 1-Near Mean Cordial Graph.

Proof. Let G = (V, E) be a simple graph.

Let G be $K_{m,n}$.

Let $V(G) = \{u_i : 0 \le i \le m-1, v_j : 0 \le j \le n-1\}$ and

$E(G)=\{(u_iv_j : 0 \le i \le m-1, 0 \le j \le n-1\}$

Define $f : V(G) \to \{0, 1, ..., p-1\}$ by

$f(u_i) = i, \quad 0 \le i \le m-1$

$f(v_j) = m + j, \quad 0 \le j \le n-1$

The induced edge labeling are

Case(i): when m is even and n is even or odd

$$f^*(u_{2i-1}v_j) = \begin{cases} 0 & j \equiv 1 \ mod\ 2 \\ 1 & j \equiv 0 \ mod\ 2 \end{cases}$$

for $1 \le i \le \frac{m}{2}$, $0 \le j \le n-1$

$$f^*(u_{2i}v_j) = \begin{cases} 0 & j \equiv 0 \ mod\ 2 \\ 1 & j \equiv 1 \ mod\ 2 \end{cases}$$

for $0 \le i \le \frac{m}{2} - 1$, $0 \le j \le n-1$

Here, $e_f(0) = e_f(1) = \frac{mn}{2}$

Case(ii): when m is odd and n is even or odd

$$f^*(u_{2i-1}v_j) = \begin{cases} 0 & j \equiv 0 \ mod\ 2 \\ 1 & j \equiv 1 \ mod\ 2 \end{cases}$$

for $1 \le i \le \frac{m-1}{2}$, $0 \le j \le n-1$

$$f^*(u_{2i}v_j) = \begin{cases} 0 & i \equiv 1 \ mod\ 2 \\ 1 & i \equiv 0 \ mod\ 2 \end{cases}$$

for $0 \le i \le \frac{m-1}{2}$, $0 \le j \le n-1$

$$\text{Here, } e_f(0) = \begin{cases} \frac{mn}{2} & n \text{ is even} \\ \frac{mn-1}{2} & n \text{ is odd} \end{cases}$$

$$e_f(1) = \begin{cases} \frac{mn}{2} & n \text{ is even} \\ \frac{mn+1}{2} & n \text{ is odd} \end{cases}$$

Hence the graph satisfies the condition $|e_f(0) - e_f(1)| \le 1$

Therefore, the graph, $K_{m,n}$, is a 1-near mean cordial graph.

Illustration 2. The 1-near mean cordial graph of $K_{4,3}$ and $K_{3,3}$ are shown in the figure 3(a) and figure 3(b)

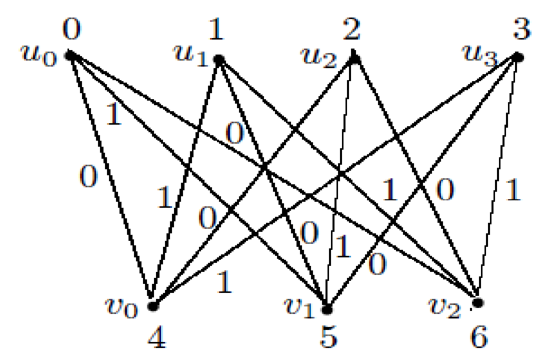

Figure 3(a): $K_{4,3}$

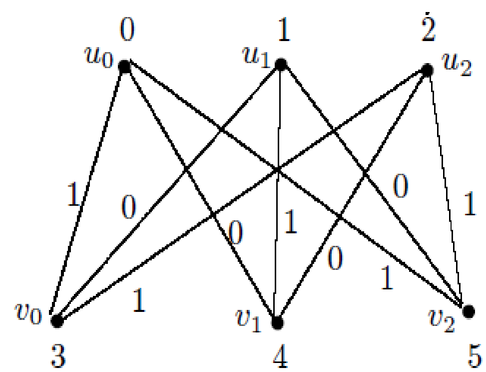

Figure 3(b): $K_{3,3}$

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Theorem 3.4. The star $K_{1,n}$ is a 1-Near Mean Cordial Graph.
Proof. Let G=(V, E) be a simple graph.
Let G be $K_{1,n}$
Let $V(G)= \{u, v_i : 1 \le i \le n\}$ and
$E(G) = \{uv_i : 1 \le i \le n\}$.
Define $f:V(G) \to \{0, 1, .., .p - 1\}$by
$f(u) = 0$
$f(v_i) = i, \quad 1 \le i \le n$
The induced edge labeling are,

$$f^*(uv_i)= \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n$$

$$\text{Here, } e_f(0)= \begin{cases} \frac{n}{2} & n \text{ is even} \\ \frac{n-1}{2} & n \text{ is odd} \end{cases}$$

$$e_f(1)= \begin{cases} \frac{n}{2} & n \text{ is even} \\ \frac{n+1}{2} & n \text{ is odd} \end{cases}$$

Hence the graph satisfies the condition $|e_f(0) - e_f(1)| \le 1$
Therefore , a star $K_{1,n}$ is a 1-near mean cordial graph.

Illustration 3. The 1-near mean cordial graph of $K_{1,5}$ is shown in the figure 4.

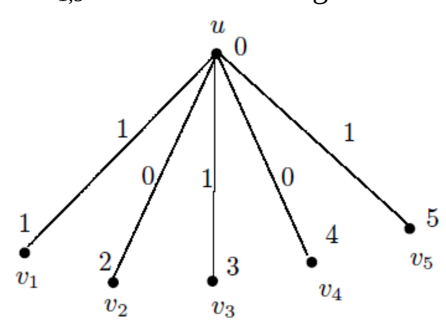

Figure 4: $K_{1,5}$

Theorem 3.5. The graph P_n+K_1 is a 1-Near Mean Cordial Graph.
Proof. Let G = (V, E) be a simple graph.
Let G be $P_n + K_1$
Let $V(G) = \{u_i : 1 \le i \le n, v\}$ and
$E(G)=\{[(u_iu_{i+1}) : 1 \le i \le n - 1] \cup [(u_iv) : 1 \le i \le n]\}$
Case:(i) when n is odd
Define $f : V(G) \to \{0, 1, ..., p - 1\}$ by
$f(u_{2i+1}) = i, \quad 0 \le i \le \frac{n-1}{2}$,
$f(u_{2i}) = n - i, \quad 1 \le i \le \frac{n-1}{2}$,
$f(v) = n$ or $p - 1$.
The induced edge labeling are

$$f^*(u_iu_{i+1})= \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n-1$$

$$f^*(u_iv)= \begin{cases} 0 & i \equiv 0 \bmod 3 \\ 1 & i \equiv 1,2 \bmod 3 \end{cases} \quad 1 \le i \le n$$

Here, $e_f(0) = n - 1, \quad e_f(1) = n$
Case(ii) when n is even
Define $f : V(G) \to \{0, 1, ..., p - 1\}$ by
$f(u_{2i+1})= i, \quad 0 \le i \le \frac{n}{2} - 1$,
$f(u_{2i}) = n - i, \quad 1 \le I \le \frac{n}{2}$,
$f(v) = n$ or $p - 1$.
The induced edge labeling are

$$f^*(u_iu_{i+1})= \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n-1$$

$$f^*(u_iv)= \begin{cases} 0 & i \equiv 0,1 \bmod 4 \\ 1 & i \equiv 2,3 \bmod 4 \end{cases} \quad 1 \le i \le n$$

Here, $e_f(0) = n - 1$, $e_f(1) = n$
Hence the graph satisfies the condition $|e_f(0) - e_f(1)| \le 1$
Therefore, the graph $P_n + K_1$ is a 1-near mean cordial graph.

Illustration 4. The 1-near mean cordial graph of $P_5 + K_1$ and $P_6 + K_1$ are shown in the figure 5(a) and 5(b)

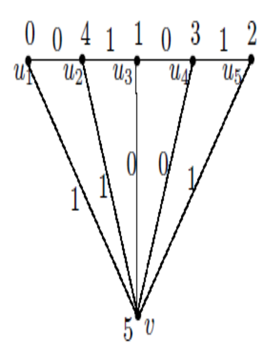

Figure 5(a): $P_5 + K_1$

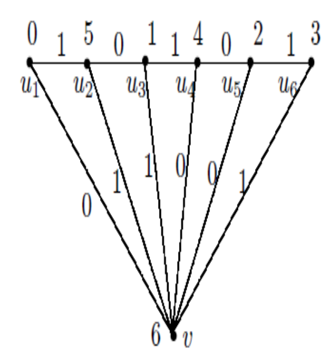

Figure 5(b): $P_6 + K_1$

ISSN: 2278 – 7798

Theorem 3.6. The graph $C_n \odot K_1$ is a 1-Near Mean Cordial Graph.

Proof. Let G = (V, E) be a simple graph.

Let G be $C_n \odot K_1$

Let V (G) = $\{u_i : 0 \leq i \leq n, v_i : 1 \leq i \leq n\}$ and E(G)=$\{[(u_iu_{i+1}) : 1 \leq i \leq n-1] \cup (u_nu1) \cup [(u_iv_i) : 1 \leq i \leq n]\}$

Define f : V (G) $\rightarrow$ {0, 1, ..., p − 1} by

$f(u_i) = 2i - 2, \quad 1 \leq i \leq n$

$f(v_i) = 2i - 1, \quad 1 \leq i \leq n$

The induced edge labeling are

$f^*(u_iu_{i+1}) = 0, \quad 1 \leq i \leq n-1$

$f^*(u_nu_1) = 0$

$f^*(u_iv_i) = 1, \quad 1 \leq i \leq n$

Here, $e_f(0) = e_f(1) = n$

Hence the graph satisfies the condition $| e_f(0) - e_f(1) | \leq 1$

Therefore, the graph, $C_n \odot K_1$ is a 1-near mean cordial graph.

Illustration 5. The 1-near mean cordial graph of $C_7 \odot K_1$ is shown in the figure 6

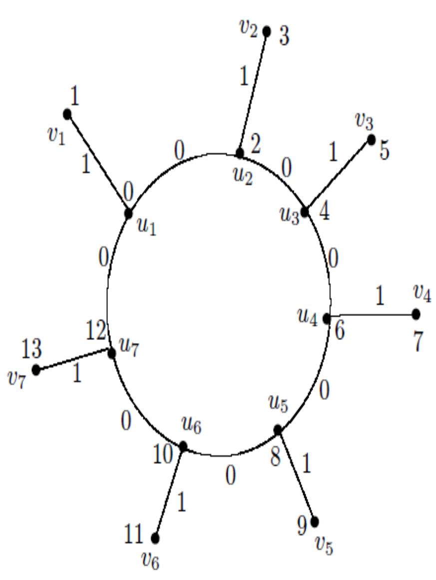

Figure 6: $C_7 \odot K_1$

Theorem 3.7. The graph $P_n \odot K_1$ is a 1-Near Mean Cordial Graph.

Proof. Let G = (V, E) be a simple graph.

Let G be $P_n \odot K_1$

Let V (G) = $\{u_i : 1 \leq i \leq n, v_i : 1 \leq i \leq n\}$ and E(G)=$\{[(u_iu_{i+1}) : 1 \leq i \leq n-1] \cup [(u_iv_i) : 1 \leq i \leq n]\}$

Define f : V (G) $\rightarrow$ {0, 1, ..., p − 1} by

$f(u_i) = 2i - 2, \quad 1 \leq i \leq n$

$f(v_i) = 2i - 1, \quad 1 \leq i \leq n$

The induced edge labeling are

$f^*(u_iu_{i+1}) = 0, \quad 1 \leq i \leq n-1$

$f^*(u_iv_i) = 1, \quad 1 \leq i \leq n$

Here, $e_f(0) = n - 1, \quad e_f(1) = n.$

Hence the graph satisfies the condition $| e_f(0) - e_f(1) | \leq 1$

Therefore, the graph, $P_n \odot K_1$, is a 1-near mean cordial graph.

Illustration 6. The 1-near mean cordial graph of $P_6 \odot K_1$ is shown in the figure 7

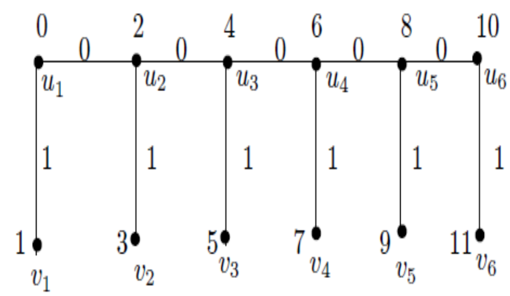

Figure 7: $P_6 \odot K_1$

Theorem 3.8. The graph $D_2(P_n)$ is a 1-Near Mean Cordial Graph.

Proof. Let G = (V, E) be a simple graph.

Let G be $D_2(P_n)$

Let V(G)=$\{u_i : 0 \leq i \leq n-1, v_i : 0 \leq i \leq n-1\}$ and E(G)=$\{[(u_iu_{i+1}),(v_iv_{i+1}) : 0 \leq i \leq n-2] \cup [(u_iv_{i+1}), (v_iu_{i+1}) : 0 \leq i \leq n-2]\}$

Define f : V (G) $\rightarrow$ {0, 1, ..., p − 1} by

$f(u_i) = 2i, \quad 0 \leq i \leq n-1$

$f(v_i) = 2i + 1, \quad 0 \leq i \leq n-1$

The induced edge labeling are

$f^*(u_iu_{i+1}) = 0, \quad 0 \leq i \leq n-2$

$f^*(v_iv_{i+1}) = 0, \quad 0 \leq i \leq n-2$

$f^*(u_iv_{i+1}) = 1, \quad 0 \leq i \leq n-2$

ISSN: 2278 – 7798

$f^*(v_iu_{i+1}) = 1, \quad 0 \leq i \leq n-2$

Here, $e_f(0) = 2n - 2 = e_f(1)$

Hence the graph satisfies the condition $|e_f(0) - e_f(1)| \leq 1$

Therefore, the graph, $D_2(P_n)$, is a 1-near mean cordial graph.

Illustration 7. The 1-near mean cordial graph of $D_2(P_3)$ is shown in the figure 8

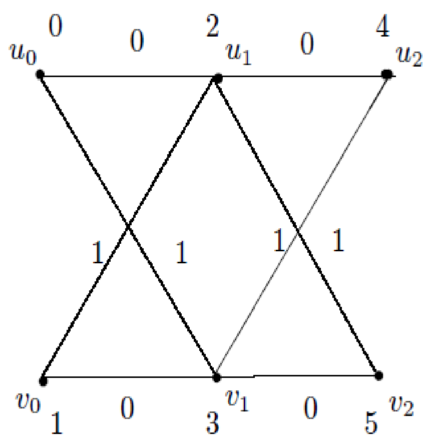

Figure 8: $D_2(P_3)$

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ISSN: 2278 – 7798