SQUARE DIFFERENCE 3-EQUITABLE LABELING FOR SOME GRAPHS

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Abstract:
A square difference 3-equitable labeling of a graph G with vertex set V is a bijection f from V to {1, 2, ..., | V |} such that if each edge uv is assigned the label −1 if $| [f(u)]^2 - [f(v)^2] | \equiv -1 (\text{mod } 4)$, the label 0 if $| [f(u)]^2 - [f(v)]^2 | \equiv 0(\text{mod } 4)$ and the label 1 if $| [f(u)]^2 - [f(v)]^2 | \equiv 1(\text{mod } 4)$, then the number of edges labeled with i and the number of edges labeled with j differ by atmost 1 for $-1 \leq i, j \leq 1$. If a graph has a square difference 3-equitable labeling, then it is called square difference 3-equitable graph. In this paper, we investigate the square difference 3-equitable labeling behaviour of comb, triangular snake, crown, star, wheel.

Keywords: Square difference 3-equitable labeling, square difference 3-equitable graphs.
AMS Subject Classification (2010): 05C78.

1 INTRODUCTION

All graphs considered here are finite, simple and undirected. Gallian has given a dynamic survey of labeling [1]. For graph and theoretic terminologies and notations we follow Harary [2]. The concept of 3-equitable labeling was introduced by S.K. Vaidya and N.H. Shah in the year 2012[7] . A ternary vertex labeling of a graph G is called a 3-equitable labeling if $| v_f (i) - v_f (j) | \leq 1$ and $| e_f (i) - e_f (j) | \leq 1$ for all $-1 \leq i, j \leq 1$. A graph G is 3-equitable if it admits 3-equitable labeling. The concept of square difference labeling was introduced by J. Shiama in the year of 2012 and 2013[3, 4, 5, 6]. The concept of square difference 3-equitable labeling of pahs and cycle was introduced by S. Murugesan and J. Shiama in the year of 2015[8].

2 PRELIMINARIES

Definition 2.1. Let G = (V, E) be a graph. A mapping f : V (G) → {−1, 0, 1} is called ternary vertex v of G and f(v) is called the label of the vertex v of G under f.

For an edge e = uv, he induced edge labeling is given by f* : E(G) →{−1, 0, 1}. Let $v_f (-1)$, $v_f (0)$, $v_f (1)$ be the number of vertices of G having labels −1, 0, 1 respectively under f and $e_f (-1)$, $e_f (0)$, $e_f (1)$ be the number of edges having labels −1, 0, 1 respectively under f^*.

Definition 2.2. A ternary vertex labeling of a graph G is called a **3-equitable labeling** if $| v_f (i) - v_f (j) | \leq 1$ and $| e_f (i) - e_f (j) | \leq 1$ for all $-1 \leq i, j \leq 1$. A graph G is 3-equitable if it admits 3-equitable labeling.

Definition 2.3. A square difference 3-equitable labeling of a graph G wih vertex set V (G) is a bijection f : V (G) → {1, 2, 3, ..., | V |} such that the induced edge labeling f*: E(G) → {−1, 0, 1} is defined by

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f*(e=uv)

$$= \begin{cases} -1 \mid [f(u)]2 - [f(v)]2 \mid \equiv -1(\text{mod } 4) \\ 0 \mid [f(u)]2 - [f(v)]2 \mid \equiv 0(\text{mod } 4) \\ 1 \mid [f(u)]2 - [f(v)]2 \mid \equiv 1(\text{mod } 4) \end{cases}$$

Definition 2.4. The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has p1 points) and p1 copies of G_2 and then joining the i^{th} copy of G_1 to every point in the i^{th} copy of G_2.The graph $P_n \odot K_1$ is called a **comb.**

Definition 2.5. A **Triangular Snake** is obtained from a path $v_1, v_2, ...v_n$ joining v_i and v_{i+1} to a new vertex w_i for $1 \leq i \leq n - 1$. That is, every edge of a path is replaced by a triangle C_3.

Definition 2.6. C_n^+ is a graph obtained from G by attaching a pendent vertex from each vertex of the graph Cn is called **Crown.**

Definition 2.7. The graph $K_{1,r}$, $r \geq 1$ is called a **star** at the vertex has degree r is called center.

Definition 2.8. A graph C_n+K_1 is called a **wheel** with n spokes and is denoted by W_n.

3 MAIN RESULTS

Theorem 3.1. The comb P_n^+ admits square difference 3-equitable labeling.

Proof. Let G be a comb obtained from a path $P_n = v_1, v_2, ..., v_n$ by joining a vertex u_i to v_i. The labeling of P_1^+ and P_2^+ are given as follows.

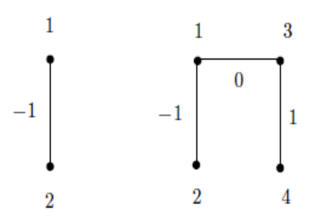

Fig 1 . Square difference 3-equitable labeling of P_1^+ and P_2^+.

If $n \geq 3$, we consider the following cases.

Case-1: $n \equiv 1(\text{mod } 6)$

Define

$f(v_1) = 2$

$f(v_2) = 3$

$f(v_3) = 1$

$f(v_4) = 4$,

for $1 \leq i \leq \frac{n-7}{6}$,

$f(v_{6i-1}) = 6i + 1$

$f(v_{6i}) = 6i - 1$

$f(v_{6i+1}) = 6i$

$f(v_{6i+2}) = 6i + 2$

$f(v_{6i+3}) = 6i + 3$

$f(v_{6i+4}) = 6i + 4$

and

$f(v_{n-2}) = n$

$f(v_{n-1}) = n - 2$

$f(v_n) = n - 1$.

Then

$| [f(v_1)]^2 - [f(v_2]^2 | \equiv 1(\text{mod}4)) \Rightarrow f(v_1v_2) = 1$

$| [f(v_2)]^2 - [f(v_3]^2 | \equiv 0(\text{mod}4)) \Rightarrow f(v2v3) = 0$

$|[f(v_3)]^2 - [f(v_4]^2 | \equiv -1(\text{mod}4) \Rightarrow f(v_3v_4) = -1$,

for $1 \leq i \leq \frac{n-7}{6}$,

$|[f(v_{6i-2}]^2 - [f(v_{6i-1})]^2| \equiv 1(\text{mod}4)$

$\Rightarrow f^*(v_{6i} - v_{6i-1}) = 1$

$|[f(v_{6i-1}]^2 - [f(v_{6i})]^2 | \equiv 0(\text{mod}4)$

$\Rightarrow f^*(v_{6i-1}v_{6i}) = 0$

$|[f(v_{6i}]^2 - [f(v_{6i+1})]^2| \equiv -1(\text{mod}4)$

$\Rightarrow f^*(v_{6i}v_{6i+1}) = -1$

$|[f(v_{6i+1}]^2 - [f(v_{6i+2})]^2 \equiv 0(\text{mod}4)$

$\Rightarrow f^*(v_{6i+1}v_{6i+2}) = 0$

$|[f(v_{6i+2})]^2 - [f(v_{6i+3})]^2 \equiv 1(\text{mod}4)$

$\Rightarrow f^*(v_{6i+2}v_{6i+3}) = 1$

$|[f(v_{6i+3})]^2 - [f(v_{6i+4})]^2 \equiv -1(\text{mod}4)$

$\Rightarrow f^*(v_{6i+3}v_{6i+4}) = -1$

and

$|[f(v_{n-3})]^2 - [f(v_{n-2}]^2| \equiv 1(\text{mod}4)$

$\Rightarrow f^*(v_{n-3}v_{n-2}) = 1$

$|[f(v_{n-2})]^2 - [f(v_{n-1}]^2| \equiv 0(\text{mod}4)$

$\Rightarrow f^*(v_{n-2}v_{n-1}) = 0$

$|[f(v_{n-1})]^2 - [f(v_n)]^2 | \equiv -1(\text{mod}4)$

$\Rightarrow f^*(v_{n-1}v_n) = -1$.

$f(u_j) = n + j, 1 \leq j \leq 5$

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$$f(u_j) = \begin{cases} n+j+1 & j \equiv 0 (mod\ 3) \\ n+j-1 & j \equiv 1 (mod\ 3), \\ n+j & j \equiv 2(mod\ 3) \end{cases} \quad 6 \le j \le n$$

Then

$e_f(-1) = \left\lceil \frac{2n+1}{3} \right\rceil$ and $e_f(0) = e_f(1) = \left\lfloor \frac{2n-1}{3} \right\rfloor$.

Case-2: $n \equiv 2(mod 6)$

Define

$f(v_1) = 2$
$f(v_2) = 3$
$f(v_3) = 1$
$f(v_4) = 4$,
for $1 \le i \le \frac{n-8}{6}$,
$f(v_{6i-1}) = 6i + 1$
$f(v_{6i}) = 6i - 1$
$f(v_{6i+1}) = 6i$
$f(v_{6i+2}) = 6i + 2$
$f(v_{6i+3}) = 6i + 3$
$f(v_{6i+4}) = 6i + 4$
and
$f(v_{n-3}) = n - 1$
$f(v_{n-2}) = n - 3$
$f(v_{n-1}) = n - 2$
$f(v_n) = n$

$$f(u_j) = \begin{cases} n+j & j \equiv 0,1,2,5(mod\ 6) \\ n+j+1 & j \equiv 3(mod\ 6) \\ n+j-1 & j \equiv 4(mod\ 6) \end{cases}$$

Then

$e_f(-1) = e_f(0) = e_f(1) = \frac{2n-1}{3}$.

Case-3: $n \equiv 3(mod 6)$

Define

$f(v_1) = 2$
$f(v_2) = 3$
$f(v_3) = 1$
$f(v_4) = 4$,
for $1 \le i \le \frac{n-9}{6}$,
$f(v_{6i-1}) = 6i + 1$
$f(v_{6i}) = 6i - 1$
$f(v_{6i+1}) = 6i$
$f(v_{6i+2}) = 6i + 2$
$f(v_{6i+3}) = 6i + 3$
$f(v_{6i+4}) = 6i + 4$.
and
$f(v_{n-4}) = n - 2$
$f(v_{n-3}) = n - 4$
$f(v_{n-2}) = n - 3$
$f(v_{n-1}) = n - 1$
$f(v_n) = n$.

$$f(u_j) = \begin{cases} n+j+1 \\ n+j-1 \end{cases} \quad for\ j = 1,2$$

$f(u_j) = n + j, \quad 3 \le j \le n$.

Then

$e_f(-1) = e_f(1) = \frac{2n}{3}$ and $e_f(0) = \left\lfloor \frac{2n-1}{4} \right\rfloor$.

Case-4: $n \equiv 4(mod 6)$

Define

$f(v_1) = 2$
$f(v_2) = 3$
$f(v_3) = 1$
$f(v_4) = 4$,
for $1 \le i \le \frac{n-4}{6}$,
$f(v_{6i-1}) = 6i + 1$
$f(v_{6i}) = 6i - 1$
$f(v_{6i+1}) = 6i$
$f(v_{6i+2}) = 6i + 2$
$f(v_{6i+3}) = 6i + 3$
$f(v_{6i+4}) = 6i + 4$.
and

$$f(u_j) = \begin{cases} n+j & j \equiv 0,1,4,5(mod\ 6) \\ n+j+1 & j \equiv 2(mod\ 6) \\ n+j-1 & j \equiv 3(mod\ 6) \end{cases}$$

Then

$e_f(-1) = e_f(1) = \left\lfloor \frac{2n-1}{3} \right\rfloor$ and $e_f(0) = \frac{2n+1}{3}$.

Case-5: $n \equiv 5(mod 6)$

Define

$f(v_1) = 2$
$f(v_2) = 3$
$f(v_3) = 1$
$f(v_4) = 4$,
for $1 \le i \le \frac{n-5}{6}$,
$f(v_{6i-1}) = 6i + 1$
$f(v_{6i}) = 6i - 1$
$f(v_{6i+1}) = 6i$
$f(v_{6i+2}) = 6i + 2$
$f(v_{6i+3}) = 6i + 3$
$f(v_{6i+4}) = 6i + 4$.
and
$f(v_n) = n$
and

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$f(u_j) = n + j, \quad 1 \leq j \leq n$

Then

$e_f(-1) = e_f(0) = e_f(1) = \left\lfloor \frac{2n}{3} \right\rfloor$.

Case-6: $n \equiv 0 (mod 6)$

Define

$f(v_1) = 2$

$f(v_2) = 3$

$f(v_3) = 1$

$f(v_4) = 4$,

$f(v_i) = i, \quad 5 \leq i \leq n$

and

$$f(u_j) = \begin{cases} n + j & j \equiv 1,2,3,4 (mod\ 6) \\ n + j + 1 & j \equiv 5 (mod\ 6) \\ n + j - 1 & j \equiv 0 (mod\ 6) \end{cases}$$

Then

$e_f(-1), e_f(0), e_f(1) \in \left\{ \left\lfloor \frac{2n-1}{3} \right\rfloor, \left\lceil \frac{2n+1}{3} \right\rceil \right\}$.

Thus in all cases , $| e_f(i) - e_f(j) | \leq 1$ for all $-1 \leq i, j \leq 1$ and therefore P_n^+ is a square difference 3-equitable labeling graph.

Illustration 1. The Square difference 3-equitable labeling of P_{12}^+ is shown below.

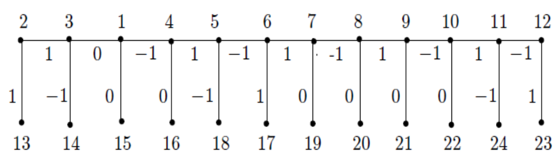

Fig 2. The Square difference 3-equitable labeling of P_{12}^+

Theorem 3.2. The Triangular Snake Tn admits a square difference 3-equitable labeling.

Proof. Let T_n be a path $v_1, v_2, ..., v_n$ joining v_i and v_{i+1} to a new vertex w_i.

Define $f(v_i) = i$, $1 \leq i \leq n$ and $f(w_i) = n + i$, $1 \leq i \leq n - 1$.

$e_f(-1) = e_f(1) = n - 1$ and $e_f(0) = n - 2$.

Thus $| e_f(i) - e_f(j) | \leq 1$ for all $-1 \leq i, j \leq 1$ and therefore T_n is a square difference 3-equitable labeling.

Illustration 2. The Square difference 3-equitable labeling of T_7 is shown below.

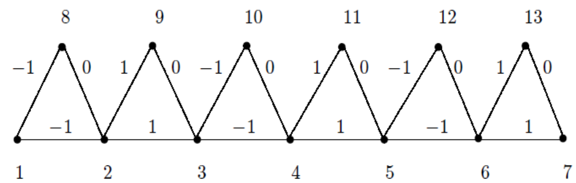

Fig 3. Square difference 3-equitable labeling of T_7.

Theorem 3.3. The Crown C_n^+ admits a square difference 3-equitable labeling except $n \equiv 3(mod 6)$.

Proof. Let $v_1v_2v_3...v_nv_1$ be the cycle C_n.

Let u_i be the vertex which is adjacent to v_i, $1 \leq i \leq n$.

Case-1: $n \equiv 1(mod 6)$

Define

$f(v_1) = 2$

$f(v_2) = 3$

$f(v_3) = 1$

$f(v_4) = 4$

for $1 \leq i \leq \frac{n-7}{6}$,

$f(v_{6i-1}) = 6i + 1$

$f(v_{6i}) \quad = 6i - 1$

$f(v_{6i+1}) = 6i$

$f(v_{6i+2}) = 6i + 2$

$f(v_{6i+3}) = 6i + 3$

$f(v_{6i+4}) = 6i + 4$

and

$f(v_{n-2}) = n$

$f(v_{n-1}) = n - 2$

$f(v_n) \quad = n - 1$

and

$f(v_1u_1) = 2n$

$f(v_2u_2) = n + i - 1$

$$f(v_iu_i) = \begin{cases} n + i \\ n + i - 2 \end{cases} \quad 3 \leq i \leq n - 1$$

$f(v_nu_n) = n + i - 1$

Then

$e_f(-1), e_f(0), e_f(1) \in \left\{ \left\lfloor \frac{2n}{3} \right\rfloor, \left\lceil \frac{2n}{3} \right\rceil \right\}$.

Case-2: $n \equiv 2(mod 6)$

Define

$f(v_1) = 2$

$f(v_2) = 1$

$f(v_3) = 4$

$f(v_4) = 5$

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$f(v_5) = 3$
for$1 \leq i \leq \frac{n-8}{6}$,
$f(v_{6i}) = 6i + 2$
$f(v_{6i+1}) = 6i$
$f(v_{6i+2}) = 6i + 1$
$f(v_{6i+3}) = 6i + 3$
$f(v_{6i+4}) = 6i + 4$
$f(v_{6i+5}) = 6i + 5$
and
$f(v_{n-2}) = n$
$f(v_{n-1}) = n - 2$
$f(v_n) = n - 1$
and
$f(v_1u_1) = 2n$

$$f(v_iu_i) = \begin{cases} n+i-1 & i \equiv 4,5 (\text{mod } 6) \\ n+i & i \equiv 0,2 (\text{mod } 6) \\ n+i-2 & i \equiv 1,3 (\text{mod } 6) \end{cases}$$

$f(v_nu_n) = n + i - 1$
Then
$e_f(-1), e_f(0), e_f(1) \in \left\{\left\lfloor\frac{2n}{3}\right\rfloor, \left\lceil\frac{2n}{3}\right\rceil\right\}$.

Case-3: $n \equiv 4(\text{mod}6)$
Define
$f(v_1) = 2$
$f(v_2) = 3$
$f(v_3) = 1$
$f(v_4) = 4$
for $1 \leq i \leq \frac{n-4}{6}$,
$f(v_{6i-1}) = 6i + 1$
$f(v_{6i}) = 6i - 1$
$f(v_{6i+1}) = 6i$
$f(v_{6i+2}) = 6i + 2$
$f(v_{6i+3}) = 6i + 3$
$f(v_{6i+4}) = 6i + 4$
and

$$f(v_iu_i) = \begin{cases} n+i & i \equiv 0,1,2,5 (mod\ 6) \\ n+i+1 & i \equiv 3 (mod\ 6) \\ n+i-1 & i \equiv 4 (mod\ 6) \end{cases}$$

Then
$e_f(-1), e_f(0), e_f(1) \in \left\{\left\lfloor\frac{2n}{3}\right\rfloor, \left\lceil\frac{2n}{3}\right\rceil\right\}$.

Case-4: $n \equiv 5(\text{mod}6)$
Define
$f(v_1) = 2$
$f(v_2) = 1$
$f(v_3) = 4$
$f(v_4) = 5$
$f(v_5) = 3$
for$1 \leq i \leq \frac{n-5}{6}$,
$f(v_{6i}) = 6i + 2$
$f(v_{6i+1}) = 6i$
$f(v_{6i+2}) = 6i + 1$
$f(v_{6i+3}) = 6i + 3$
$f(v_{6i+4}) = 6i + 4$
$f(v_{6i+5}) = 6i + 5$
and
$f(v_1u_1) = 2n$
$f(v_nu_n) = n + 2$

$$f(v_iu_i) = \begin{cases} n+i-1 & i \equiv 2,4 (mod\ 6) \\ n+i & i \equiv 0,5 (mod\ 6) \\ n+i+1 & i \equiv 1,3 (mod\ 6) \end{cases}$$

Then
$e_f(-1), e_f(0), e_f(1) \in \left\{\left\lfloor\frac{2n}{3}\right\rfloor, \left\lceil\frac{2n}{3}\right\rceil\right\}$. .

Case-5: $n \equiv 0(\text{mod}6)$
Define
$f(v_1) = 2$
$f(v_2) = 3$
$f(v_3) = 5$
$f(v_4) = 4$
$f(v_5) = 1$
$f(v_6) = 6$
for $1 \leq i \leq \frac{n-6}{6}$
$f(v_{6i+1}) = 6i + 3$
$f(v_{6i+2}) = 6i + 1$
$f(v_{6i+3}) = 6i + 2$
$f(v_{6i+4}) = 6i + 4$
$f(v_{6i+5}) = 6i + 5$
$f(v_{6i+6}) = 6i + 6$
and

$$f(v_iu_i) = \begin{cases} n+i & i \equiv 1,2,3,4 (mod\ 6) \\ n+i+1 & i \equiv 5 (mod\ 6) \\ n+i-1 & i \equiv 0 (mod\ 6) \end{cases}$$

Then
$e_f(-1), e_f(0), e_f(1) \in \left\{\left\lfloor\frac{2n}{3}\right\rfloor, \left\lceil\frac{2n}{3}\right\rceil\right\}$.

Thus in above cases, $|e_f(i) - e_f(j)| \leq 1$ for all $-1 \leq i, j \leq 1$ and therefore C_n is a square difference 3-equitable labeling graph.

Case-6: $n \equiv 3(\text{mod}6)$
We see that $|e_f(i) - e_f(j)|=0$ or 1 but
$|e_f(i) - e_f(j)|=2$.

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Thus C_n^+ is not square difference 3-equitable labeling.

Hence C_n^+ is square difference 3-equitable labeling except $n \equiv 3 (mod 6)$.

Illustration 3. The square difference 3-equitable labeling of C_8^+ is shown below.

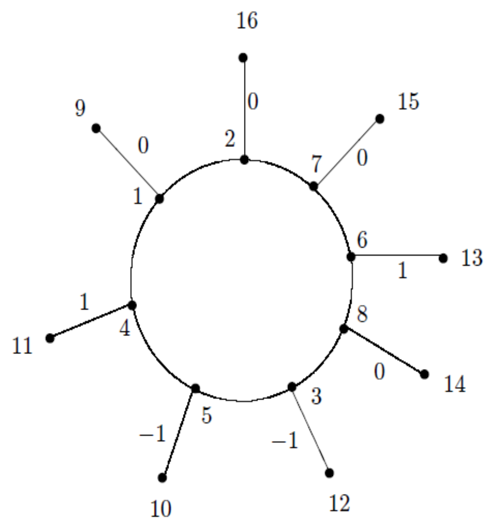

Fig 4. Square difference 3-equitable labeling of C_8^+.

Theorem 3.4. The Star graph $K_{1,n}$ is not square difference 3-equitable labeling for $n \geq 9$ except $n = 10$.

Proof. Let v be the central vertex and let $v_1, v_2, ..., v_n$ be the end vertices of the star $K_{1,n}$.

Case-(1) : n = 1, 2, 3

The square difference 3-equitable labeling of $K_{1,1}$,$K_{1,2}$ and $K_{1,3}$ are given as follows.

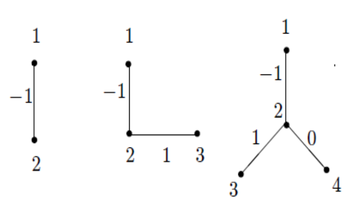

Fig 5. Square difference 3-equitable labeling of $K_{1,1}, K_{1,2}$ and $K_{1,3}$

Case-(2) : $4 \leq n \leq 8$ and n = 10

Assign the label 4 to the vertex v and the remaining labels to the vertices $v_1, v_2, ..., v_n$.

We see that $|e_f (i) - e_f (j)| \leq 1$ for all $-1 \leq i, j \leq 1$.

Case-(3) : n = 9

We see that $|e_f (-1) - e_f (0)|=2$ and hence $K_{1,9}$ is not square difference 3-equitable labeling.

Case-(4) : $n \geq 11$

We see that $|e_f (-1) - e_f (0)| \geq 3$.

Thus $K_{1,n}$ is not square difference 3-equitable labeling for $n \geq 9$ except n = 10.

Illustration 4. The square difference 3-equitable labeling of $K_{1,8}$ is shown below.

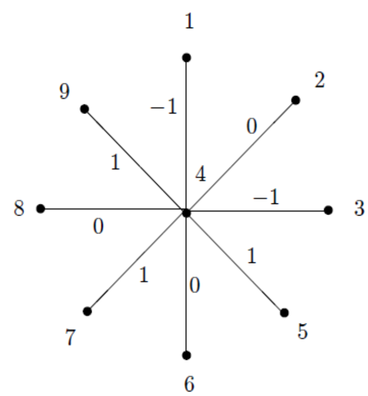

Fig 6. Square difference 3-equitable labeling of $K_{1,8}$

Theorem 3.5. The wheel graph W_n admits a square difference 3-equitable labeling if $n \equiv 1, 2 (mod 6)$.

Proof. Let $W_n = C_n + K_1$.

Let C_n be a cycle $u_1, u_2, ..., u_n, u_1$ and $V (K_1) = \{u\}$.

If $n \geq 5$, we consider the following cases.

Case-(1) : $n \equiv 1 (mod 6)$

Define

$f(u_1) = 1$

$f(u_2) = 3$

$f(u_3) = 2$

$f(u_4) = 5$

for $1 \leq i \leq \frac{n-7}{6}$,

Subcase-(i): i is odd

$f(u_{6i-1}) = 6i - 2$

$f(u_{6i}) = 6i + 1$

$f(u_{6i+1}) = 6i$

$f(u_{6i+2}) = 6i + 2$

$f(u_{6i+3}) = 6i + 5$

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$f(u_{6i+4}) = 6i + 3$
Subcase-(ii): i is even
$f(u_{6i-1}) = 6i - 2$
$f(u_{6i}) = 6i + 1$
$f(u_{6i+1}) = 6i$
$f(u_{6i+2}) = 6i + 4$
$f(u_{6i+3}) = 6i + 2$
$f(u_{6i+4}) = 6i + 3$
and
$f(u_{n-2}) = n - 3$
$f(u_{n-1}) = n$
$f(u_n) = n - 1$
and
$f(u) = n + 1$
Then
$e_f(-1), e_f(0), e_f(1) \in \left\{\left\lfloor \frac{2n}{3} \right\rfloor, \left\lceil \frac{2n}{3} \right\rceil\right\}$.
Case-(2) : $n \equiv 2(\text{mod}6)$
Subcase-(i) : n = 8
Define
$f(u_1) = 2$
$f(u_2) = 3$
$f(u_3) = 1$
$f(u_4) = 4$
$f(u_5) = 7$
and
$f(u_{n-2}) = n - 3$
$f(u_{n-1}) = n - 2$
$f(u_n) = n + 1$
Subcase-(ii) : n > 8
$f(u_1) = 2$
$f(u_2) = 1$
$f(u_3) = 3$
$f(u_4) = 4$
$f(u_5) = 7$
for $1 \le i \le \frac{n-8}{6}$,
Subcase-(a) : i is odd
$f(u_{6i}) = 6i - 1$
$f(u_{6i+1}) = 6i$
$f(u_{6i+2}) = 6i + 3$
$f(u_{6i+3}) = 6i + 4$
$f(u_{6i+4}) = 6i + 8$
$f(u_{6i+5}) = 6i + 7$
Subcase-(b):i is even
$f(u_{6i}) = 6i - 1$
$f(u_{6i+1}) = 6i$
$f(u_{6i+2}) = 6i + 3$
$f(u_{6i+3}) = 6i + 4$
$f(u_{6i+4}) = 6i + 7$
$f(u_{6i+5}) = 6i + 8$
and
$f(u_{n-2}) = n - 3$
$f(u_{n-1}) = n - 2$
$f(u_n) = n + 1$
and
$f(u) = 8$
Then
$e_f(-1), e_f(0), e_f(1) \in \left\{\left\lfloor \frac{2n}{3} \right\rfloor, \left\lceil \frac{2n}{3} \right\rceil\right\}$.
Case-(3) : $n \equiv 0, 3, 4, 5(\text{mod}6)$
We see that $|e_f(i) - e_f(j)|=0$ or 1 but $|e_f(i) - e_f(j)|\geq 2$.
Thus W_n is not square difference 3-equitable labeling.
Hence W_n is square difference 3-equitable labeling if $n \equiv 1, 2(\text{mod}6)$.

Illustration 5. The square difference 3-equitable labeling of W_8 is shown below.

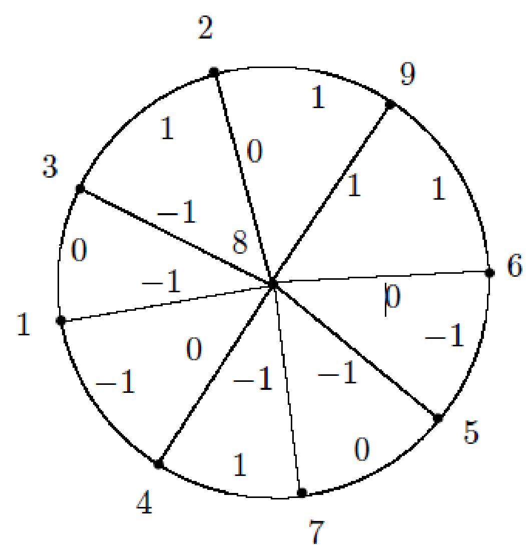

Fig 7. Square difference 3-equitable labeling of W_8.

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