

GAIN SCHEDULING CONTROL OF INFANTE AUV USING LMI

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Abstract:

The paper addresses the problem of Autonomous Underwater Vehicles which is an under actuated system that traverses autonomously without any external interference and find extensive applications in defence organizations for underwater mine detection and region surveillance. The system is modelled using INFANTE AUV hydrodynamic parameters that are controlled by path following control using MATLAB. The paper is concerned with depth control of AUV using Model Predictive Control without considering disturbances. The depth control is concerned with the design of control laws that force a vehicle to reach and maintain a fixed position in vertical plane. The depth and pitch angle control of body fixed z-axis to a fixed point using MPC toolbox of MATLAB is shown in the paper. Also, an application is made to the control of a prototype AUV in the vertical plane using nonlinear gain-scheduling control, whereby a set of linear, dynamic, reduced order output feedback controllers are designed and scheduled on the vehicle's forward speed. The paper summarizes the controller design steps, describes a technique for its practical implementation, and presents experimental results obtained with the INFANTE AUV using MATLAB.

1. INTRODUCTION

About 70% of the Earth's surface is covered with water which is an empire of natural resources. In order to utilize these resources, mankind depends on developing underwater vehicles and employing them [1]. The underwater vehicles can be categorized as manned and unmanned systems. In manned system, there are military submarines and non-military submersibles operated for underwater investigations and assessment.

Unmanned submersibles can be further classified as (i) Remotely Operated Vehicle (ROV) and (ii) Autonomous Underwater Vehicle (AUV). A ROV is a remotely operated vehicle usually connected with the mother ship or base station through a tethered wire whereas AUV is an Autonomous Underwater Vehicle which traverses autonomously without any external interference. As opposed to ROV, control of an AUV is difficult because it is an underactuated system.

1.1 Autonomous Underwater Vehicle(AUV)

AUV refers to an autonomous robot equipped with suitable sensors and actuators which enable it to navigate in the subsea environment. It is an undersea system which has its own power and controlled by an onboard computer while doing a pre-defined task [2]. They are compact, self contained, low drag profile crafts powered by a single underwater DC power thruster. The vehicle uses on-board computers, power packs and vehicle payloads for automatic control, navigation and guidance. They have been operated in a semi autonomous mode under human supervision, which requires them to be tracked, monitored, or even halted during a mission so as to change the mission plan.

1.2 AUV Structure

Most of the applications of AUV require that it should follow a desired path like pipeline for the applications like navigation.

1.2.1 Navigation System

Navigation system is meant for obtaining the position and orientation of the vehicle using INS, GPS, or other acoustic sensors. But the navigation of an AUV is difficult because the unavailability of the sensors for giving accurate position and orientation measurements. In the navigation system, usually the sensor data get corrupted by external noises. So signal processing has a major role in navigation system. If a state is unavailable then an estimator can be employed to generate the missing or unmeasured state.

1.2.2 Guidance System

The guidance system deals with the desired path generation from AUVs current position to the desired position. The major challenge in the guidance control is to generate an optimum path for the AUV considering the obstacles between the paths. It is also necessary for the AUV to follow the optimum path successfully, for this, the path feasibility for the particular AUV should be determined.

1.2.3 Control Structure

Control structure determines the required control forces necessary for steering the AUV along the desired path. While developing a control law, it is necessary to check the stability of the AUV states, and also the generated control forces should reside within its maximum limit. Design of control law for a fully-actuated system is simpler than an under-actuated system. In an under-actuated system, it is challenging to develop a control law together with ensuring the system stability. For both the cases it is necessary to show the robustness and adaptation of the control structure for the external disturbances.

The focus of this paper is to develop control algorithms for an AUV to accomplish path following of a desired path. Also, the non linear coefficients of AUV dynamics are linearized and the depth is controlled by putting constraints on pitch rate and yaw velocity using a linear controller such as a Model Predictive Controller technique. The robustness of the vehicle is analysed using gain scheduling controller by applying it to the Linear Matrix Inequalities toolbox.

2. AUV KINEMATICS AND DYNAMICS

Kinematics and dynamics of an AUV are described in Fig.1 where the transformation matrix T represents the transformation of body frame to earth fixed frame. The AUV parameter block represents the added mass and hydrodynamic coupled parameters. For implementing the path following control in x-y domain, only three **Degree of Freedom** is considered i.e. surge equation of motion is along x-direction, sway equation of motion is along y-direction and yaw equation of motion is angular movement along z-direction. The corresponding kinematic equations are also considered.

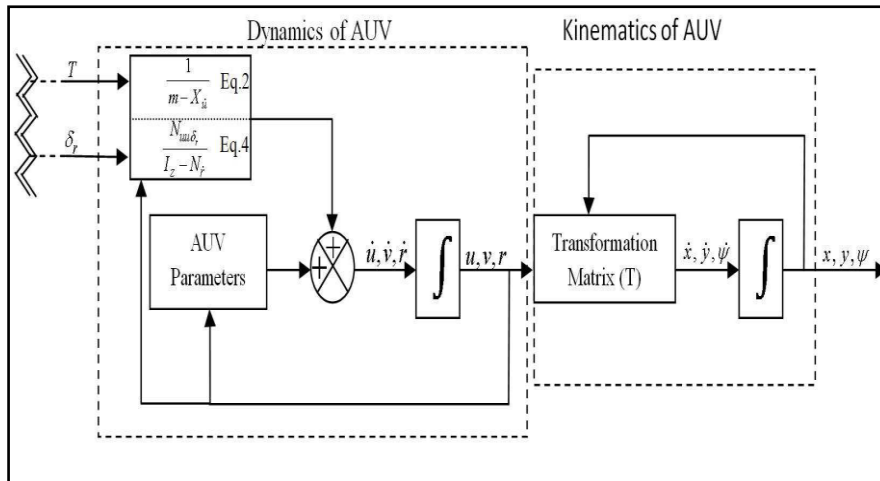


Fig.1: Kinematics and Dynamics structure of AUV

To study the motion of marine vehicle 6 degrees of freedom are required since to describe independently the complete position and orientation of the vehicle we require 6 independent coordinates. To describe position and translation motion first three sets of coordinates and their time derivatives are required. While for orientation and rotational motion last three sets of coordinates and their time derivatives are required.

DOF	MOTION	Forces	Linear and angular velocity	Position
1	Motion in x-direction(surge)	X	u	x
2	Motion in y-direction(sway)	Y	v	y
3	Motion in z-direction(heave)	Z	w	z
4	Rotation in x-direction(roll)	K	p	ϕ
5	Rotation in y-direction(pitch)	M	q	θ
6	Rotation in z-direction(yaw)	N	r	ψ

Table 1: Notation used for AUV modelling

The kinematics equations of AUV are generally represented using two coordinate frames i.e. earth-fixed frame and body-fixed frame [24]. The velocity parameters of the AUV are determined from the body-fixed frame and using a transformation matrix, the velocity in the earth-fixed frame is determined.

The transformation matrix $J_1(\eta)$ and $J_2(\eta)$ are defined as follows,

$$J_1(\eta_2) = \begin{bmatrix} \cos(\Psi) & -\sin(\Psi) & 0 \\ \sin(\Psi) & \cos(\Psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \quad (1)$$

$$J_2(\eta_2) = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)/\cos(\theta) & \cos(\phi)/\cos(\theta) \end{bmatrix} \quad (2)$$

where, $J_1(\eta_2)$ is utilized for the conversion of body fixed linear velocities(u, v, w) to earth fixed linear velocities ($\dot{x}, \dot{y}, \dot{z}$) and $J_2(\eta_2)$ is used for converting the body- fixed angular velocities (p, q, r) to earth fixed angular velocities ($\dot{\theta}, \dot{\phi}, \dot{\Psi}$).

The complete transformation between body-fixed and earth-fixed frames represent the kinematics equation of the AUV which is given as follows,

$$\begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} I^{1(\eta_2)} & 0_{3 \times 3} \\ 0_{3 \times 3} & J^{2(\eta_2)} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (3)$$

where, $\eta_1 = [\dot{x}, \dot{y}, \dot{z}]^T$ and $\eta_2 = [\dot{\theta}, \dot{\phi}, \dot{\psi}]^T$ represents the AUV velocities in the earth fixed frame. The corresponding body-fixed velocities of the AUV are $v_1 = [u, v, w]$ and $v_2 = [p, q, r]$.

Dynamics of the AUV consists of nonlinearity and coupling between various terms, accordingly following are the dynamic equation along the respective axis.

- Surge Motion:

$$m[\dot{u} - vr + \omega q - x_g(q^2 + r^2) + y_g(pq - \dot{r}) + z_g(pr + \dot{q})] = X \quad (4)$$

- Sway motion :

$$m[\dot{v} - wp + ur - y_g(p^2 + r^2) + z_g(qr - \dot{p}) + x_g(pq + \dot{r})] = Y \quad (5)$$

- Heave motion:

$$m[\dot{w} - uq + vp - z_g(q^2 + p^2) + x_g(pr - \dot{q}) + y_g(rq + \dot{p})] = Z \quad (6)$$

- Roll motion:

$$I_x \dot{p} + (I_z - I_y)qr - (\dot{r} + pq)I_{xz} + (r^2 - q^2)I_{yz} + (pr - \dot{q})I_{xy} + m[y_g(\dot{w} - u\dot{q} + vp) - z_g(\dot{v} - wp + ur)] = K \quad (7)$$

- Pitch motion:

$$I_y \dot{q} + (I_x - I_z)pr - (\dot{p} + qr)I_{xz} + (r^2 - q^2)I_{yz} + (pr - \dot{q})I_{xy} + m[z_g(\dot{u} - v\dot{r} + wq) - y_g(\dot{w} - uq + vp)] = M \quad (8)$$

- Yaw Motion:

$$I_z \dot{r} + (I_y - I_x)pq - (\dot{q} + rp)I_{yz} + (q^2 - p^2)I_{xz} + (rq - \dot{p})I_{zx} + m[x_g(\dot{v} - w\dot{p} + ur) - y_g(\dot{u} - vr + wq)] = N \quad (9)$$

The first three equations correspond to translational motion of the vehicle while the last three equations deal with the rotational motion of the vehicle.

The parameter X, Y, Z, K, M, N are the external forces and moments, which includes Hydrostatic force, drag force, Lift force, Propeller Thrust, Added Mass and also the effect of stern plane and rudder planes. These external parameters are defined as follows,

$$X = X_{HS} + X_{u|u}|u| + X_{\dot{u}}\dot{u} + X_{wq}wq + X_{qq}qq + X_{vr}vr + X_{rr}rr + X_{prop} \quad (10)$$

$$Y = Y_{HS} + Y_{v|v}|v| + Y_{r|r}|r| + Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_{ur}ur + Y_{wp}wp + Y_{pq}pq + Y_{uv}uv + Y_{uu\delta_r}u^2\delta_r \quad (11)$$

$$Z = Z_{HS} + Z_{w|w}|w| + Z_{q|q}|q| + Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_{uq}uq + Z_{vp}vp + Z_{rp}rp + Z_{uw}uw + Z_{uu\delta_r}u^2\delta_r \quad (12)$$

$$K = K_{HS} + K_{p|p}|p| + Z_p\dot{p} + K_{prop} \quad (13)$$

$$M = M_{HS} + M_{w|w}|w| + M_{q|q}|q| + M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} + M_{uq}uq + M_{vp}vp + M_{rp}rp + M_{uw}uw + M_{uu\delta_r}u^2\delta_r \quad (14)$$

$$N = N_{HS} + N_{v|v}|v| + N_{r|r}|r| + N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_{ur}ur + N_{wp}wp + N_{pq}pq + N_{uv}uv + N_{uu\delta_r}u^2\delta_r \quad (15)$$

the parameters used in the equations above are the external components which affect the overall dynamics of the Autonomous Underwater Vehicle.

The hydrodynamic coefficients used in the equation above are used in the transformation matrices as follows:

$C_{x\dot{u}} = \frac{\rho}{2}L^3X_{\dot{u}}$	-141.9	$C_{X_{rr}} = \frac{\rho}{2}L^4X_{rr}$	832	$C_{y\dot{v}} = \frac{\rho}{2}L^3Y_{\dot{v}}$	-1715.4
$C_{Y_{rr}} = \frac{\rho}{2}L^4Y_{rr}$	-32.5	$C_{y\dot{r}} = \frac{\rho}{2}L^4Y_{\dot{r}}$	186.9	$C_{Y_{vv}} = \frac{\rho}{2}L^2Y_{vv}$	-667.5
$C_{z\dot{w}} = \frac{\rho}{2}L^3Z_{\dot{w}}$	-4617	$C_{N_{vv}} = \frac{\rho}{2}L^3N_{vv}$	433.8	$C_{z\dot{q}} = \frac{\rho}{2}L^4Z_{\dot{q}}$	-1701.9
$C_{N_{rr}} = \frac{\rho}{2}L^5N_{rr}$	-310	$C_{K_{\dot{q}}} = \frac{\rho}{2}L^4K_{\dot{q}}$	-40.6	$C_{X_{wq}} = \frac{\rho}{2}L^3X_{wq}$	137
$C_{M_{\dot{w}}} = \frac{\rho}{2}L^4M_{\dot{w}}$	-2090.4	$C_{X_{w\delta s}} = \frac{\rho}{2}L^2X_{w\delta s}$	-221.7	$C_{M_{\dot{q}}} = \frac{\rho}{2}L^5M_{\dot{q}}$	-1692.3
$C_{X_{\delta s\delta s}} = \frac{\rho}{2}L^2X_{\delta s\delta s}$	-455	$C_{N_{\dot{v}}} = \frac{\rho}{2}L^4N_{\dot{v}}$	957	$C_{X_{\delta r\delta r}} = \frac{\rho}{2}L^2X_{\delta r\delta r}$	-80.3
$C_{N_{\dot{r}}} = \frac{\rho}{2}L^5N_{\dot{r}}$	-1349	$C_{X_{q\delta s}} = \frac{\rho}{2}L^3X_{q\delta s}$	-308.9	$C_{X_{uu}} = \frac{\rho}{2}L^2X_{uu}$	-35.4
$C_{Y_{\delta r}} = \frac{\rho}{2}L^2Y_{\delta r}$	117.2	$C_{X_{vv}} = \frac{\rho}{2}L^2X_{vv}$	-128	$C_{Z_{\delta s}} = \frac{\rho}{2}L^2Z_{\delta s}$	-689.7
$C_{X_{ww}} = \frac{\rho}{2}L^2X_{ww}$	-89.48	$C_{M_{\delta s}} = \frac{\rho}{2}L^3M_{\delta s}$	-791.3	$C_{X_{qq}} = \frac{\rho}{2}L^4X_{qq}$	9587.4
$C_{N_{\delta r}} = \frac{\rho}{2}L^3N_{\delta r}$	-266				

Table 2: INFANTE AUV Hydrodynamic Coefficients

The kinematics and dynamics equation can be simplified by considering only body relative surge, sway, yaw rate & earth relative position, heading & yaw angle. and again neglecting all out of plane terms results in:

$$m[\dot{u} - vr - x_g r^2 + y_g \dot{r}] = X \quad (16)$$

$$m[\dot{v} + ur - y_g r^2 + x_g (\dot{r})] = Y \quad (17)$$

$$I_z \dot{r} + m[x_g (\dot{v} + ur) - y_g (\dot{u} - vr)] = N \quad (18)$$

Here, X,Y& N are vehicle parameters and are combination of various external forces such as added mass, hydrodynamic damping, hydrostatics etc.

3. PATH FOLLOWING CONTROL STRATEGY FOR AN INDIVIDUAL AUV

Let the desired path, P which the AUV is to follow Fig.2. It is intended to design a control law such that the AUV will follow the desired path P. A path following controller for an under actuated AUV is to be designed such that it steers the AUV towards the desired path P while maintaining a constant velocity in the forward motion.

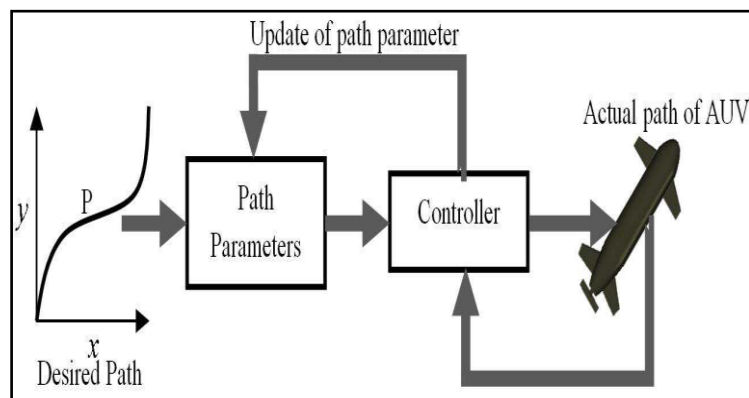


Fig. 2: Path following controller implementation

Using the linearized hydrodynamics coefficients as described in equations 1-10, the dynamics simulation of the AUV is obtained assuming a fixed value of propeller thrust & rudder angle. The vehicle has to follow a circular path which is given as an input to the vehicle in parametric fashion. The results after simulation as obtained is shown in the result section as described below in the paper.

4. MODEL PREDICTIVE CONTROLLER

It is a type of control in which the current control signal is determined such that a desirable output behaviour results in the future. This future behaviour is a function of past inputs to the process as well as the inputs that we are considering to take in the future. In MPC structure there is a feedback or feed forward path to compute the process measurements.

There are mainly three components available in MPC structure (i). The process model (ii). The cost function (iii). The optimizer

The information about the controlled process and prediction of the response of the process values according to the manipulated control variables are done by the process model. Then the error is reduced by the minimization of the cost function. The general structure of Model Predictive Controller is shown in Fig.3.

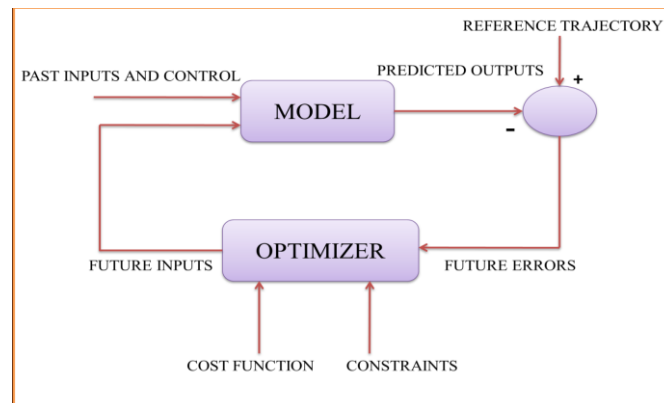


Fig.3. General Structure of Model Predictive Controller

It provides functions, an application, and Simulink blocks for systematically analysing, designing, and tuning model predictive controllers. The toolbox enables us to diagnose issues that could lead to runtime failures and provides advice on changing weights and constraints to improve performance and robustness.

The process output is predicted by using a model of the process to be controlled. Any model that describes the relationship between the input and the output of the process can be used. Further if the process is subject to disturbances, a disturbance or noise model can be added to the process model. In order to define how well the predicted process output tracks the reference trajectory, a criterion function is used. Typically the criterion is the difference between the predicted process output and the desired reference trajectory.

The MPC control strategy was simulated using MPC toolbox which is a MATLAB-based toolbox. The Cost function is given as

$$J = \sum_{i=1}^N \left(\sum_{j=1}^{n_y} (w_j^y e_{yij})^2 + \sum_{j=1}^{n_u} \left[(w_j^u e_{uij})^2 + (w_j^{\Delta u} \Delta u_{ij})^2 \right] \right) \quad (19)$$

Where

N = number of controller sampling intervals in the scenario

n_y = number of controlled outputs

n_u = number of manipulated variables

e_{yij} = set point (or reference) tracking error i.e. the difference between output j and its set point at time step i

e_{uij} = deviation of manipulated variable j from its target value at time step i

Δu_{ij} = change in manipulated variable j at time step i

w_j^y = performance weight for output j

w_j^u = performance weight for manipulated variable j

The selection of MPC to control an AUV is attributed to several factors. Some of them are listed below.

- The concept is equally applicable to single-input, single-output (SISO) as well as multi-input, multi-output systems (MIMO).
- MPC can be applied to linear and nonlinear systems.
- It can handle constraints in a systematic way during the controller design.

5. CONTROL SYNTHESIS

To develop a depth controller, the vehicle's forward nominal speed $u = u_0$ is assumed to be constant and the vertical plane model is formally written as

$$\frac{dx_v}{dt} = F_v(x_v, u_v),$$

where $x_v = [w, q, \theta]^T \in \mathbb{R}^3$ is the state vector, $u_v = [\delta_b, \delta_s]^T \in \mathbb{R}^2$ is the input vector, and $F_v : \mathbb{R}^3 \times \mathbb{R}^2 \rightarrow \mathbb{R}^3$ is a nonlinear function that is easily obtained from the surge, and pitch equations of motion, together with kinematics depth and pitch relationships described in detail in the section above stated. The model for the vertical plane was linearized about the equilibrium point determined by $[w_0; q_0; z_0; y_0]^T = [0, 0, 0, 0]^T$ and $u_0 = [\delta_b, \delta_s]^T = [0, 0]^T$. The resulting linearized model Eigen values are presented in Fig.7.

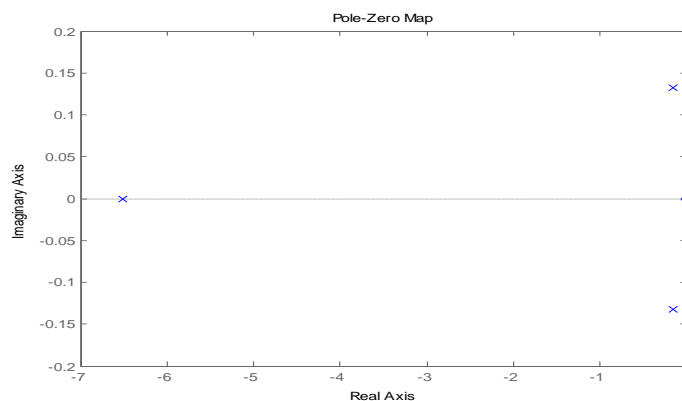


Fig.4.Linearized model Eigen values

The model exhibits an Eigen value at zero and three stable Eigen values that link together the variables w, q and y. The state space linearized dynamics and input matrices for the forward velocity of 2.0 m/s are represented below:

$$A = \begin{bmatrix} -1.4 & 2.763 & 0 & 0.078 \\ 2.108 & -5.419 & 0 & -0.312 \\ 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.797 & -0.201 \\ 1.588 & -0.809 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The state space matrix A has one second-order mode with a natural frequency of 0.132 rad/s, a real Eigen value at -6.5 rad/s, and a zero Eigen value. Notice that the variable y does not exhibit a pure integration effect, due to the existence of a restoring torque caused by the combined effect of buoyancy and gravity. In the input matrix, the bow and stern plane deflections δ_b and δ_s affect directly the state variables w and q.

The synthesis of MPC was carried out in the following steps. First, the parameter of MPC is chosen considering the given linearized model. The prediction horizon and control horizon are chosen to be $H_p = 10$ & $H_u = 2$, respectively. The time elapsed between control moves is 0.01 sec. The constraints for the input and state variables are given as:

For the input variable, the pitch is constrained for vertical control is limited by

$$-0.2 \leq \theta \leq 0.2$$

For the state variables, the pitch rate(rad/sec) is constrained to

$$-20 \leq q \leq 20$$

The forward velocity u(m/s) is limited by

$$-25 \leq u \leq 25$$

while, the heave velocity w and the surge velocity v are not constrained.

The second step is determining the input and output weight parameters. For the input variable, θ no weight is assigned meaning that θ is allowed to vary freely between its minimum value and maximum value. However, the rate weight of θ must be assigned non--zero value since in reality the rate of change of pitch angle is limited. The rate weight of θ is chosen to be 0.3993. The output weight of 0.246 is assigned to depth and pitch rate. No weight is assigned to the other state variables. Overall, the choice of weight is guided by the trade-off between the robustness and the combined disturbance rejection and set-point tracking. The control synthesis is performed using MPC design tool in MATLAB. The Simulink diagram using the above stated constraints is as follows:

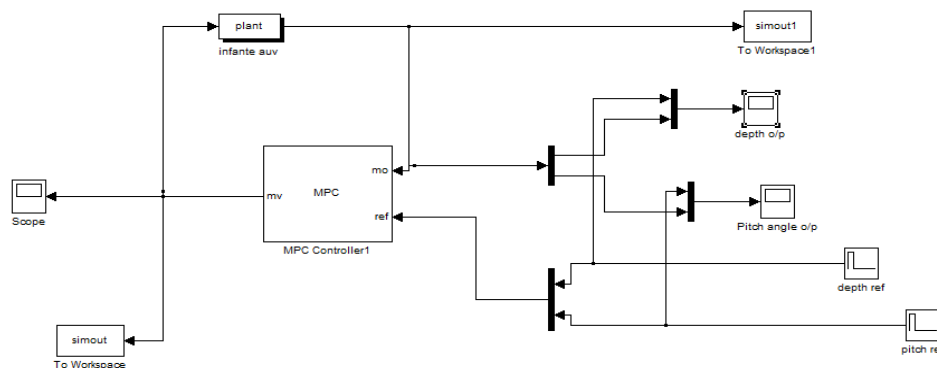


Fig.5. : Simulink diagram for depth and pitch control

The step response is chosen as a reference setpoint for the depth and the pitch angle and the results obtained after the simulation are described in the result section in the paper as detailed below.

6. GAIN-SCHEDULING CONTROL OF DEPTH OF AUV

This is followed by control system design for precise manoeuvring in the vertical plane. The technique adopted for controller design is gain scheduling [23]. Using this approach, a set of linear controllers is first derived for a finite number of linearized models of the plant at selected operating points. The resulting controllers are then interpolated on the vehicle's forward speed. In fact, the resulting controllers exhibit only the dynamics introduced by appended integrators as well as extra dynamics that act as shaping filters to limit the actuation bandwidth. The importance of output feedback control strategies cannot be overemphasized: in practice, it is often impossible to measure the full state vector of a given plant. This motivates the development of controllers that rely on output variables only, effectively increasing the simplicity and thus the reliability of the control laws adopted. In the case of the INFANTE AUV, for example, it is difficult to measure the angle of attack in the vertical plane. However, it is crucial to achieve stabilization and good manoeuvring performance in that plant, thus justifying the use of output feedback control techniques to meet desired stability and performance criteria.

It can be seen theoretically that the reduced order output feedback control problem can be converted into a static output feedback (SOF) problem for a related augmented system. However, in spite of the availability of necessary and sufficient conditions for plant stabilization by SOF, "no algorithm is currently available which guarantees to compute a stabilizing gain or determine if such a gain exists". These problems can be solved by Linear Matrix Inequalities (LMIs). However, difficulties arise when designing (sub-optimal) SOF controllers because this problem can only be cast in terms of an equivalent one that involves bilinear matrix inequalities (BMIs) [23]. Because the resulting problem is no longer convex, no efficient numerical procedures exist for its solution as in the case of LMIs.

In the work reported here, a finite number of ROF controllers were developed for linearized plant models obtained at different operating conditions determined by the vehicle's forward speed. The controller parameters were then interpolated and scheduled on speed (that is, dynamic pressure). The final implementation of the resulting nonlinear gain-scheduled controller was done using the methodology described in [21] that guarantees a fundamental linearization property and avoids the need to feedforward the values of the state variables and inputs at trimming.

7. CONTROL SYSTEM DESIGN AND IMPLEMENTATION

This section describes the design of a depth control system for the AUV INFANTE, based on the dynamic model presented in equations (10)&(11). The methodology adopted for controller design is nonlinear gain-scheduled control, whereby the design of a controller to achieve stabilization and adequate performance of a given nonlinear plant (system to be controlled) involves the following steps[22]:

- i. Linearizing the plant about a finite number of representative operating points,
- ii. Designing linear controllers for the plant linearization at each operating point,

- iii. Interpolating the parameters of the linear controllers of Step (ii) to achieve adequate performance of the linearized closed-loop systems at all points where the plant is expected to operate. The interpolation is performed according to an external scheduling variable (vehicle's forward speed), and the resulting family of linear controllers is referred to as a gain-scheduled controller,
- iv. Implementing the gain-scheduled controller on the original nonlinear plant.

In what follows a brief summary is given of the work done at each of the design steps, leading to the development of a controller for the vehicle that is scheduled on forward speed. For the sake of brevity, the linear design methodology is illustrated for the case of a single operating condition that corresponds to a forward speed of 2.0 m/s.

The depth of AUV is controlled by non linear gain-scheduling control, whereby a set of linear finite reduced order output feedback controllers were designed using LMI based techniques. The Simulink diagram is as follows:

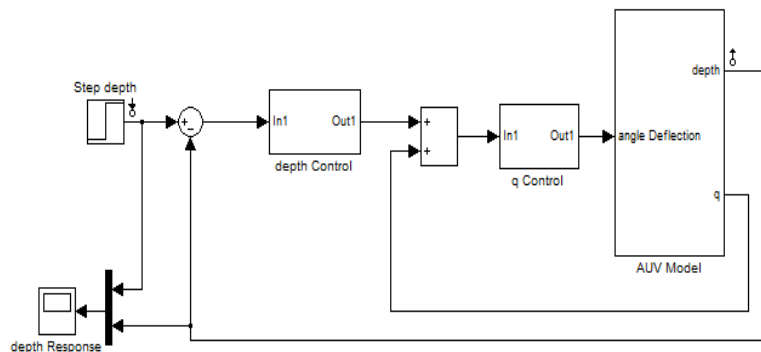


Fig.6: Gain Scheduling Control of depth of AUV

The results obtained after simulation is shown in the sections given below.

8. LINEAR MATRIX INEQUALITIES

The methodology selected for linear control system design was reduced order output feedback with an H1 criterion [23]. This method rests on a firm theoretical basis and leads naturally to an interpretation of control design specifications in the frequency domain.

The design tools adopted are LMIs, which are steadily becoming a standard tool for the design of advanced control systems. As explained in [29], the story of LMIs can be traced back to the work of Lyapunov, who showed that the origin of the linear time-invariant system $dx(t)/dt = Ax(t)$ is asymptotically stable if and only if there exists a positive definite matrix P such that $A'P + PA < 0$. Let P_1, P_2, \dots, P_m be a basis for the space of $n \times n$ positive definite matrices, with $m = n \times (n + 1)/2$. Further let $F_0 = 0$ and $F_i = A'P + P_i A$. Then, finding P positive definite that satisfies the equation above (or determining that none exists) is equivalent to determining whether a vector $x = (x_1, \dots, x_n) \in R^n$ exists such that

$$F(x) = F_0 + \sum_{i=1}^m x_i F_i < 0 \quad (21)$$

is satisfied.

An LMI is any constraint $F(x)$ of the form introduced above. In the general case, the vector $x = (x_1, \dots, x_n)' \in \mathbb{R}^n$ represents the free design variables (also called decision variables) and the symmetric matrices $F_i = F_i' \in \mathbb{R}_{n \times n}$ are given. The inequality symbol in Eq. (23) means that $F(x)$ is negative definite, i.e., for all nonzero $u \in \mathbb{R}^n$, $u'F(x)u < 0$. Solving this inequality consists of finding x such that (23) holds.

LMIs present the following appealing property for computational purposes: finding a feasible x such that $F(x) < 0$ is a convex optimization problem for which efficient interior point algorithms have been developed and implemented in the Matlab LMI Toolbox.

In general, multi-objective/multi-criteria control problems, where mixed time and frequency domain specifications must be met, are extremely difficult to solve. However, within the LMI framework, multi-objective problems involving simultaneous H_2 and H_∞ performance criteria as well as regional Eigen value placement, settling time, saturation, and initial conditions response specifications, can be formulated and solved using advanced numerical tools. The key idea in the LMI approach to multi-objective controller design consists of converting each closed-loop control objective or specification into an additional constraint on the class of admissible closed-loop Lyapunov functions. This design technique expresses the desired closed-loop control objectives and specifications in terms of a set of LMIs (involving a single Lyapunov function that guarantees simultaneous achievement of the different closed-loop requirements, possibly at the expense of being conservative).

In summary, LMIs provide a powerful formulation as well as a versatile design technique for a wide variety of linear control problems. Since solving LMIs is a convex optimization problem for which numerical solvers are now available, reducing a control problem to an LMI can be seen as a practical solution for many control problems.

9. RESULTS

The parameters of AUV are tracked and calculated in such a way that a prototype has to strictly follow a circular path. Using the linearized hydrodynamics coefficients as described in equations 1-10, the dynamics simulation of the AUV is obtained assuming a fixed value of propeller thrust & rudder angle. The simulation of the MATLAB program shows that the vehicle is tracking a perfect circle as the input is provided to the vehicle.

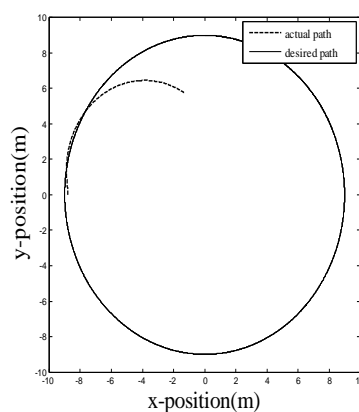


Fig.7. Dynamics Simulation of AUV

The simulation of AUV is done in next using MPC tool, considering depth and pitch angle parameters as described in the section 5, where the control synthesis of AUV is done and the transformation matrices are generated using various parameters of INFANTE AUV. It should be noted down that the depth and pitch angle of AUV is controlled without considering the effect of external disturbances.

As a case study the AUV is to follow the pitch angle and depth setpoint defined as a pulse at $t= 10$ s with the amplitude of 8 m and period of 100 sec. The weight is tuned in order to increase input rate penalties relative to setpoint penalties.

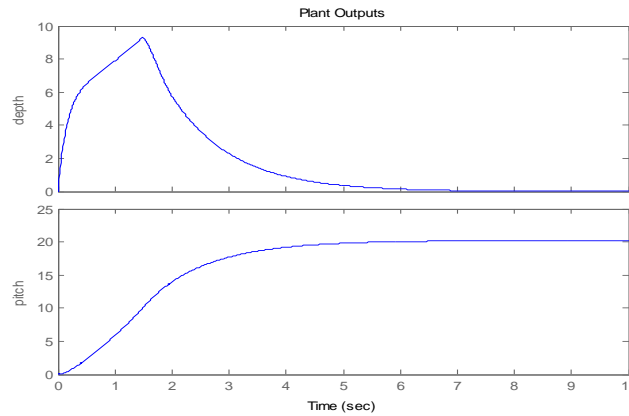


Fig.8. Response of the predicted outputs controlled by MPC.

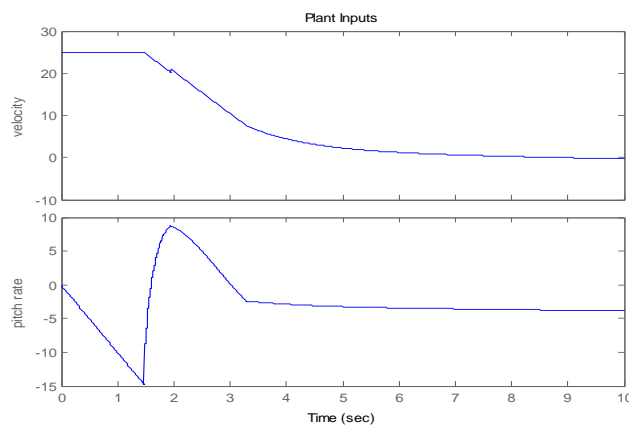


Fig.9: Response of the predicted manipulated input variables.

The simulation results using MPC controller shows that the depth and pitch angle is tracked properly with a deviation at 1.5 sec and afterwards the stability is achieved between setpoint tracking and disturbance rejection such that it minimises the tracking error and reduces it to zero.

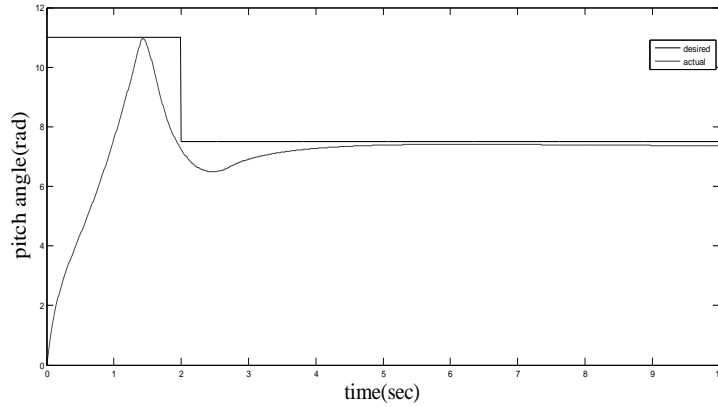


Fig.10: O/p response of pitch angle after simulation

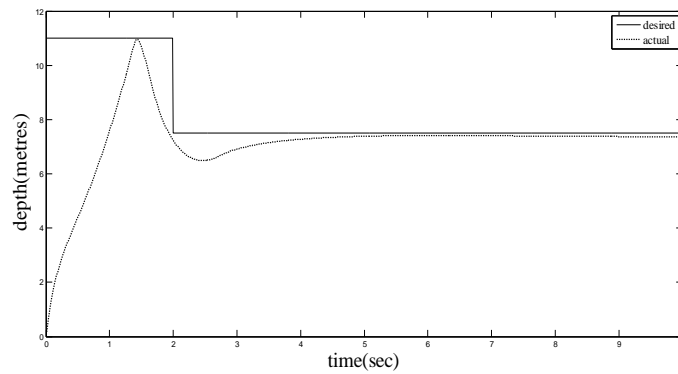


Fig.11 : O/p response of depth after simulation

The optimization of the control input is done considering the weighted tracking error between the predicted output and reference trajectory and the consumption of control energy subject the constraints in the ranges of actuators and their slew rates. The proposed MPC controller is shown to be robust against disturbance while maintaining an acceptable setpoint tracking performance.

The depth of AUV is controlled by non linear gain-scheduling control, whereby a set of linear finite reduced order output feedback controllers were designed using LMI based techniques. The Fig. 12 shown below shows the commanded and the measured depth, the INFANTE AUV was switched to command to dive a step response of amplitude 200m by adopting the methodology of gain scheduling control and linear matrix inequalities. In the figure shown below, the dashed lines and the solid lines represent the desired and simulation results, respectively.

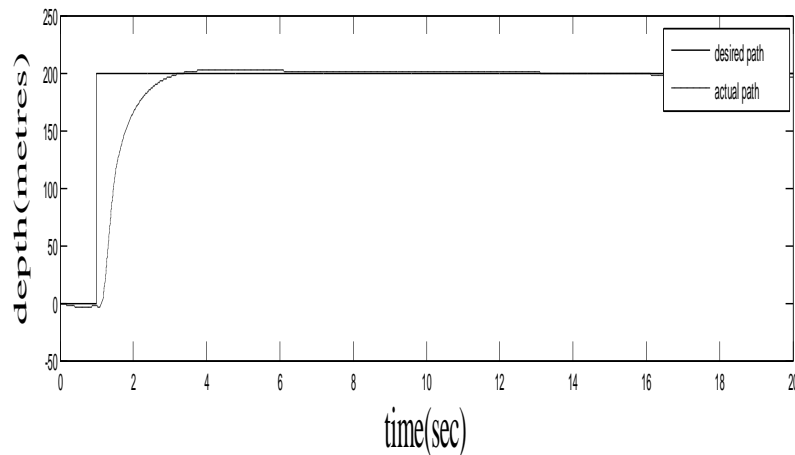


Fig.12. Response of depth control using gain scheduling controller

The figure shows the design and experimental testing of a control system for the INFANTE AUV in the vertical plane. The general setup adopted for controller design was nonlinear gain-scheduling control, whereby a set of linear finite reduced order output feedback controllers were designed using linear matrix inequality (LMI) based techniques. The controllers implemented have proven extremely reliable over a long series of missions with the INFANTE AUV.

10. CONCLUSIONS

This paper addressed the path following control problem of an Autonomous Underwater Vehicle and also the formation control of multiple Autonomous Underwater Vehicles. As discussed in the sections above, the controllers for path following problem is developed.

The development of path following controller for an AUV has been successfully implemented using MATLAB & SIMULINK considering the nonlinearities and coupling terms in the dynamic equation. Lyapunov theory and backstepping concept have been used to develop path following controller for an AUV where the coupling of rudder angle between sway motion and yaw motion has been considered. Also the control of forward motion i.e. surge motion is included for forward motion control. The gains of the controller are also adapted according to the error derived while following the path.

A new approach to control the yaw angle of an AUV using MPC has been demonstrated. The simple LOS guidance scheme is used to generate the reference heading. The results produced are for stationary targets and are quite encouraging as the actuator constraints are handled in an efficient way. Dealing non-stationary targets using the proposed algorithm is an area of active research.

The general setup adopted for controller design was nonlinear gain-scheduling control for the depth control design when the disturbances are present, whereby a set of linear finite reduced order output feedback controllers were designed using linear matrix inequality (LMI) based techniques. The methodology adopted addressed explicitly the fact that one of the vehicle states is not easily accessible for measurement. Furthermore, it is well rooted in recent theoretical advances in control theory and numerical analysis. The controllers implemented have proven extremely reliable over a long series of missions with the INFANTE AUV. Further

problems that warrant further research include AUV control close to the surface in the presence of strong wave action and AUV terrain following.

In the work described here, 6DoF is considered in the dynamic equations and these are used for implementation of different controllers. The effects of ocean current has not been considered in the development of control law of the AUV. Hence, for real-world situation one has to certainly consider the above effect in the control development. Further, to address the uncertainties in the AUV parameters such as hydrodynamic effect and oceanic current a robust controller can be developed.

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