CO – TOTAL DOMINATION ON STRONG LINE CORPORATE GRAPHS

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Abstract

In this paper, we introduce the co – total domination on strong line corporate graphs and we investigate the relation and bounds between the strong line corporate domination number and the total strong line corporate domination number. Also we establish the co – total strong line corporate domination number for some standard graphs.

Keywords:

Co- total strong line corporate dominating set, strong line corporate dominating set, total strong line corporate dominating set.

1 INTRODUCTION

All graphs in this paper are finite, simple and undirected. Let $G=(V,E)$ be a graph where the symbols V and E denote the vertex set and edge set of G. For all other terminology and notations, we follow Harray[1] and the definitions related to dominations are referred from T.W. Haynes & Kulli[2][3].

M.H. Muddebihal[4] introduced inverse Line Domination graph. S.Pethanachi Selvam, S.Padmashini[5] introduced Inverse Complementary domination graph. S.Pethanachi Selvam, S.Padmashini[6] introduced line corporate domination graph.

By the motivation of the papers, we introduce co – total domination on strong line corporate graph, co – total strong line corporate domination number and total strong line corporate domination number.

In this paper we prove the bounds and properties related to co – total strong line and total strong line corporate domination number of G.

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2 PRELIMINARIES

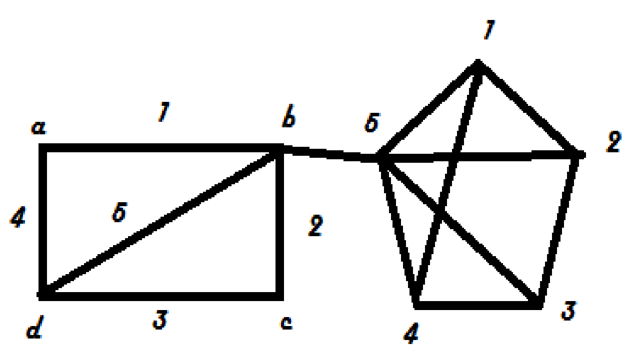
Definition 2.1. Let $G_1(= (p, q))$ and $G_2(= (s, t))$ be any two graphs. A graph obtained by joining the graphs G_1 and G_2 by a bridge is called corporate graph and is denoted by *C(G).*

Definition 2.2. Let $G_1(= (p, q))$ and $G_2(= (s, t))$ be any two graphs. A graph obtained by joining the maximum degree vertex of G_1 and the maximum degree vertex of G_2 by a bridge is called strong corporate graph and is denoted by *SC(G)* .

Definition 2.3. If we take $G_2 = L(G_1)$ in corporate graph then it is called line corporate graph and is denoted by *LC(G).*

Definition 2.4. If we take $G_2 = L(G_1)$ in Strong corporate graph then it is called strong line corporate graph and is denoted by *SLC(G).*

Example 2.5.



Defintion 2.6.

Let *SLC(G)* be a strong line corporate graph. A set $D \subseteq V(SLC(G))$ is said to be strong line corporate dominating set (shortly written as *SLCDS)* if every vertex in $V - D$ is adjacent to some vertex in D.

The minimum cardinality of vertices in such a set is called strong line corporate domination number of *SLC(G)* and is denoted by $\gamma(SLC(G))$.

Defintion 2.7. Let *D* be a strong line corporate dominating set *(SLCDS)*. If $V \subseteq (SLC(G)) - D$ contains another *SLCDS* namely D^{-1} , then D^{-1} is called the inverse strong line corporate dominating set *(shortly by ISLCDS) w.r.t D.*

The minimum cardinality of vertices in *ISLCDS* is called Inverse strong line corporate domination number and is denoted by $\gamma^{-1}(SLC(G))$.

Example 2.8.

In the above example 2.5, the strong line corporate dominating set is {b,5} and the inverse strong line corporate dominating set is {d,3,4}

3 CO – TOTAL DOMINATION

Definition 3.1. A strong line corporate dominating set $S \subseteq V(SLC(G))$ is said to be co - total strong line corporate dominating set of G if the induced subgraph $< V(SLC(G) - S >$ has no isolated vertices.

The minimum cardinality of a co - total strong line corporate dominating set of *G* is the co - total domination number of *SLC(G)* and is denoted by $\gamma_{ct}((SLC(G))$.

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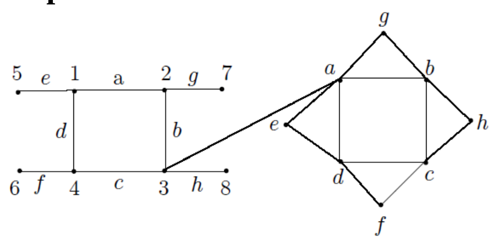
Example 3.2.

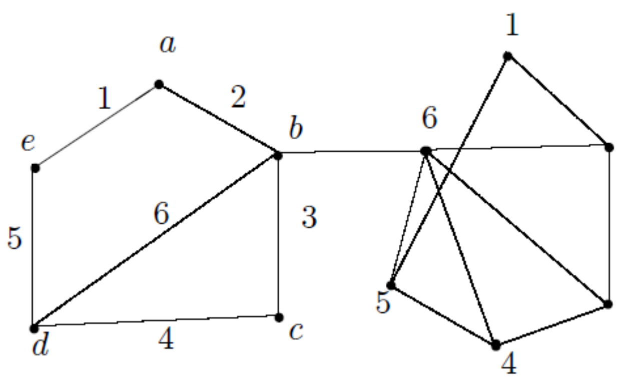


Here, the co - total strong line corporate dominating set of G is {5, 6, 7, 8, b, d}.

Definition 3.3. A strong line corporate dominating set $T \subseteq V(SLC(G))$ is said to be total strong line corporate dominating set of G, if the induced subgraph $<T>$ has no isolated vertices.

The minimum cardinality of a total strong line corporate dominating set of G is the total domination number and is denoted by $\gamma_t((SLC(G))$.

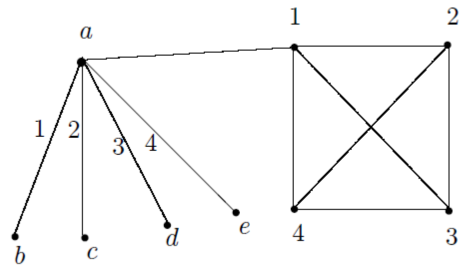
Example 3.4.



Here, the total strong line corporate dominating set of G is $\{a, b, 6, 5\}$ and $\gamma_t(SLC(G)) = 4$.

Note 3.5. Every *CTSLCDS* is also *SLCDS*. But the converse need not be true.

Example 3.6.



Here, the co - total strong line corporate dominating set {b, c, d, e, 4} which is also strong line corporate dominating set. But, {a,1} is a strong line corporate dominating set which is not the co - total strong line corporate dominating set.

4 MAIN RESULTS.

Theorem 4.1. (a) For any cycle C_p , $p \geq 3$

$$\gamma_{ct}(SLC(Cp)) = \begin{cases} p - \left\lceil \frac{p}{3} \right\rceil & if\ n \equiv 1(mod\ 3) \\ 2\left\lceil \frac{p}{3} \right\rceil & if\ n \equiv 2(mod\ 3) \\ \frac{2p}{3} & if\ n \equiv 0(mod\ 3) \end{cases}$$

(b) For any star graph $K_{1,n}$,

$$\gamma_{ct}(SLC(K_{1,n})) = n + 1$$

(c) For any path P_p, $\gamma_{ct}(SLC(P_p)) = \left\lceil \frac{p+q}{2} \right\rceil$

(d) For any complete bipartite graph $K_{n,n}$,

$$\gamma_{ct}(SLC(K_{n,n})) = 2.$$

(e) For any wheel graph , p>3,

$$\gamma_{ct}(SLC(W_p)) = \begin{cases} 2 + \left\lceil \frac{p-2}{3} \right\rceil & if\ p \equiv 0(mod\ 3) \\ 2 + \left\lceil \frac{p-3}{2} \right\rceil & otherwise \end{cases}$$

Theorem 4.2. If v is an end vertex of SLC(G), then v is in minimal co - total dominating set of SLC(G).

Proof. Let $D = \{v_1, v_2, ..., v_{p+q}\} \subseteq V((SLC(G))$ be the minimal co - total dominating set of $SLC(G)$.

Let $v_i = v$ be the end vertex of $SLC(G)$, then either $v_i \in G$ or $v_i \in L(G)$.

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Our claim is $v_i \in D$.
Suppose not, then consider the following cases.
Case(i): Consider any two vertices u and w such that $u,w \in D$.
Since $v_i \in (V(SLC(G) - D)$, v_i is in every $u - w$ path in $SLC(G)$ which is a contradiction to v_i is an end vertex of $SLC(G)$.
Case(ii): Consider any two vertices u and w such that $u,w \notin D$, then v is adjacent to at least two vertices which is a contradiction to $deg(v_i) = 1$. Hence $vi \in D$.

Note 4.3. If $SLC(G)$ has isolated vertices, then $i(SLC(G)) \subseteq CTSLC(G)$ where $i(SLC(G))$ is the isolated vertices of $SLC(G)$ but total strong line corporate dominating set of G does not exist , since the induced subgraph of the dominating set having isolates vertices.

Theorem 4.4. Let $SLC(G)$ be a strong line corporate graph. If $\gamma_{ct}(SLC(G))$ and $\gamma_t(SLC(G))$ exists and if $v_i \in CTSLCDS$ but $v_i \notin TSLCDS$, then

$$\left\lfloor \frac{p+q+\Delta(SLC(G)-(\Delta(G)+\Delta(L(G))}{\delta(G)+\delta(L(G)} \right\rfloor \leq \gamma_{ct}(SLC(G))+\gamma_t(SLC(G)) \leq p+q.$$

Proof. Let $X = \{v_1, v_2, ..., v_{p+q}\}$ be the set of vertices of SLC(G) .
Let $D = \{v_1, v_2, ..., v_i\} \subseteq V(SLC(G))$ and $S = \{v_1, v_2, ..., v_j\} \subseteq V(SLC(G))$ be the minimum set of vertices in the co-total and total dominating sets of $SLC(G)$.
Then, $|D| + |S| \leq i + j \leq p + q$.
Next, we have to prove that the lower bound.
Since, $\gamma_{ct}(SLC(G)) \geq 2$ and $\gamma_t(SLC(G)) \geq 2$, $\gamma_{ct}(SLC(G)) + \gamma_t(SLC(G)) \geq 4$.

To Prove,

$$\left\lfloor \frac{p+q+\Delta(SLC(G)-(\Delta(G)+\Delta(L(G))}{\delta(G)+\delta(L(G)} \right\rfloor \leq 4$$

Suppose

$$\left\lfloor \frac{p+q+\Delta(SLC(G)-(\Delta(G)+\Delta(L(G))}{\delta(G)+\delta(L(G)} \right\rfloor > 4,$$

then
$p + q + \Delta(SLC(G)) - (\Delta(G) + \Delta(L(G)) > 4(\delta(G) + \delta(L(G))$
$p + q + \Delta(SLC(G)) > 4(\delta(G) + \delta(L(G)) + \Delta(G) + \Delta(L(G)$ which is a contradiction for a graph having co-total and total strong line corporate dominating set , since $\Delta(SLC(G)) \leq p$ or q, $\delta(G) \leq p - 1$, $\delta(L(G)) \leq q - 1$, $\Delta(G) \leq p - 1$, $\Delta(L(G) \leq q - 1$.
Hence,

$$\left\lfloor \frac{p+q+\Delta(SLC(G)-(\Delta(G)+\Delta(L(G))}{\delta(G)+\delta(L(G)} \right\rfloor \leq \gamma_{ct}(SLC(G))+\gamma_t(SLC(G)) \leq p+q.$$

Theorem 4.5. Let $SLC(G)$ be a strong line corporate graph. If $\gamma_{ct}(SLC(G))$ and $\gamma_t(SLC(G))$ exists and if $v_i \in CTSLCDS$ but $v_i \notin TSLCDS$, then $\gamma_{ct}(SLC(G)) + \gamma_t(SLC(G) - \gamma(G) \leq \left\lceil \frac{p+2q}{2} \right\rceil$

Proof. Let $X = \{v_1, v_2, ..., v_{p+q}\}$ be the set of vertices of SLC(G) .
Let $\{v_1, v_2, ..., v_i\} \subseteq V(SLC(G))$ and $\{v_1, v_2, ..., v_j\} \subseteq V(SLC(G))$ be the minimum set of vertices in the co-total and total dominating sets of SLC(G) and let $\{v_1, v_2, ..., v_k\} \subseteq V(SLC(G))$ be the minimum set of vertices in the dominating set of SLC(G) .
Since $\gamma_{ct}(SLC(G))$ and $\gamma_t(SLC(G))$ exists , by the above theorem we have $\gamma_{ct}(SLC(G)) + \gamma_t(SLC(G) \leq p + q$ and since, $\gamma(G) \leq \frac{p}{2}$. we have,
$\gamma_{ct}(SLC(G)) + \gamma_t(SLC(G) - \gamma(G) \leq p + q - \frac{p}{2} \leq \left\lceil \frac{p+2q}{2} \right\rceil$

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Hence , $\gamma_{ct}(SLC(G)) + \gamma_t(SLC(G) - \gamma(G) \leq \left\lceil \frac{p+2q}{2} \right\rceil$

Theorem 4.6. Let *SLC(G)* be the strong line corporate graph. Then $\gamma_{ct}(SLC(G)) \leq p + q$ equality holds for P_2 .

Proof. Let $X = \{v_1, v_2, ..., v_{p+q}\}$ be the set of vertices of *SLC(G)* .

Let $D = \{v_1, v_2, ..., v_n\} \subseteq V(SLC(G))$ be the minimal co – total dominating set of *SLC(G)* .

Suppose $|V(SLC(G)) - D| = 0$,then the equality holds immediately.

That is ,in P_2 ,$SLC(G) = P_3$ .We have , $\gamma_{ct}(SLC(G)) = 3.$

Further , if $|V(SLC(G)) - D| \geq 2$ then $V(SLC(G)) – D$ has at least two vertices such that $|D| \leq |V(SLC(G))|$.

Since $|V(SLC(G)) - D| \geq 2,$

$|V(SLC(G))| - |D| \geq 2$

$|D| \leq |V(SLC(G))| - 2$

$|D| \leq p + q - 2 \leq p + q.$

Hence, $\gamma_{ct}(SLC(G)) \leq p + q.$

Theorem 4.7. Let SLC(G) be any connected graph. Then

$\gamma_{ct}(SLC(G)) + \gamma(SLC(G)) \leq p + q + 1.$

Proof. Let $X = \{v_1, v_2, ..., v_{p+q}\}$ be the set of vertices of *SLC(G)* .

Let $D \subseteq X$ be the dominating set of SLC(G) such that $deg(v_i) \geq 2, \forall v_i \in D,$ $1 \leq i \leq p + q,$ then there exists a minimal set of vertices $D_1 \subseteq D$ which covers all the vertices in *SLC(G)* .

Clearly, D_1 is the minimal dominating set of *G*.

Suppose $< V(SLC(G)) – D_1 >$ contains no isolated vertices, then D_1 itself is a co-total strong line corporate dominating set of G.

Otherwise , let us consider the following cases.

Case(i): If *SLC(G)* contains end vertices.

In this case , let $S = \{v_1, v_2, ..., v_i\}$ be the set of all end vertices.

Subcase(i): Suppose these end vertices cover all the vertices , then S is the dominating set such that $< V(SLC(G))- S >$ has no isolated vertices.

Thus ,$\gamma_{ct}(SLC(G)) + \gamma(SLC(G)) \leq p + q + 1.$

Subcase(ii): If $N[S] \neq V(SLC(G)),$ then let $T \subseteq D$ be the set of vertices such that the vertices in T is not adjacent to the vertices in D .

Take the minimal set of vertices T_1 among T in which $< V(SLC(G)) - S - T >$ has no isolated vertices .

Clearly , $T_1 \cup S$ forms co - total strong line corporate dominating set such that

$\gamma_{ct}(SLC(G)) \leq p + 1.$ Also we have, $\gamma(SLC(G)) \leq q.$

Case(ii): If *SLC(G)* does not contains any end vertex , then take the minimum set of vertices K having minimum degree such that $< V(SLC(G)) -K >$ has no isolated vertices.

Then, $\gamma_{ct}(SLC(G)) \leq \left\lceil \frac{p+q}{2} \right\rceil$

Also we have, $\gamma(SLC(G)) \leq \frac{p+q}{2}$.

From all the above cases , we have $\gamma_{ct}(SLC(G)) + \gamma(SLC(G)) \leq p + q + 1.$

Remark 4.8. From the proof of the above theorem we see that,

$\gamma(SLC(G)) \leq \gamma_{ct}(SLC(G))$.

Example 4.9.

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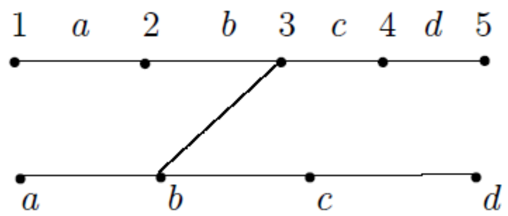

Here , the strong line corporate dominating set is {2,4,b,c} and the co – total strong line corporate dominating set is {1,4,5,a,d}. $\gamma(SLC(G)) = 4$ and

$\gamma_{ct}(SLC(G)) = 5$.

Theorem 4.10. Let *SLC(G)* be any connected strong line corporate graph. Then $\gamma_{ct}(SLC(G))+\gamma^{-1}(SLC(G)) \leq 2p + q + 1$.

Proof. Let $X = \{v_1, v_2, ..., v_{p+q}\}$ be the vertex set of *SLC(G)* .

Let $D \subseteq V(SLC(G))$ be the minimum dominating set of the strong line corporate graph of G.

If D^{-1} exists, then $|D^{-1}| \leq \left\lceil \frac{p+q}{2} \right\rceil$

By using the *theorem 4.7*,we have $S \subseteq V(SLC(G))$ be the co - total dominating set of *SLC(G)* such that $|S| \leq p + q$.

Then, $|D^{-1}| + |S| \leq p + q + \frac{p+q+1}{2}$

$$\leq \frac{2p+2q+p+q+1}{2}$$

$$\leq \frac{3p+3q+1}{2}$$

$$\leq 2p + q + 1.$$

Hence,

$\gamma_{ct}(SLC(G))+\gamma^{-1}(SLC(G)) \leq 2p + q + 1$.

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