ISSN: 2278 – 7798
International Journal of Science, Engineering and Technology Research (IJSETR)
Volume 5, Issue 4, April 2016

Comparative Study of Two Standby Systems with Concept of Priority to Failed Unit

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Abstract— : **The present paper gives the comparative study with respect to MTSF between two models for compressor standby systems. where Model 1 comprises two units and Model 2 comprises three units. In Model 1, initially one unit is in operative state and other is in standby state where as in Model 2 initially two units are in operative state and third unit is in standby state. In both models priority is given to failed unit for service , repair and replacement than any new failure of an operative unit. For comparative analysis of the unit real failure as well as repair data from milk plant have been collected and measures of system effectiveness i.e MTSF for both the models has been computed graphically as well as numerically by using semi Markov process and regenerative point techniques**

Index Terms— **Compressor unit, Regenerative point technique, Refrigeration System , Semi-Markov process**

INTRODUCTION

Standby systems are of great interest in the field of reliability and therefore have been discussed by various researchers including [1-7] under various assumptions/considerations. For graphical study, they have taken assumed as well as real/observed values for failure and repair rates. Though contribution of various researchers in the field of reliability modelling and analysis has made the field very rich, yet comparative study between two types of systems is reported very less in literature. A potential application of the reliability concepts has been recently explored in terms comparative study of standby compressor systems with and without provision of priority to failed compressor unit, thereby achieving some reliability measures of the system effectiveness which in turn are meaningful in understanding the comparative analysis of such systems by U. Sharma and J. kaur. [8].

In present paper both standby systems has been studied under the assumption of priority to failed unit for service repair and replacement . Model 1 consist of two compressor units, where one unit is in operative and other is in standby state. At least one unit should be in operative state for functioning of system. Model 2 consist of three units, where two units are in operative and one unit is in standby state. At least two units should in operative state for functioning of system. Various measures of system effectiveness has been calculated by using semi-Markov process and regenerative point techniques. For practical utility of our proposed model previous data of different years from milk plant has been gathered and is used for graphical analysis .

Manuscript received April, 2016.

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Notations

OI,OII First , Second Compressor are in Operative State

SII, SIII Second and Third Compressors are in Standby state

λ_{i1}, λ_{i2}, λ_{i3}, Failure rate when failure is of serviceable , repairable and replaceable for first , second and third compressor respectively (i= 1,2,3and i symbol used for compressor unit)

α_{i1}, α_{i2}, α_{13} Repair rates when failure is of serviceable , repairable and replaceable type for first, second and third compressor respectively (i=1,2,3)

$G_{i1}(t)$,$g_{i1}(t)$ c.d.f and p.d.f of time for service when failure is of serviceable type for first, second and third compressor respectively

$G_{i2}(t)$,$g_{i2}(t)$ c.d.f and p.d.f of time for service when failure is of repairable type for first, second and third compressor respectively

$G_{i3}(t)$,$g_{i3}(t)$ c.d.f and p.d.f of time for replacement when failure is of replaceable type for first , second and third compressor respectively

Q_{ij} , q_{ij} c.d.f and p.d.f of first passage time from a regenerative state i to j or to a failed state j in (0, t]

p_{ij},p_{ij}^{k} probability of transition from regenerative state i to regenerative state j without visiting any other state in (0,t],visiting state k once in (0,t]

q_{ij}^{k} p.d.f of first passage from regenerative state i to regenerative state j or to failed state j visting k once in (0,t]

$ø_i(t)$ c.d.f of the first passage time from regenerative state i to a failed state.

FsI , FsII ,FsIII Failure category of serviceable type for first, Second and third compressor

FrI,FrII,FrIII Failure category of repairable type for first ,second and third compressor

FrepI ,FrepII ,FrepIII Failure category of replaceable type for First , Second and third compressor

FwrI,FwsI,FwrepI First compressor is waiting for Repair, Service, Replacement respectively

Models Description and Assumptions

1) The unit is initially operative at state 0 and its transition depends upon the type of failure category to any of the three states 1 to 3 with different failure rates.

2) All failure times are assumed to have exponential distribution

3) After each servicing/ repair/replacement at states the unit works as good as new.

4) Priority given to failed unit for service , repair and replacement

Model 1

Transition Probabilities and Mean Sojourn Times

A state transition diagram showing the various states of transition of the system is shown in Fig. 1.The epochs of entry into states 0,1,2,3,13,14 and 15 are regenerative states. States 4,5,6,7,8,9,10,11 and 12 are down states. The non zero elements p_{ij} are given below:

$$p_{01}=\frac{\lambda_{11}}{\lambda^*},p_{02}=\frac{\lambda_{12}}{\lambda^*},p_{03}=\frac{\lambda_{13}}{\lambda^*} where \lambda^*=\lambda_{11}+\lambda_{12}+\lambda_{13}$$

$$p_{10}=g_{11}^*(\lambda),p_{20}=g_{12}^*(\lambda),p_{30}=g_{13}^*(\lambda)$$

$$where \lambda=\lambda_{21}+\lambda_{22}+\lambda_{23};\ p_{27},p_{2,13}^7=\frac{\lambda_{21}}{\lambda}(1-g_{12}^*(\lambda))$$

$$p_{28},p_{2,14}^8=\frac{\lambda_{22}}{\lambda}(1-g_{12}^*(\lambda));\ p_{29},p_{2,15}^9=\frac{\lambda_{23}}{\lambda}(1-g_{12}^*(\lambda))$$

$$p_{14},p_{1,13}^4=\frac{\lambda_{21}}{\lambda}(1-g_{11}^*(\lambda));\ p_{15},p_{1,14}^5=\frac{\lambda_{22}}{\lambda}(1-g_{11}^*(\lambda))$$

$$p_{16},p_{1,15}^6=\frac{\lambda_{23}}{\lambda}(1-g_{11}^*(\lambda));\ p_{3,10},p_{3,13}^{10}=\frac{\lambda_{21}}{\lambda}(1-g_{13}^*(\lambda))$$

$$p_{3,11},p_{3,14}^{11}=\frac{\lambda_{22}}{\lambda}(1-g_{13}^*(\lambda));\ p_{3,12},p_{3,15}^{12}=\frac{\lambda_{23}}{\lambda}(1-g_{13}^*(\lambda))$$

$$p_{10}+p_{14}+p_{15}+p_{16}=1, p_{10}+p_{1,13}^4+p_{1,14}^5+p_{1,15}^6=1$$

$$p_{20}+p_{27}+p_{28}+p_{29}=1, p_{20}+p_{2,13}^7+p_{2,14}^8+p_{2,15}^9=1$$

$$p_{30}+p_{3,10}+p_{3,11}+p_{3,12}=1, p_{30}+p_{3,13}^{10}+p_{3,14}^{11}+p_{3,15}^{12}=1$$

The mean sojourn time (μ_i) in the regenerative state 'i' is defined as time of stay in that state before transition to any other state:

$$\mu_0=\frac{1}{\lambda_{11}+\lambda_{12}+\lambda_{13}},\mu_1=\frac{1}{\lambda_{21}+\lambda_{22}+\lambda_{23}}$$

$$\mu_2=\frac{1}{\lambda_{21}+\lambda_{22}+\lambda_{23}},\mu_3=\frac{1}{\lambda_{21}+\lambda_{22}+\lambda_{23}}$$

$$\mu_4=\int_0^\infty \bar{G}_{21}(t)dt,\mu_5=\int_0^\infty \bar{G}_{22}(t)dt,\mu_6=\int_0^\infty \bar{G}_{23}(t)dt$$

$$\mu_7=\int_0^\infty \bar{G}_{21}(t)dt,\mu_8=\int_0^\infty \bar{G}_{22}(t)dt,\mu_9=\int_0^\infty \bar{G}_{23}(t)dt$$

$$\mu_{10}=\int_0^\infty \bar{G}_{21}(t)dt,\mu_{11}=\int_0^\infty \bar{G}_{22}(t)dt,\mu_{12}=\int_0^\infty \bar{G}_{23}(t)dt$$

The unconditional mean time taken by the system to transit for any regenerative state 'j' when it (time) is counted from the epoch of entrance into state 'i' is mathematically state as:

$$mij=\int_0^\infty t dQ_{ij}(t)=-q_{ij}'^*(0), m_{01}+m_{02}+m_{03}=\frac{1}{(\lambda^*)}=\mu_0$$

$$m_{10}+m_{14}+m_{15}+m_{16}=\mu_1(1-g_{11}^*(\lambda)$$

$$m_{20}+m_{27}+m_{28}+m_{29}=\mu_2(1-g_{12}^*(\lambda)$$

$$m_{30}+m_{3,10}+m_{3,11}+m_{3,12}=\mu_3(1-g_{13}^*(\lambda)$$

Mean Time to System Failure

To determine the mean time to system failure (MTSF) of the system, we regard the failed states of the system as absorbing states. Now mean time to system failure (MTSF) when unit started at the beginning of state 0 is

$$\text{MTSF}=T_0=\lim_{s\to O}\frac{1-\varnothing_0^{**}(s)}{s}=\frac{N}{D},\varnothing_0^{**}(s)=\frac{N(s)}{D(s)}$$

$$\text{Where } N(s)=\varnothing_{03}^{**}(s)\varnothing_{3,10}^{**}(s)+\varnothing_{03}^{**}(s)\varnothing_{3,11}^{**}(s)$$
$$+\varnothing_{03}^{**}(s)\varnothing_{3,12}^{**}(s)+\varnothing_{01}^{**}(s)\varnothing_{14}^{**}(s)+\varnothing_{01}^{**}(s)\varnothing_{15}^{**}(s)$$
$$+\varnothing_{01}^{**}(s)\varnothing_{16}^{**}(s)+\varnothing_{02}^{**}(s)\varnothing_{27}^{**}(s)+\varnothing_{02}^{**}(s)\varnothing_{28}^{**}(s)$$
$$+\varnothing_{02}^{**}(s)\varnothing_{29}^{**}(s)$$
$$D(s)=1-\varnothing_{01}^{**}(s)\varnothing_{10}^{**}(s)-\varnothing_{02}^{**}(s)\varnothing_{20}^{**}(s)-\varnothing_{03}^{**}(s)\varnothing_{30}^{**}(s)$$

Model 2

Transition Probabilities and Mean Sojourn Times

A state transition diagram showing the various states of transition of the system is shown in Fig. 2.The epochs of entry into states 0,1,2,3,22,23,24,25,26 and 27 are regenerative states.

States 4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19, 20 and 21 are down states. The non zero elements p_{ij} are given below:

$$p_{01}=\frac{\lambda_{11}}{\lambda^*},p_{02}=\frac{\lambda_{12}}{\lambda^*},p_{03}=\frac{\lambda_{13}}{\lambda^*} where \lambda^*=\lambda_{11}+\lambda_{12}+\lambda_{13}$$

$$p_{10}=g_{11}^*(\lambda),p_{20}=g_{12}^*(\lambda),p_{30}=g_{13}^*(\lambda)$$

$$p_{14},p_{1,22}^4=\frac{\lambda_{21}}{\lambda}(1-g_{11}^*(\lambda));\ p_{15},p_{1,23}^5=\frac{\lambda_{22}}{\lambda}(1-g_{11}^*(\lambda))$$

$$p_{16},p_{1,24}^6=\frac{\lambda_{23}}{\lambda}(1-g_{11}^*(\lambda));\ p_{17},p_{1,25}^7=\frac{\lambda_{31}}{\lambda}(1-g_{11}^*(\lambda))$$

$$p_{18},p_{1,26}^8=\frac{\lambda_{32}}{\lambda}(1-g_{11}^*(\lambda));\ p_{19},p_{1,27}^9=\frac{\lambda_{33}}{\lambda}(1-g_{11}^*(\lambda))$$

$$p_{2,10},p_{2,22}^{10}=\frac{\lambda_{21}}{\lambda}(1-g_{12}^*(\lambda));\ p_{2,11},p_{2,23}^{11}=\frac{\lambda_{22}}{\lambda}(1-g_{12}^*(\lambda))$$

$$p_{2,12},p_{2,24}^{12}=\frac{\lambda_{23}}{\lambda}(1-g_{12}^*(\lambda)) where \lambda=\lambda_{21}+\lambda_{22}+\lambda_{23}+\lambda_{31}+\lambda_{32}+\lambda_{33}$$

$$p_{2,13},p_{2,25}^{13}=\frac{\lambda_{31}}{\lambda}(1-g_{12}^*(\lambda));\ p_{2,14},p_{2,26}^{14}=\frac{\lambda_{32}}{\lambda}(1-g_{12}^*(\lambda))$$

$$p_{2,15},p_{2,27}^{15}=\frac{\lambda_{33}}{\lambda}(1-g_{12}^*(\lambda));\ p_{3,16},p_{3,22}^{16}=\frac{\lambda_{21}}{\lambda}(1-g_{13}^*(\lambda))$$

$$p_{3,17},p_{3,23}^{17}=\frac{\lambda_{21}}{\lambda}(1-g_{13}^*(\lambda));\ p_{3,18},p_{3,24}^{18}=\frac{\lambda_{21}}{\lambda}(1-g_{13}^*(\lambda))$$

$$p_{3,19},p_{3,25}^{19}=\frac{\lambda_{31}}{\lambda}(1-g_{13}^*(\lambda));\ p_{3,20},p_{3,26}^{20}=\frac{\lambda_{31}}{\lambda}(1-g_{13}^*(\lambda))$$

$$p_{3,21},p_{3,27}^{21}=\frac{\lambda_{31}}{\lambda}(1-g_{13}^*(\lambda));\ p_{01}+p_{02}+p_{03}=1$$

$$p_{10}+p_{14}+p_{15}+p_{16}+p_{17}+p_{18}+p_{19}=1,$$

$$p_{10}+p_{1,22}^4+p_{1,23}^5+p_{1,24}^6+p_{1,25}^7+p_{1,26}^8+p_{1,27}^9=1$$

$$p_{20}+p_{2,10}+p_{2,11}+p_{2,12}+p_{2,13}+p_{2,14}+p_{2,15}=1$$

$$p_{20}+p_{2,22}^{10}+p_{2,23}^{11}+p_{2,24}^{12}+p_{2,25}^{13}+p_{2,26}^{14}+p_{2,27}^{15}=1$$

$$p_{30}+p_{3,16}+p_{3,17}+p_{3,18}+p_{3,19}+p_{3,20}+p_{3,21}=1$$

$$p_{30}+p_{3,22}^{16}+p_{3,23}^{17}+p_{3,24}^{18}+p_{3,25}^{19}+p_{3,26}^{20}+p_{3,27}^{21}=1$$

The mean sojourn time (μ_i) in the regenerative state 'i' is defined as time of stay in that state before transition to any other state:

$$\mu_0=\frac{1}{\lambda^*},\mu_1=\frac{1}{\lambda}$$

$$\mu_2=\frac{1}{\lambda},\mu_3=\frac{1}{\lambda}$$

$$\mu_{22}=\int_0^\infty \bar{G}_{21}(t)dt,\mu_{23}=\int_0^\infty \bar{G}_{22}(t)dt,\mu_{24}=\int_0^\infty \bar{G}_{23}(t)dt$$

$$\mu_{25}=\int_0^\infty \bar{G}_{21}(t)dt,\mu_{26}=\int_0^\infty \bar{G}_{22}(t)dt,\mu_{27}=\int_0^\infty \bar{G}_{23}(t)dt$$

The unconditional mean time taken by the system to transit for any regenerative state 'j' when it (time) is counted from the epoch of entrance into state 'i' is mathematically state as:

$$mij = \int_0^\infty t dQ_{ij}(t) = -q_{ij}'^*(0), m_{01} + m_{02} + m_{03} = \frac{1}{(\lambda^*)} = \mu_0$$

$$m_{10} + m_{14} + m_{15} + m_{16} + m_{17} + m_{18} + m_{19} = \mu_1(1 - g_{11}^*(\lambda))$$

$$m_{10} + m_{1,22}^4 + m_{1,23}^5 + m_{1,24}^6 + m_{1,25}^7 + m_{1,26}^8 + m_{1,27}^9 = \mu_1(1 - g_{11}^*(\lambda))$$

$$m_{20} + m_{2,10} + m_{2,11} + m_{2,12} + m_{2,13} + m_{2,14} + m_{2,15} = \mu_2(1 - g_{12}^*(\lambda))$$

$$m_{20} + m_{2,22}^{10} + m_{2,23}^{11} + m_{2,24}^{12} + m_{2,25}^{13} + m_{2,26}^{14} + m_{2,27}^{15} = \mu_2(1 - g_{12}^*(\lambda))$$

$$m_{30} + m_{3,16} + m_{3,17} + m_{3,18} + m_{3,19} + m_{3,20} + m_{3,21} = \mu_3(1 - g_{13}^*(\lambda))$$

$$m_{30} + m_{3,22}^{16} + m_{3,23}^{17} + m_{3,24}^{18} + m_{3,25}^{19} + m_{3,26}^{20} + m_{3,27}^{21} = \mu_3(1 - g_{13}^*(\lambda))$$

Mean Time to System Failure

To determine the mean time to system failure (MTSF) of the system, we regard the failed states of the system as absorbing states. Now mean time to system failure (MTSF) when unit started at the beginning of state 0 as

$$\text{MTSF} = T_0 = \lim_{s \to O} \frac{1 - \varnothing_0^{**}(s)}{s} = \frac{N}{D}$$

$$\varnothing_0^{**}(s) = \frac{N(s)}{D(s)}$$

Where $N(s) = \varnothing_{03}^{**}(s)\varnothing_{3,16}^{**}(s) + \varnothing_{03}^{**}(s)\varnothing_{3,17}^{**}(s)$
$+\varnothing_{03}^{**}(s)\varnothing_{3,18}^{**}(s) + \varnothing_{03}^{**}(s)\varnothing_{3,19}^{**}(s) + \varnothing_{03}^{**}(s)\varnothing_{3,20}^{**}(s)$
$+\varnothing_{03}^{**}(s)\varnothing_{3,21}^{**}(s) + \varnothing_{01}^{**}(s)\varnothing_{14}^{**}(s) + \varnothing_{01}^{**}(s)\varnothing_{15}^{**}(s)$
$+\varnothing_{01}^{**}(s)\varnothing_{16}^{**}(s) + \varnothing_{01}^{**}(s)\varnothing_{17}^{**}(s) + \varnothing_{01}^{**}(s)\varnothing_{18}^{**}(s)$
$+\varnothing_{01}^{**}(s)\varnothing_{19}^{**}(s) + \varnothing_{02}^{**}(s)\varnothing_{2,10}^{**}(s) + \varnothing_{02}^{**}(s)\varnothing_{2,11}^{**}(s)$
$+\varnothing_{02}^{**}(s)\varnothing_{2,12}^{**}(s)$
$D(s) = 1 - \varnothing_{01}^{**}(s)\varnothing_{10}^{**}(s) - \varnothing_{02}^{**}(s)\varnothing_{20}^{**}(s)$
$-\varnothing_{03}^{**}(s)\varnothing_{30}^{**}(s)$

For graphical representation ,let us suppose that

$$g_{11}(t) = \alpha_{11} e^{-\alpha_{11} t}, g_{12}(t) = \alpha_{12} e^{-\alpha_{12} t}, g_{13}(t) = \alpha_{13} e^{-\alpha_{13} t}$$

using the above particular case, the following values are estimated as

$\alpha_{11} = 0.006896, \alpha_{12} = 0.000586, \alpha_{13} = 0.04166$

$\alpha_{21} = 0.0000983, \alpha_{22} = 0.0001347, \alpha_{23} = 0.00015873$

$\alpha_{31} = 0.000345209, \alpha_{32} = 0.0010162602$

$\alpha_{33} = 0.0006648936 \lambda_{11} = 0.00003868,$

$\lambda_{12} = 0.00003879, \lambda_{13} = 0.00003865$

$\lambda_{21} = 0.0007359, \lambda_{22} = 0.0007367, \lambda_{23} = 0.0007352$

$\lambda_{31} = 0.0000456079, \lambda_{32} = 0.0000456089$

$\lambda_{33} = 0.0000456071$

$N = 8907.480982, D = 0.6391011951 (Model1)$

$N = 8887.14331, D = 0.631107 (Model2)$

Graphical Interpretations

Graphs represents the behaviour of MTSF (Model 1 and Model 2)with failure rate λ_{12} having variation in λ_{22} (Model 1) and failure rate λ_{12} having variation in λ_{22} and λ_{32}(Model 2).It is clear that as failure rate λ_{12} increases MTSF (Model 1 and Model 2) decreases.

The measures of system effectiveness are obtained as:

Mean time to unit/compressor

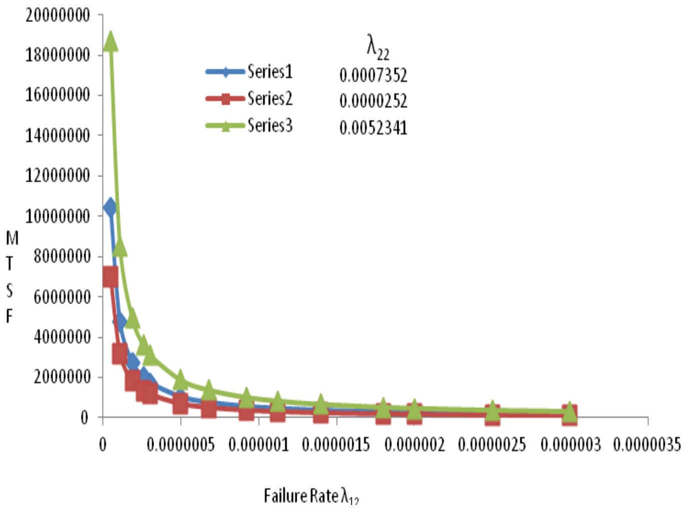
MTSF (Model 1) =13973.51263 Hrs

Mean time to unit/compressor

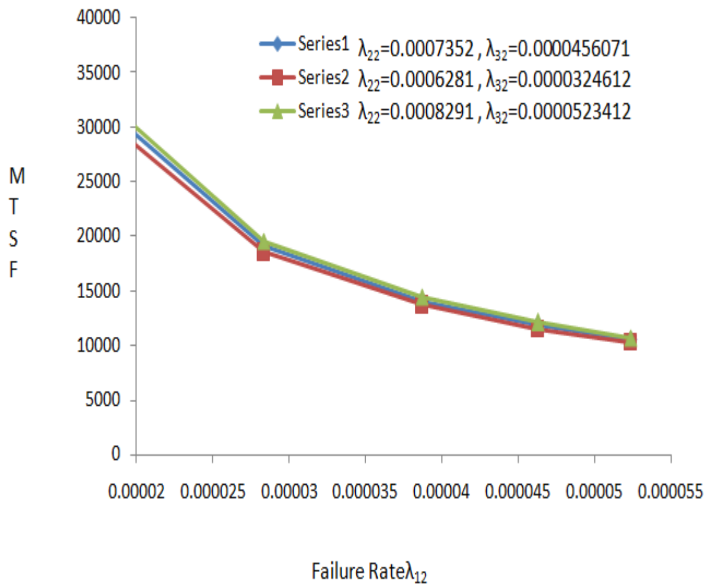
MTSF (Model 2) =14081.8271 hrs.

It can be conclude that for the given system MTSF for Model 2 is greater than Model 1

Graph between MTSF and λ_{12} with variation in λ_{22}. (Model 1)



Graph between MTSF and λ_{12} with variation in λ_{22}, λ_{32} (Model 2)



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Model 1

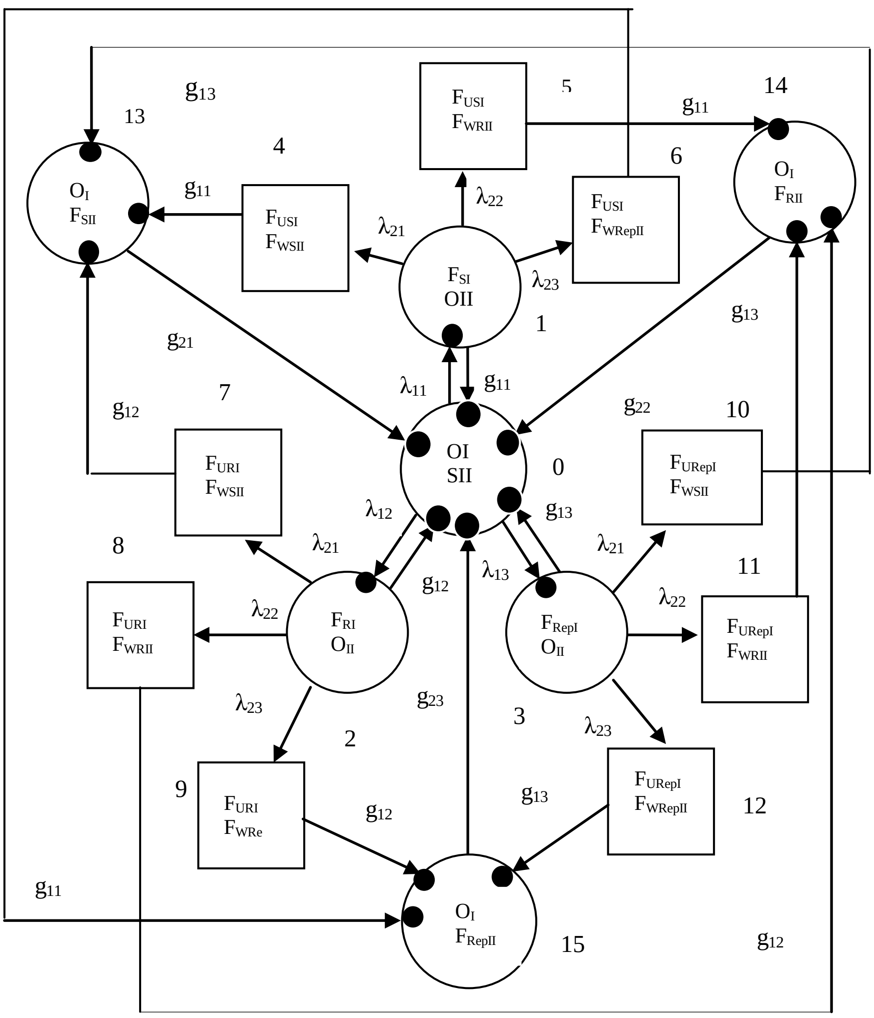

Fig 1

ISSN: 2278 – 7798
International Journal of Science, Engineering and Technology Research (IJSETR)
Volume 5, Issue 4, April 2016

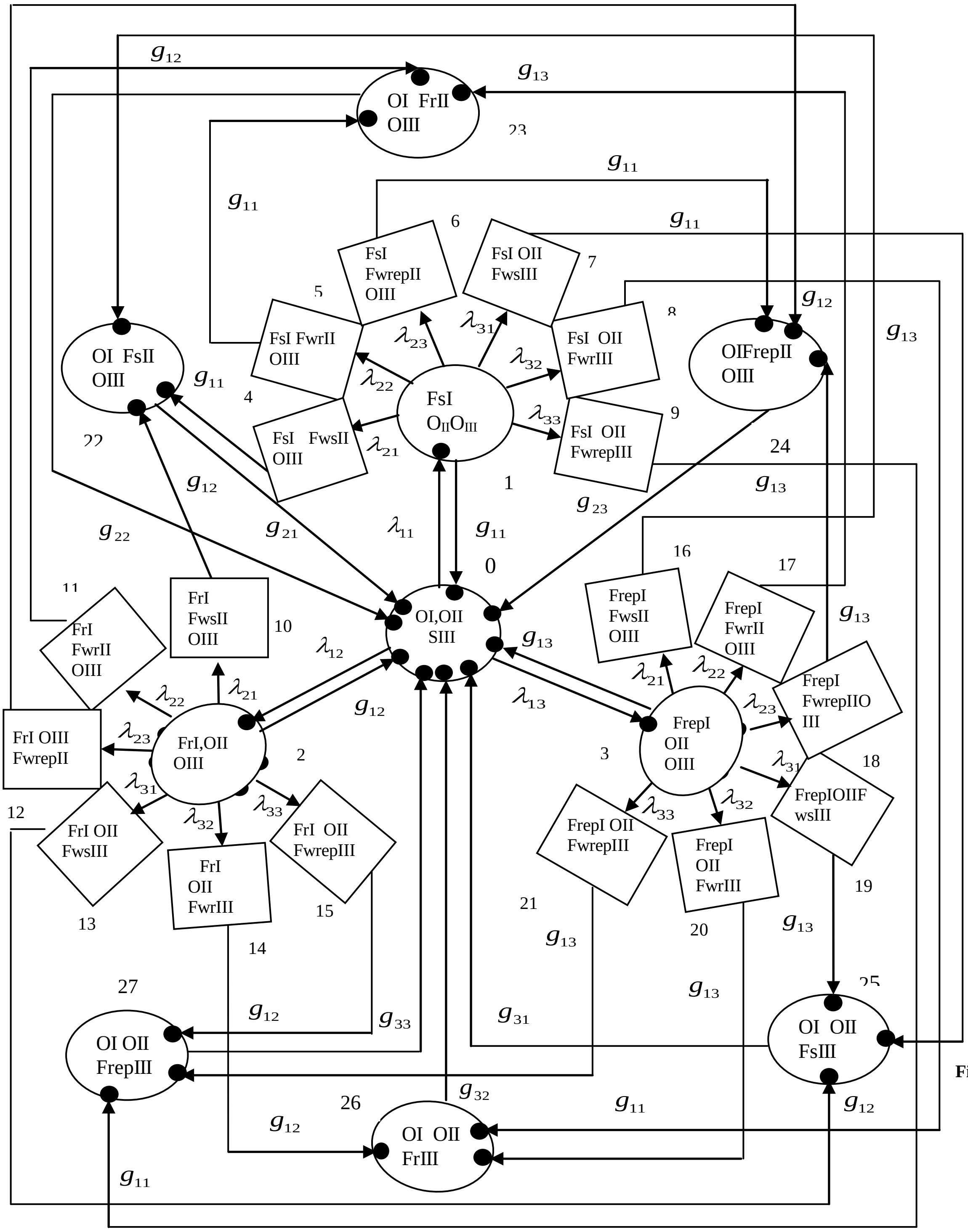

Fig 2

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