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3 - sum cordial labeling of some graphs

N. Deepa[1] and R. Sridevi[2]

[1]M.Phil Scholar, PG and Research Department of Mathematics,
Sri S.R.N.M.College, Sattur-626 203, India.

[2]PG and Research Department of Mathematics, Sri S.R.N.M.College,
Sattur-626 203, India.

ABSTRACT

In this paper, we proved that wheel, helm, fan, n - pan, triangular snake and crown are 3 - sum cordial graphs.

Keywords: Cordial labeling, Sum cordial labeling, 3 - Sum cordial labeling.

1 INTRODUCTION

In this paper, all graphs are finite, simple and undirected. Let $G = (V,E)$ be a graph with $| V | = p$ vertices and $| E | = q$ edges. For all other terminology and notations we follow Harary [1]. A detailed study on variety of applications on graph labeling is carried out in Bloom and Golomb [2]. Cordial graphs were first introduced by Cahit [3] as a weaker version of both graceful graphs and harmonious graphs.

Sum cordial labeling was introduced by J. Shiama [4]. In this paper, we investigate 3 - sum cordial labeling of some graphs like wheel, helm, fan, n - pan, triangular snake and crown.

2 PRELIMINARIES

Definition 2.1. A mapping $f : V (G) \rightarrow \{0, 1\}$ is called **binary vertex labeling** of G and $f(v)$ is called the label of the vertex v of G under f. The induced edge labeling $f^* : E(G) \rightarrow \{0, 1\}$ is given by $f^*(e = uv) = | f(u)-f(v) |$. Let us denote $v_f (0)$, $v_f (1)$ be the number of vertices of G having labels 0 and 1 respectively under f and $e_f (0)$, $e_f (1)$ be the number of edges of G having labels 0 and 1 respectively under f^*.

Definition 2.2. A labeling f of G where $N = \{0, 1\}$ and the induced edge labeling $\bar{f}$ is given by $\bar{f} (u, v) = | f(u) - f(v) |$, $\bar{N} = \{0, 1\}$. We call such a labeling cordial, if the following condition is satisfied: $| v_f (1) - v_f (0) | \leq 1$, $| e_f (1) - e_f (0) | \leq 1$, where $v_f (i)$ and $e_f (i)$, $i = \{0, 1\}$ is the number of vertices and edges of G respectively, with label i (under f and $\bar{f}$ respectively). A graph G is called cordial, if it admits a **cordial labeling**.

Definition 2.3. A binary vertex labeling of a graph G with induced edge labeling $f^* : E(G) \rightarrow \{0, 1\}$ defined by $f^*(uv) = (f(u) + f(v))(mod\ 2)$ is called **sum cordial labeling**, if $| v_f (0) - v_f (1) | \leq 1$ and $| e_f (0) - e_f (1) | \leq 1$. A graph G is **sum cordial**, if it admits sum cordial labeling.

Definition 2.4. Let f be a map from $V (G)$ to $\{0, 1, 2\}$. For each edge uv, assign the label

$(f(u) + f(v))(mod\ 3)$. The map f is called **3 - sum cordial labeling** of G, if $|\ v_f(i) - v_f(j)\ | \leq 1$ and $|\ e_f(i) - e_f(j)\ | \leq 1$ for $i \neq j$ and $i,\ j \in \{0, 1, 2\}$. Any graph which satisfies the 3 - sum cordial labeling is called **3 - sum cordial graph**.

Definition 2.5. The **wheel graph** W_n is defined as the join of $K_1 + C_n$. The vertex corresponding to K_1 is said to be apex vertex, the vertices corresponding to cycle are called rim vertices. The edges corresponding to cycle are called the rim edges and edges joining apex and vertices of the cycle are called spoke edges.

Definition 2.6. The **helm** H_n is the graph obtained from a wheel W_n by attaching a pendant edge to each rim vertex.

Definition 2.7. The **fan** F_n $(n \geq 2)$ is obtained by joining all vertices of the path P_n to a further vertex called the center, and contains $n + 1$ vertices and $2n - 1$ edges.

Definition 2.8. The n - **pan graph** is the graph obtained by joining a cycle graph C_n to a singleton graph K_1 with a bridge.

Definition 2.9. The **triangular snake** T_n is obtained from the path P_n by replacing every edge of a path by a triangle C_3.

Definition 2.10. The **crown** $(C_n \odot K_1)$ is obtained by joining a pendant edge to each vertex of cycle C_n.

3 MAIN RESULTS

Theorem 3.1. Any wheel W_n $(n \geq 3)$ is a 3 - sum cordial graph.

Proof. Let $G = W_n$ be a graph with the vertices $v, v_1, v_2, ..., v_n$.
Then $|\ V(G)\ | = n + 1$ and $|\ E(G)\ | = 2n$.
We define $f : V(G) \rightarrow \{0, 1, 2\}$ as follows :

$f(v) = 2$ and for $1 \leq i \leq n$

$$f(v_i) = \begin{cases} 0, & \text{if } i \equiv 1\ (mod\ 3) \\ 1, & \text{if } i \equiv 2\ (mod\ 3) \\ 2, & \text{if } i \equiv 0\ (mod\ 3) \end{cases}$$

Thus, $|\ v_f(i) - v_f(j)\ | \leq 1$ and $|\ e_f(i) - e_f(j)\ | \leq 1$ for $i \neq j$ and $i, j \in \{0, 1, 2\}$.
Therefore, W_n has 3 - sum cordial labeling for $n \geq 3$.
Hence, W_n $(n \geq 3)$ is a 3 - sum cordial graph.

Illustration 1. 3 - sum cordial labeling of W_6 is shown in the figure 1.

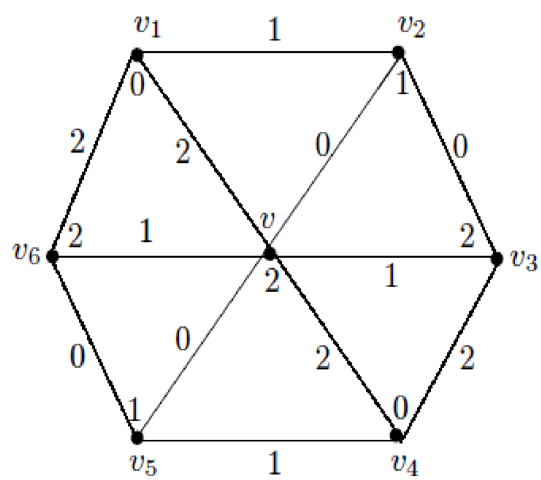

Figure 1

Theorem 3.2. Any helm H_n $(n \geq 3)$ is a 3 - sum cordial graph.

Proof. Let $G = H_n$ be a graph. Let v be an apex vertex and $v_1, v_2, ..., v_n$ are rim vertices. we denote the pendant vertices by $w_1, w_2, ..., w_n$.
Then $|\ V(G)\ | = 2n + 1$ and $|\ E(G)\ | = 3n$.
We define $f : V(G) \rightarrow \{0, 1, 2\}$ as follows :

Case (i) : $n \equiv 0, 2\ (mod\ 3)$

$f(v) = 2$ and for $1 \leq i, j \leq n$

$$f(v_i) = f(w_j) = \begin{cases} 0, & \text{if } i, j \equiv 1 \ (mod\ 3) \\ 1, & \text{if } i, j \equiv 2 \ (mod\ 3) \\ 2, & \text{if } i, j \equiv 0 \ (mod\ 3) \end{cases}$$

Case (ii) : $n \equiv 1 \ (mod\ 3)$

$f(v) = 1, f(w_n) = 2$ and

For $1 \le i \le n$ and $1 \le j \le n - 1$

$$f(v_i) = f(w_j) = \begin{cases} 0, & \text{if } i, j \equiv 1 \ (mod\ 3) \\ 1, & \text{if } i, j \equiv 2 \ (mod\ 3) \\ 2, & \text{if } i, j \equiv 0 \ (mod\ 3) \end{cases}$$

In both cases, $| v_f (i) - v_f (j) | \le 1$ and $| e_f (i) - e_f (j) | \le 1$ for $i \ne j$ and $i, j \in \{0, 1, 2\}$.
Therefore, H_n has 3 - sum cordial labeling for $n \ge 3$.
Hence, $H_n \ (n \ge 3)$ is a 3 - sum cordial graph.

Illustration 2. 3 - sum cordial labeling of H_3 is shown in the figure 2.

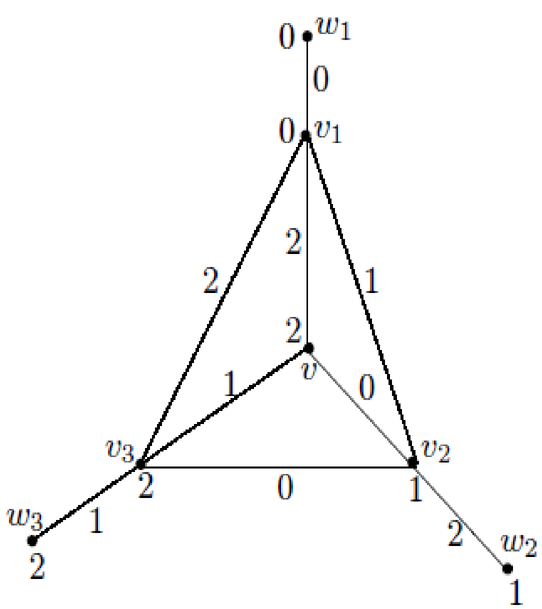

Figure 2

Illustration 3. 3 - sum cordial labeling of H_4 is shown in the figure 3.

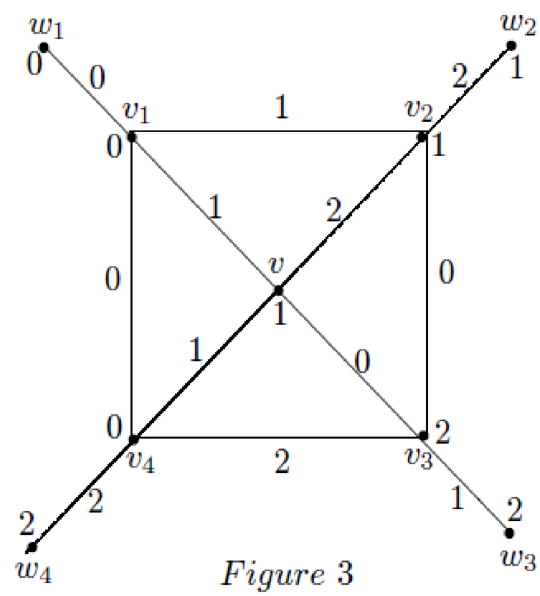

Figure 3

Theorem 3.3. Any fan $F_n \ (n > 2)$ is a 3 - sum cordial graph.

Proof. Let $G = F_n$ be a graph with the vertices $v, v_1, v_2, ..., v_n$.

Then $| V(G) | = n + 1$ and $| E(G) | = 2n - 1$.
We define $f : V(G) \to \{0, 1, 2\}$ as follows :

$f(v) = 1$ or 2 and for $1 \le i \le n$

$$f(v_i) = \begin{cases} 0, & \text{if } i \equiv 1 \ (mod\ 3) \\ 1, & \text{if } i \equiv 2 \ (mod\ 3) \\ 2, & \text{if } i \equiv 0 \ (mod\ 3) \end{cases}$$

Thus, $| v_f (i) - v_f (j) | \le 1$ and $| e_f (i) - e_f (j) | \le 1$ for $i \ne j$ and $i, j \in \{0, 1, 2\}$.
Therefore, F_n has 3 - sum cordial labeling for $n > 2$.
Hence, $F_n \ (n > 2)$ is a 3 - sum cordial graph.

Note 1. For $n = 2$, fix $f(v) = 2$.

Illustration 4. 3 - sum cordial labeling of F_5 is shown in the figure 4.

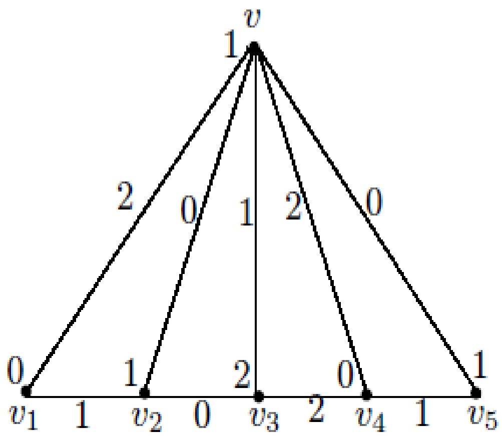

Figure 4

Illustration 5. 3 - sum cordial labeling of F_2 is shown in the figure 5.

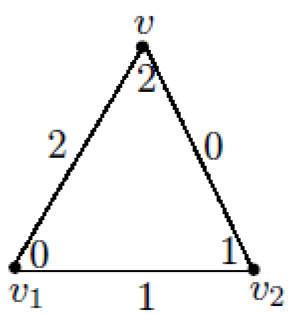

Figure 5

Theorem 3.4. Any n - pan $(n \geq 3)$ is a 3 - sum cordial graph.

Proof. Let G be a graph with the vertices $v, v_1, v_2, \ldots, v_n$.
Then $|V(G)| = n + 1$ and $|E(G)| = n + 1$.
We define $f : V(G) \rightarrow \{0, 1, 2\}$ as follows :

Case (i) : $n \equiv 0, 1 \ (mod\ 3)$

$$f(v = \begin{cases} 0 \text{ or } 2, & \text{if } n \equiv 0 (mod\ 3) \\ 2, & \text{if } n \equiv 1 (mod\ 3) \end{cases}$$

$$f(v_i) = \begin{cases} 0, & \text{if } i \equiv 1\ (mod\ 3) \\ 1, & \text{if } i \equiv 2\ (mod\ 3) \\ 2, & \text{if } i \equiv 0\ (mod\ 3) \end{cases} ;\ 1 \leq i \leq n$$

Case (ii) : $n \equiv 2 \ (mod\ 3)$
Subcase (i) : n is odd

$f(v) = 2$ and for $1 \leq i \leq n$

$$f(v_i) = \begin{cases} 0, & \text{if } i \equiv 1,2\ (mod\ 3) \\ 1, & \text{if } i \equiv 3,4\ (mod\ 3) \\ 2, & \text{if } i \equiv 5,0\ (mod\ 3) \end{cases}$$

Subcase (ii) : n is even

$f(v) = 2$, fix

$$f(v_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq 3 \\ 1, & \text{if } 4 \leq i \leq 6 \\ 2, & \text{if } 7 \leq i \leq 9 \end{cases}$$

and for $10 \leq i \leq n$
`

$$f(v_i) = \begin{cases} 0, & \text{if } i \equiv 4,5\ (mod\ 3) \\ 1, & \text{if } i \equiv 0,1\ (mod\ 3) \\ 2, & \text{if } i \equiv 2,3\ (mod\ 3) \end{cases}$$

In the above cases, $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for $i \neq j$ and $i, j \in \{0, 1, 2\}$.
Therefore, n - pan has 3 - sum cordial labeling for $n \geq 3$.
Hence, n - pan $(n \geq 3)$ is a 3 - sum cordial graph.

Illustration 6. 3 - sum cordial labeling of 4 - pan is shown in the figure 6.

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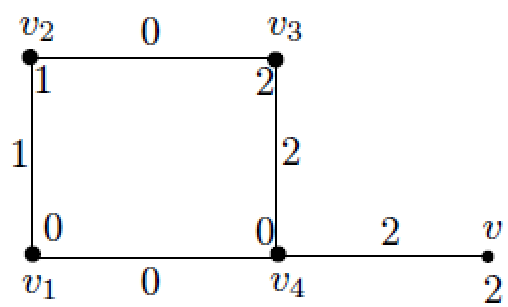

Figure 6

Illustration 7. 3 - sum cordial labeling of 14 - pan is shown in the figure 7.

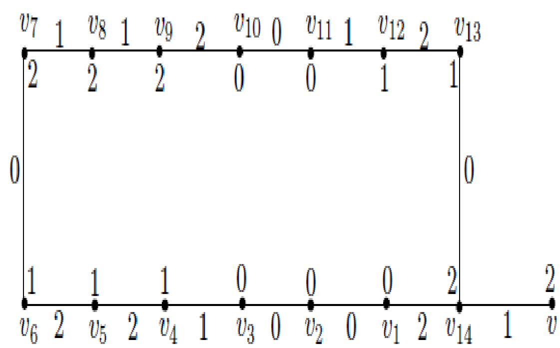

Figure 7

Theorem 3.6. The triangular snake T_n , $(n \geq 2)$ is a 3 - sum cordial graph.

Proof. Let $v_1, v_2, ..., v_n$ and $e_1, e_2, ..., e_{n-1}$ be the vertices and edges of the path P_n respectively.
To construct a triangular snake T_n from the path P_n, join v_i and v_{i+1} to new vertex v_i' by edges $e_{2i-1}' = v_i v_i'$ and $e_{2i}' = v_{i+1} v_i'$ for $i = 1, 2, ..., n - 1$.
Then $| V(T_n) | = 2n - 1$ and $| E(T_n) | = 3n - 3$.
We define $f : V(T_n) \rightarrow \{0, 1, 2\}$ as follows :

For $1 \leq i \leq n$

$$f(v_i) = \begin{cases} 0, & \text{if } i \equiv 1 \ (mod\ 3) \\ 1, & \text{if } i \equiv 2 \ (mod\ 3) \\ 2, & \text{if } i \equiv 0 \ (mod\ 3) \end{cases}$$

and for $1 \leq i \leq n - 1$

$$f(v_i') = \begin{cases} 2, & \text{if } i \equiv 1 \ (mod\ 3) \\ 0, & \text{if } i \equiv 2 \ (mod\ 3) \\ 1, & \text{if } i \equiv 0 \ (mod\ 3) \end{cases}$$

Thus, $| v_f(i) - v_f(j) | \leq 1$ and $| e_f(i) - e_f(j) | \leq 1$ for $i \neq j$ and $i, j \in \{0, 1, 2\}$.
Therefore, T_n has 3 - sum cordial labeling for $n \geq 2$.
Hence, T_n $(n \geq 2)$ is a 3 - sum cordial graph.

Illustration 8. 3 - sum cordial labeling of T_5 is shown in the figure 8.

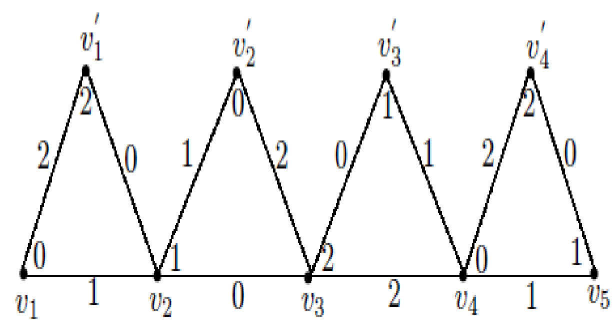

Figure 8

Theorem 3.7. The crown $C_n \odot K_1$ $(n \geq 3)$ is a 3 - sum cordial graph.

Proof. Let $G = C_n \odot K_1$ be a graph. Let $v_1, v_2, ..., v_n$ be the vertices of the cycle C_n and $w_1, w_2, ..., w_n$ be the pendant vertices of $C_n \odot K_1$.
Then $| V(G) | = 2n$ and $| E(G) | = 2n$.
We define $f : V(G) \rightarrow \{0, 1, 2\}$ as follows :

Case (i) : $n \equiv 0 \ (mod\ 3)$

For $1 \leq i , j \leq n$

$$f(v_i) = f(w_j) = \begin{cases} 0, & \text{if } i,j \equiv 1 \ (mod\ 3) \\ 1, & \text{if } i,j \equiv 2 \ (mod\ 3) \\ 2, & \text{if } i,j \equiv 0 \ (mod\ 3) \end{cases}$$

Case (ii) : $n \equiv 1, 2 \ (mod\ 3)$
Subcase (i) : n is odd

For $1 \le i, j \le n$

$$f(v_i) = \begin{cases} 0, & \text{if } i \equiv 1,2 \ (mod\ 3) \\ 1, & \text{if } i \equiv 3,4 \ (mod\ 3) \\ 2, & \text{if } i \equiv 5,0 \ (mod\ 3) \end{cases}$$

$$f(w_j) = \begin{cases} 2, & \text{if } j \equiv 1 \ (mod\ 3) \\ 1, & \text{if } j \equiv 2 \ (mod\ 3) \\ 0, & \text{if } j \equiv 0 \ (mod\ 3) \end{cases}$$

Subcase (ii) : n is even

$f(w_n) = 2$

For $1 \le i \le n$

$$f(v_i) = \begin{cases} 0, & \text{if } i \equiv 1 \ (mod\ 3) \\ 1, & \text{if } i \equiv 2 \ (mod\ 3) \\ 2, & \text{if } i \equiv 0 \ (mod\ 3) \end{cases}$$

and for $1 \le j \le n - 1$

$$f(w_j) = \begin{cases} 2, & \text{if } j \equiv 1 \ (mod\ 3) \\ 0, & \text{if } j \equiv 2 \ (mod\ 3) \\ 1, & \text{if } j \equiv 0 \ (mod\ 3) \end{cases}$$

In the above cases, $| v_f(i) - v_f(j) | \le 1$ and $| e_f(i) - e_f(j) | \le 1$ for $i \neq j$ and $i, j \in \{0, 1, 2\}$.
Therefore, the crown $C_n \odot K_1$ has 3 - sum cordial labeling for $n \ge 3$.
Hence, the crown $C_n \odot K_1$ $(n \ge 3)$ is a 3 - sum cordial graph.

Illustration 9. 3 - sum cordial labeling of $C_3 \odot K_1$ is shown in the figure 9.

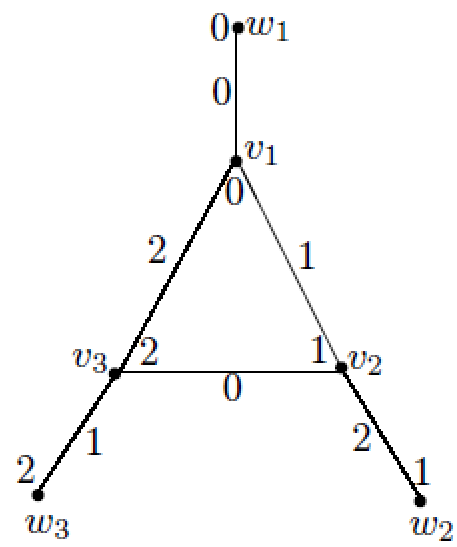

Figure 9

Illustration 10. 3 - sum cordial labeling of $C_4 \odot K_1$ is shown in the figure 10.

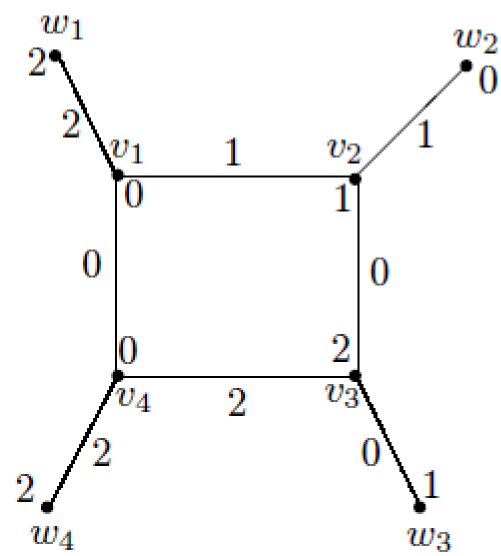

Figure 10

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