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Static Flexural Analysis of Thick Beam Using Hyperbolic Shear Deformation Theory

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Abstract **- In the present study, a hyperbolic shear deformation theory is developed for static flexural analysis of thick isotropic beams. Simply supported thick isotropic beams analyzed for the axial displacement, Transverse displacement, Axial bending stress and transverse shear stress. In this theory the hyperbolic sine and cosine function is used in the displacement field to represent the shear deformation effect and satisfy the zero transverse shear stress condition at top and bottom surface of the beams. The Governing differential equation and boundary conditions of the theory are obtained by using Principle of virtual work. The numerical results have been computed for various lengths to thickness ratios of the beams and the results obtained are compared with those of Elementary, Timoshenko, trigonometric and other higher order refined theories and with the available solution in the literature.**

Key Words **- Isotropic beam, hyperbolic shear deformation theory, principle of virtual work, shear deformation, static flexure, transverse shear stress, thick beam.**

I. INTRODUCTION

The wide spread use of shear flexible materials in aircraft, automotive, shipbuilding and other industries has stimulated interest in the accurate prediction of structural behaviour of beams. Theories of beams involve the reduction of a three dimensional problems of elasticity theory to a one-dimensional problems. Since the thickness dimension is much smaller than the longitudinal dimension, it is possible to approximate the distribution of the displacement, strain and stress components in the thickness dimension. The various methods of development of refined theories based on the reduction of the three dimensional problems of mechanics of elastic bodies are discussed by Gol denveizer [1], Kil chevskiy [2], Donnell [3], Vlasov and Leontev [4], Sayir and Mitropoulos [5].

It is well-known that elementary theory of bending of beam based on Euler-Bernoulli hypothesis that the plane sections which are perpendicular to the neutral layer before bending remain plane and perpendicular to the neutral layer after bending, implying that the transverse shear and transverse normal strains are zero. Thus, the theory disregards the effects of the shear deformation. It is also known as classical beam theory. The theory is applicable to slender beams and should not be applied to thick or deep beams. When elementary theory of beam (ETB) is used for the analysis thick beams, deflections are underestimated and natural frequencies and buckling loads are overestimated. This is the consequence of neglecting transverse shear deformations in ETB. Rankine [6], Bresse [7] were the first to include both the rotatory inertia and shear flexibility effects as refined dynamical effects in beam theory. This theory is, referred as Timoshenko beam theory as mentioned in the literature by Rebello, et.al. [8] and based upon kinematics it is known as first-order shear deformation theory (FSDT).

Stephen and Levinson [9] have introduced a refined theory incorporating shear curvature, transverse direct stress and rotatory inertia effects. The limitations of the elementary theory of bending (ETB) of beams and first order shear deformation theory (FSDT) for beams forced the development of higher order shear deformation theories.

Ghugal and Dahake [10] have developed a trigonometric shear deformation theory for flexure of thick or deep beams, taking into account transverse shear deformation effect. The number of variables in the present theory is same as that in the first order shear deformation theory. The sinusoidal function is used in displacement field in terms of thickness coordinate to represent the shear deformation effects. This theory obviates the need of shear correction factor. Ghugal and Sharma, Sayyad and Ghugal developed a variationally consistent refined hyperbolic shear deformation theory for flexure and free vibration of thick isotropic beam. This theory takes into account transverse shear deformations effects. In this paper, a hyperbolic shear deformation theory is developed for static flexural analysis of thick isotropic beams. The theory is applied to a Simply supported thick isotropic beams to analysed the axial displacement, Transverse displacement, Axial bending stress and transverse shear stress. The numerical results obtained for various lengths to thickness ratios of the beams and the results obtained are compared with those of Elementary, Timoshenko, Trigonometric and other higher order refined theories and with the available solution in the literature.

II. FORMULATION OF PROBLEM

Consider a thick isotropic Cantilever beam of length L in x direction, Width b in y direction and depth h as shown in Figure-1. Where x, y, z are Cartesian coordinates. The beam is subjected to transverse load of intensity q(x) per unit length of beam. Under this condition, the axial displacement, Transverse displacement, Axial bending stress and transverse shear stress are required to be determined.

A. Assumptions made in the theoretical formulation:

1. The axial displacement (u) consist of two parts:

a. Displacement given by elementary theory of bending.

b. Displacement due to shear deformation, which is assume to be hyperbolic in nature with respect to thickness coordinate.

2. The transverse displacement (w) in z direction is assumed to be function of x coordinate.

3. One-dimensional constitutive laws are used.

4. The beam is subjected to lateral load only.

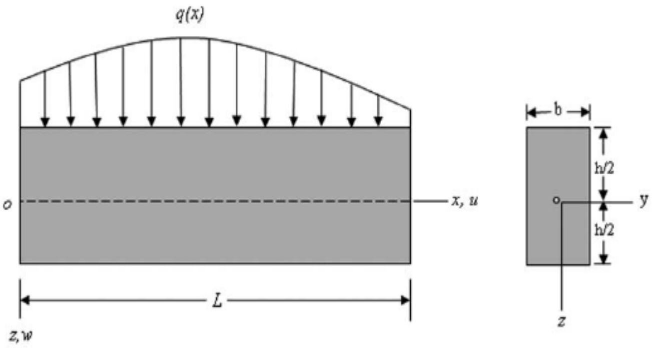

Fig-1: Cantilever beam bending under x-z plane

B. The Displacement Field:

Based on the above mentioned assumptions, the displacement field of the present beam theory can be expressed as follows. The hyperbolic function is assigned according to the shearing stress distribution through the thickness of beam.

$$u(x,z,t) = -z\frac{\partial w}{\partial x}(x,t) + \left[h\sinh\left(\frac{z}{h}\right) - \frac{4}{3}\frac{z^2}{h^2}\cosh\left(\frac{1}{2}\right)\right]\phi(x,t) \quad [01]$$

$$w(x,t) = w(x,t) \quad [02]$$

Where,
u = Axial displacement in x direction which is a function of x, z and t.
w = Transverse displacement in z direction which is function of x and t.
∅ = Rotation of cross section of beam at neutral axis due to shear which is an unknown function to be determined and it is function of x and t.

Normal strain:

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

$$\varepsilon_x = -z\frac{\partial^2 u}{\partial x^2} + \left[h\sinh\left(\frac{z}{h}\right) - \frac{4}{3}\frac{z^3}{h^2}\cosh\left(\frac{1}{2}\right)\right]\frac{\partial \phi}{\partial x} \quad [03]$$

Shear strain:

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

$$\gamma_{xz} = \left[\cosh\left(\frac{z}{h}\right) - 4\frac{z^3}{h^2}\cosh\left(\frac{1}{2}\right)\right]\phi \quad [04]$$

Stresses:
The one-dimensional Hooke's law is applied for isotropic material, stress x is related to strain x and shear stress is related to shear strain by the following constitutive relations.

$$\sigma_x = E\varepsilon_x$$

$$= -zE\frac{\partial^2 u}{\partial x^2} + E\left[h\sinh\left(\frac{z}{h}\right) - \frac{4}{3}\frac{z^3}{h^2}\cosh\left(\frac{1}{2}\right)\right]\frac{\partial \phi}{\partial x} \quad [05]$$

$$\tau_{xz} = G\gamma_{xz}$$

$$= G\left[\cosh\left(\frac{z}{h}\right) - 4\frac{z^3}{h^2}\cosh\left(\frac{1}{2}\right)\right]\phi \quad [06]$$

Where E and G are the elastic constants of the beam material.

C. Governing Differential Equations:

Governing differential equations and boundary conditions are obtained from Principle of virtual work. Using equations for stresses, strains and principle of virtual work. Using equations for stresses, strains and principle of virtual work, variationally consistent differential equations for beam under consideration are obtained. The principle of virtual work when applied to beam leads to:

$$b\int_{x=0}^{x=L}\int_{z=-h/2}^{z=h/2}\left(\sigma_x.\delta\varepsilon_x + \tau_{xz}.\delta\gamma_{xz}\right)dxdz + \rho b\int_{x=0}^{x=L}\int_{z=-h/2}^{z=h/2}\left(\frac{\partial^2 u}{\partial t^2}.\delta u + \frac{\partial^2 w}{\partial t^2}.\delta w\right)dxdz - \int_{x=0}^{x=L} q\delta w dx = 0 \quad [07]$$

Where δ = variational operator.

Employing Greens theorem in above equation successively, we obtained the coupled Euler-Langrange equations, which are the governing differential equations and associated boundary conditions of the beam. The governing differential equations obtained are as follows:

$$EI\left[\frac{\partial^4 w}{\partial x^4} - A_0\frac{\partial^3 \phi}{\partial t^3}\right] - \rho I\left[\frac{\partial^4 w}{\partial x^2 \partial t^2} - A_0\frac{\partial^3 \phi}{\partial x \partial t^2}\right] + \rho A\frac{\partial^2 w}{\partial t^2} = q(x,t) \quad [08]$$

$$EI\left[A_0\frac{\partial^3 w}{\partial x^3} - B_0\frac{\partial^2 \phi}{\partial t^2}\right] - \rho I\left[A_0\frac{\partial^3 w}{\partial x \partial t^2} - B_0\frac{\partial^2 \phi}{\partial t^2}\right] + GAC_0\phi = 0 \quad [09]$$

Where A_0, B_0 and C_0 are the stiffness coeffcients in governing equations. The associated consistent natural boundary conditions obtained are of following form along the edges x = 0 and x = L.

$$EI\left[\frac{d^3w}{dx^3}-A_0\frac{d^2\phi}{dx^2}\right]-\rho I\left[\frac{d^3w}{dxdt^2}-A_0\frac{d^2\phi}{dt^2}\right]=0 \qquad [10]$$

Where w is prescribed.

$$EI\left[\frac{d^2w}{dx^2}-A_0\frac{d\phi}{dx}\right]=0 \qquad [11]$$

Where $\frac{dw}{dx}$ is prescribed.

$$EI\left[A_0\frac{d^2w}{dx^2}-B_0\frac{d\phi}{dx}\right]=0 \qquad [12]$$

Where ∅ is prescribed.

The flexural behaviour of beam is given by solution of above equations 8 and 9 by discarding all terms containing time derivatives and satisfying the associate boundary conditions. The stiffness coefficient used in governing equations 8, 9, 10, 11 and 12 are described as below:

$$A_0=\left[12\cosh\left(\frac{1}{2}\right)-24\sinh\left(\frac{1}{2}\right)-\frac{1}{5}\cosh\left(\frac{1}{2}\right)\right] \qquad [13]$$

$$B_0=\left\{\begin{array}{l}6\left[\sinh(1)-1\right]-200\cosh^2\left(\frac{1}{2}\right)\\+432\sinh\left(\frac{1}{2}\right)\cosh\left(\frac{1}{2}\right)+\left(\frac{1}{21}\right)\cosh^2\left(\frac{1}{2}\right)\end{array}\right\} \qquad [14]$$

$$C_0=\left\{\begin{array}{l}\left(\frac{1}{2}\right)\left[\sinh(1)-1\right]+16\cosh^2\left(\frac{1}{2}\right)\\-36\sinh\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)\cosh\left(\frac{1}{2}\right)+\left(\frac{1}{5}\right)\cosh^2\left(\frac{1}{2}\right)\end{array}\right\} \qquad [15]$$

D. The General solution of Governing equilibrium equations of beam:

The general solution for transverse displacement w(x) and φ(x) can be obtained from equation 8 and 9 by discarding the terms containing time (t) derivatives. Integrating and rearranging the equation 8, we obtained the following equation

$$\frac{d^3w}{dx^3}=A_0\frac{d^2\phi}{dx^2}+\frac{Q(x)}{D} \qquad [16]$$

Where, Q(x) is generalized shear force for beam.

$$Q(x)=\int_0^x q\,dx+k_1 \qquad [17]$$

The second governing equation 9 can be written as:

$$\frac{d^3w}{dx^3}=\frac{B_0}{A_0}\frac{d^2\phi}{dx^2}-\beta\phi \qquad [18]$$

.6 and 18 a single equation in terms of φ is obtained as:

$$\frac{d^2\phi}{dx^2}-\lambda^2\phi=\frac{Q(x)}{D} \qquad [19]$$

The general solution of equation 19 is as follows:

$$\phi=k_2\cosh\lambda x+k_3\sinh\lambda x-\frac{Q(x)}{\beta D} \qquad [20]$$

Where the constants α, β, λ and D used in above equations are given below:

$$\alpha=\left(\frac{B_0}{A_0}-A_0\right),\beta=\left(\frac{GAC_0}{DA_0}\right)$$

$$\lambda^2=\frac{\beta}{\alpha},D=EI$$

The equation of transverse displacement w(x) is obtained by substituting the expression of φ(x) in equation 18 and integrating it thrice with respect to x. The general solution for w(x) is obtained as follows:

$$EIw(x)=\int\int\int\int qdxdxdxdx+\frac{D}{\lambda^3}\left(\frac{B_0}{A_0}\lambda^2-\beta\right)$$
$$\left(k_2\sinh\lambda x+k_3\cosh\lambda x\right)\frac{k_1x^3}{6}+k_4\frac{x^2}{6}+k_5x+k_6 \qquad [21]$$

Where k_1, k_2, k_3, k_4, k_5 and k_6 are the constants of integration and can be obtained by applying the boundary conditions of the beams.

3. ILLUSTARTIVE EXAMPLE

In order to prove the efficiency of the present theory, the following numerical examples are considered. The following material properties for beam are used.

Material properties:

1. Modulus of Elasticity E = 210GPa
2. Poissions ratio = 0.30
3. Density =7800 Kg/m3

A. Example 1: Cantilever beam with uniformly load q(x) = $q_0(x)$

A cantilever beam with the origin of beam on left end support at x = 0. The beam is subjected to uniformly distributed load of q(x) over the span L on surface z = h/2 acting in the z direction is given by,

$$q(x)=q_0(x)$$

The boundary conditions associated with this Problem are as follows:

At fixed end (x = L):

$$EI\frac{d^3w}{dx^3}=0 \qquad EI\frac{d^2\phi}{dx^2}=0$$
$$EI\frac{d^2w}{dx^2}=0 \qquad EI\frac{d\phi}{dx}=0$$

At free end (x = 0):

$$EI\phi=0 \qquad EI\frac{dw}{dx}=0$$
$$EIw=0$$

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General expressions obtained for w(x) and φ(x) are as follows:

$$\phi(x)=\frac{1}{2}\times\frac{A_0}{C_0}\times\frac{q_0 L}{Gbh}\times\left(\sinh\lambda x-\cosh\lambda x+1-\frac{x^2}{L^2}\right) \quad [22]$$

$$w(x)=\left(\frac{x^5}{L^5}-10\frac{x^3}{L^3}+20\frac{x^2}{L^2}\right)+10\frac{B_0}{C_0}\frac{E}{G}\frac{h^2}{L^2}\left(\frac{1}{2}\frac{x^2}{L^2}-\frac{1}{6}\frac{x^3}{L^3}\right)$$
$$+5\frac{E}{G}\frac{h^2}{L^2}\frac{A_0^2}{C_0}\times\left(\frac{\cosh\lambda x-\sinh\lambda x-1}{\lambda L}+\frac{x}{L}\right) \quad [23]$$

The axial displacement, stresses and transverse shear stress obtained based on above solutions are as follows:

$$\bar{u}=-\frac{z}{h}\frac{1}{10}\frac{L^3}{h^3}\left\{\begin{array}{l}\left(5\frac{x^4}{L^4}-30\frac{x^2}{L^2}+40\frac{x}{L}\right)+10\frac{E}{G}\frac{h^2}{L^2}\frac{B_0}{C_0}\left(\frac{x}{L}-\frac{1}{2}\frac{x^2}{L^2}\right)\\ +5\frac{E}{G}\frac{h^2}{L^2}\frac{A_0^2}{C_0}(\sinh\lambda x-\cosh\lambda x+1)\end{array}\right\}$$
$$+\frac{1}{2}\frac{A_0}{C_0}\frac{E}{G}\frac{L}{h}\left(\sinh\frac{z}{h}-\frac{4}{3}\frac{z^3}{h^3}\cosh\frac{1}{2}\right)\left(\sinh\lambda x-\cosh\lambda x+1-\frac{x^2}{L^2}\right) \quad [24]$$

$$\bar{\sigma}_x=-\frac{z}{h}\frac{1}{10}\frac{L^2}{h^2}\left\{\begin{array}{l}\left(20\frac{x^3}{L^3}-60\frac{x}{L}+40\right)+10\frac{E}{G}\frac{h^2}{L^2}\frac{B_0}{C_0}\left(1-\frac{x}{L}\right)\\ +5\frac{E}{G}\frac{h^2}{L^2}\frac{A_0^2}{C_0}(\lambda L\cosh\lambda x-\lambda L\sinh\lambda x)\end{array}\right\}$$
$$+\frac{1}{2}\frac{A_0}{C_0}\frac{E}{G}\left(\sinh\frac{z}{h}-\frac{4}{3}\frac{z^3}{h^3}\cosh\frac{1}{2}\right)\left(\lambda L\cosh\lambda x-\lambda L\sinh\lambda x-2\frac{x}{L}\right) \quad [25]$$

$$\bar{\tau}_{zx}^{EE}=\frac{1}{80}\frac{L}{h}\left(4\frac{z^2}{h^2}-1\right)\left\{\begin{array}{l}\left(60\frac{x^2}{L^2}-60\right)-10\frac{E}{G}\frac{h^2}{L^2}\frac{B_0}{C_0}\\ +5\frac{E}{G}\frac{h^2}{L^2}\frac{A_0^2}{C_0}\left(\lambda^2L^2\sinh\lambda x-\lambda^2L^2\cosh\lambda.\right.\end{array}\right.$$
$$+\frac{1}{2}\frac{A_0}{C_0}\frac{E}{G}\frac{h}{L}\left(\lambda^2L^2\sinh\lambda x-\lambda^2L^2\cosh\lambda x-2\right)\times$$
$$\left[\frac{1}{48}\cosh\left(\frac{1}{2}\right)\left(16\frac{z^4}{h^4}-1\right)+\cosh\left(\frac{1}{2}\right)-\cosh\left(\frac{z}{h}\right)\right] \quad [26]$$

$$\bar{\tau}_{zx}^{CR}=\frac{1}{2}\frac{A_0}{C_0}\frac{L}{h}\left[\cosh\left(\frac{z}{h}\right)-4\frac{z^2}{h^2}\cosh\left(\frac{1}{2}\right)\right]$$
$$\times\left(\sinh\lambda x-\cosh\lambda x+1-\frac{x^2}{L^2}\right) \quad [27]$$

IV. NUMERICAL RESULTS

The numerical results for axial displacement, transverse displacement, bending stress and transverse shear stress are presented in following non-dimensional form and the values are presented in Table-1 and Table -2

$$\bar{w}=\frac{10Ebh^3}{q_0L^4};\bar{u}=\frac{Eb}{q_0h}u$$
$$\bar{\sigma}x=\frac{b\sigma_x}{q_0};\bar{\tau}_{zx}\frac{b\tau_{zx}}{q_0}$$

Table-I: Non-Dimensional Axial Displacement $\bar{u}$ at (x=0, z=h/2), Transverse Deflection $\bar{w}$ at (x=L, z=0), Axial Stress $\bar{\sigma}_x$ at (x=0, z=h/2), Maximum Transverse Shear Stresses $\bar{\tau}_{zx}^{CR}$ and $\bar{\tau}_{zx}^{EE}$ (x=0, z=0) of the cantilever Beam Subjected to Varying Load for Aspect Ratio 4.

Source	Model	$\bar{w}$	$\bar{u}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
Present	HPSDT	12.6191	54.277	42.9196	1.5370	2.0166
Dahake	TSDT	12.6171	54.276	42.5404	1.4762	1.9477
Krishna Murty	HSDT	12.6191	54.277	42.9252	1.5379	2.0177
Timoshenko	FSDT	10.6750	48.000	32.0000	0.9772	2.9997
Bernoulli-Euler	ETB	11.0000	48.000	32.0000	----	2.9997

Table-II: Non-Dimensional Axial Displacement $\bar{u}$ at (x=0, z=h/2), Transverse Deflection $\bar{w}$ at (x=L, z=0), Axial Stress $\bar{\sigma}_x$ at (x=0, z=h/2), Maximum Transverse Shear Stresses $\bar{\tau}_{zx}^{CR}$ and $\bar{\tau}_{zx}^{EE}$ (x=0, z=0) of the cantilever Beam Subjected to Varying Load for Aspect Ratio 10.

Source	Model	$\bar{w}$	$\bar{u}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
Present	HPSDT	11.260	765.69	224.93	6.2556	3.1831
Dahake	TSDT	11.260	765.69	223.98	6.2076	2.7657
Krishna Murty	HSDT	11.260	765.69	224.94	6.2563	3.1884
Timoshenko	FSDT	10.948	750.00	200.00	15.269	7.4992
Bernoulli-Euler	ETB	11.000	750.00	200.00	----	2.9997

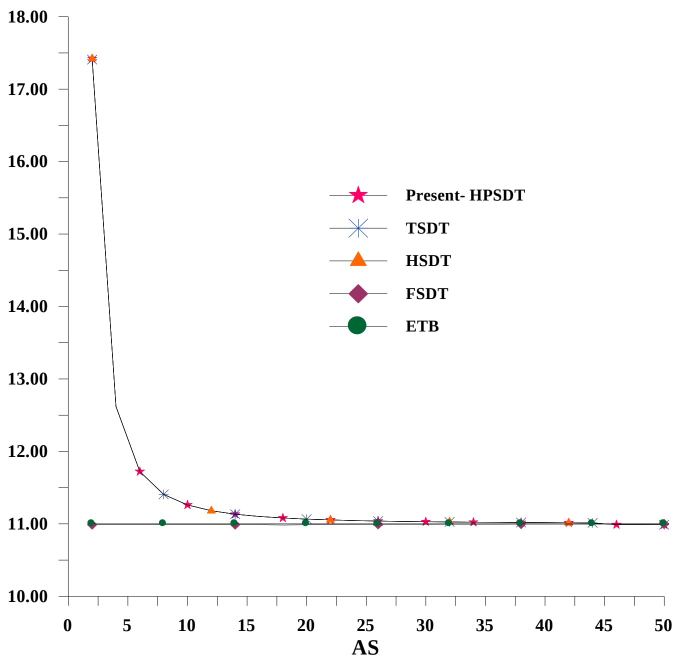

Fig-2: Variation of Transverse Displacement $\bar{w}$

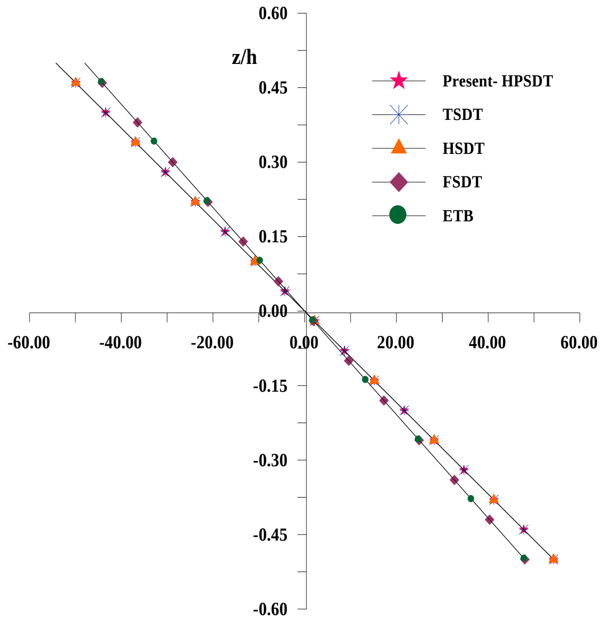

Fig-3: Variation of Maximum Axial displacement $\bar{u}$ for AS 04

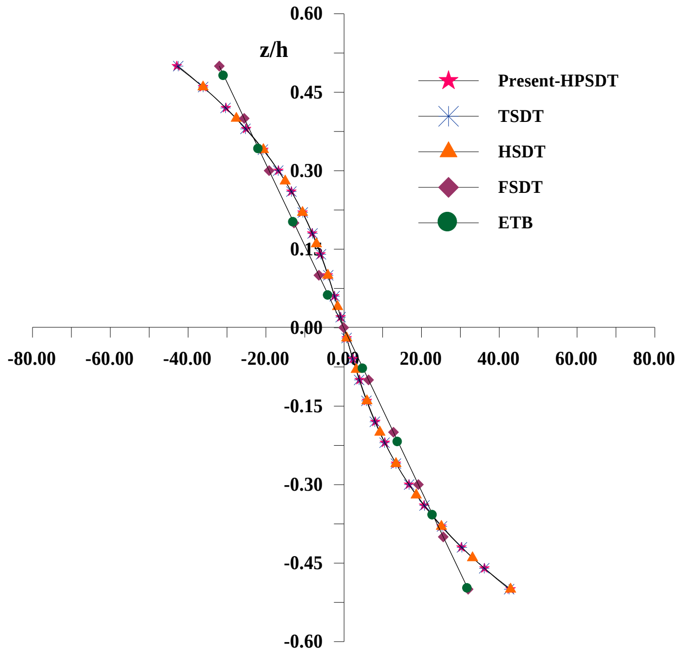

Fig-5: Variation of Maximum Axial stress $\overline{\sigma}_x$ for AS 04

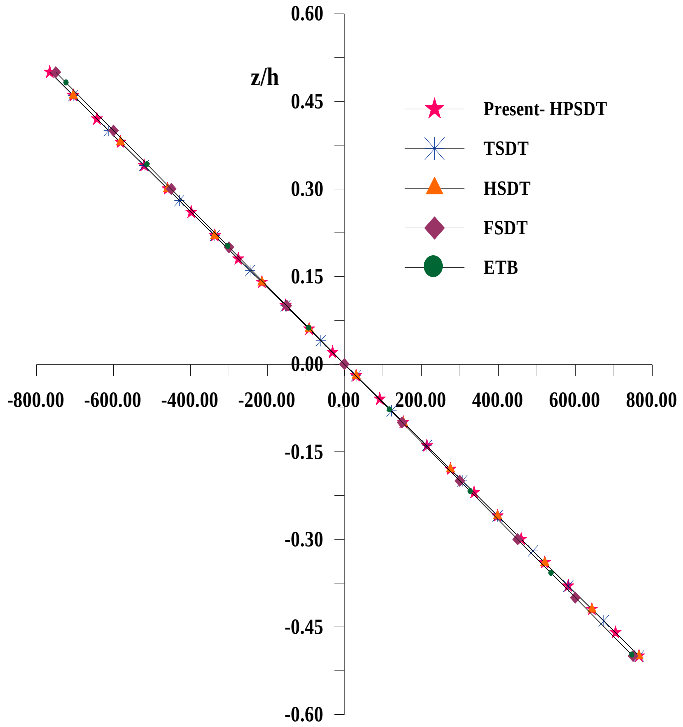

Fig-4: Variation of Maximum Axial displacement $\bar{u}$ for AS 10

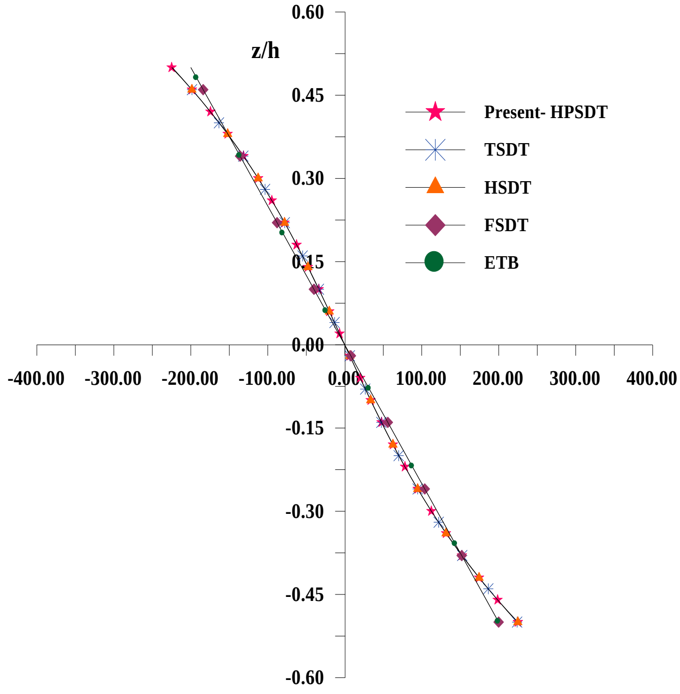

Fig-6: Variation of Maximum Axial stress $\overline{\sigma}_x$ for AS 10

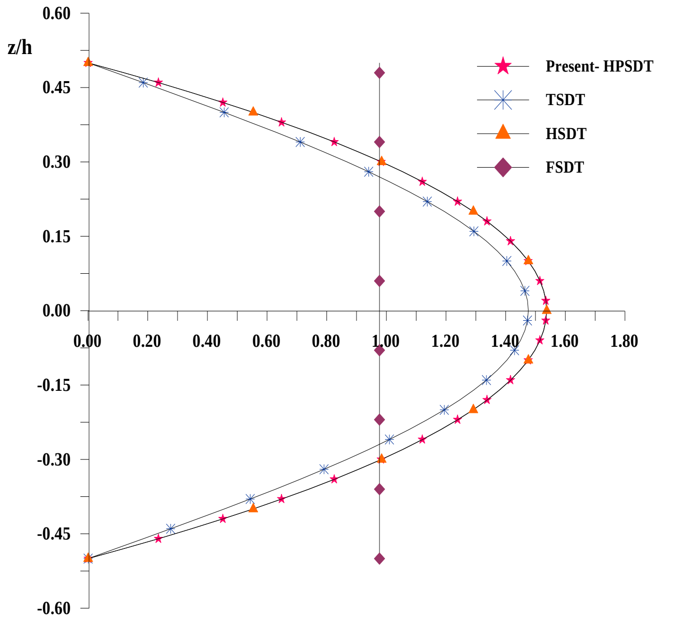

Fig-7: Variation of Transverse shear stress $\bar{\tau}_{zx}^{CR}$ for AS 04

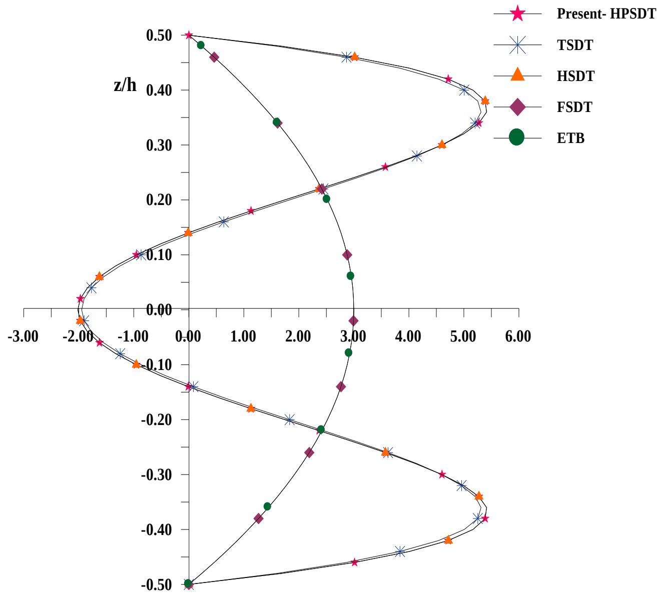

Fig-9: Variation of Transverse shear stress $\bar{\tau}_{zx}^{EE}$ for AS 04

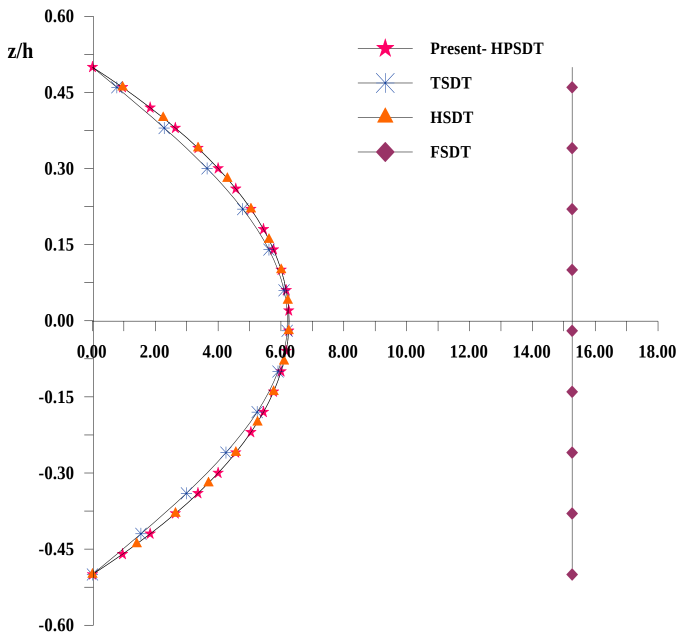

Fig-8: Variation of Transverse shear stress $\bar{\tau}_{zx}^{CR}$ for AS 10

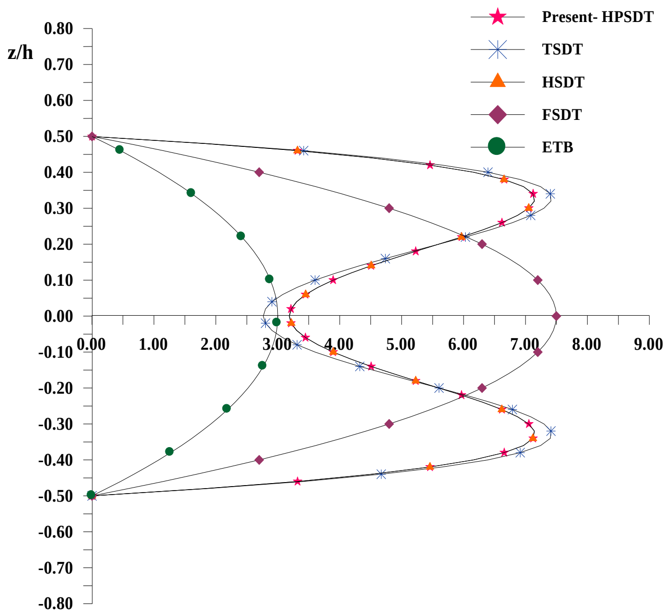

Fig-10: Variation of Transverse shear stress $\bar{\tau}_{zx}^{EE}$ for AS 10

V. CONCLUDING REMARK

From the static flexural analysis of cantilever beam following conclusion are drawn:

1. The result of maximum transverse displacement $\bar{w}$ obtained by present theory is in excellent agreement with those of other equivalent refined and higher order theories. The variation of for AS 4 and 10 are presented as shown in Fig-2.
2. From Fig-3 and Fig-4, it can be observed that, the result of axial displacement $\bar{u}$ for beam subjected to uniform load varies linearly through the thickness of beam for AS 4 and 10 respectively.
3. The maximum Non-dimensional axial stresses $\bar{\sigma}_x$ for AS 4 and 10 varies linearly through the thickness of beam as shown in Figure 5 and Figure 6 respectively.
4. The transverse shear stresses $\bar{\tau}_{zx}^{EE}$ and $\bar{\tau}_{zx}^{CR}$ are obtained directly by constitutive relation. Fig-7, 8, 9 and Fig-10 shows the through thickness variation of transverse shear stress for thick isotropic beam for AS 4 and 10. From this fig it can be observed that, the transverse shear stress satisfy the zero condition at top (z=h/2) and at bottom (z=-h/2) surface of the beam.

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