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A newsvendor framework of transaction exposure model with uniformly distributed exchange rate error and isoelastic demand

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***Abstract*— In a global supply chain consisting of one retailer and one manufacturer, both from different countries, when there is a time lag between the payments made while placing the order and the time when the order is realized, risk in the form of the exchange rate fluctuation affects the optimal pricing and order quantity decisions. We elaborate uniformly distributed exchange rate fluctuation when the retailer or manufacturer undertakes to share the exchange rate risk and the demand error is modelled in the multiplicative form in the news vendor framework. We also have compared the exchange rate effect with the generalized beta distribution error in the model given in Arcelus, Gor and Srinivasan [1]. This is accomplished by numerical example using maple software and nonlinear optimization method to compare the two scenarios of the retailer and manufacturer.**

Index Terms – transaction exposure, exchange rate error, newsvendor problem, optimal pricing and quantity

I. INTRODUCTION

Suppose two different countries having different currencies are into a business. When the exchange rate between the two currencies gets an exposure to unexpected changes, there exists a financial risk and this risk is known as foreign exchange risk (or exchange rate risk). A transaction exposure arises only when there exists a time lag between the time of the financial obligation has been incurred and the time its due to be settled. This is because of the purchase price to buyer/ retailer on the settlement day may differ from that when it was incurred, if the debt is denominated in the manufacturer currency. Arcelus, Gor and Srinivasan [1] have developed a mathematical model in news vendor framework to find optimum ordering and pricing policies for retailer/manufacturer, when the foreign exchange rate between the two countries doing the business, faces transaction exposure. The complete derivation of optimum policies and expected profit of the foreign exchange model for multiplicative demand error is given in Sanjay Patel, Ravi Gor [2]. Our main contribution in this paper is to explain the effect of uniform distribution in the exchange rate error under the isoelastic demand with multiplicative error in news vendor setting. The effect of uniform error is also compared with the generalized beta distribution error in the exchange rate.

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II. LITERATURE REVIEW

This paper fundamentally follows the model of Arcelus, Gor and Shrinivasan[1]. Cases of transaction exposure when a firm has an accounts receivable or payable denominated in a foreign currency has been reported in Goel [3]. The nature of global trade is that either the buyer or the seller has to bear what is commonly known in international finance as transaction exposure, Eitemann et al.[4], Shubita et al.[5]. The very important newsvendor framework introduced by Petruzzi and Dada [6] and the price dependent demand forms in the additive and multiplicative error structures by Mills[7] and Karlin and Carr[8] have been used. The derivation of the expected profit and optimal policies, when demand is linear, are given in Sanjay Patel, Ravi Gor [9] and for multiplicative demand error in Sanjay Patel and Ravi Gor[2]. We have also developed more general hybrid model for additive and multiplicative demand error Sanjay patel and Ravi Gor[10].

III. TRANSACTION EXPOSURE MODEL

Suppose a retailer wants to order q units from a foreign manufacturer of a certain product. The retailer does not know the demand (D) of the product, which is uncertain. But it partly depends upon the price(*p*) and partly random. In this paper we take the price dependent demand with multiplicative error which can be defined as $D(p,\in) = g(p)\in$, where $\in$ is the multiplicative error in the demand and it follows some distribution(say $f(\in)$) with mean μ in some interval [A,B] and g(*p*) is the deterministic demand. [Generally g(p) is taken as decreasing isoelastic function of *p* say, $g(p) = ap^{-b}$ in multiplicative demand error case with the restrictions *a, b* >1]

Let the exchange rate be '*r* ' in the retailer currency when the order is placed. Let w be the cost of one unit of the product in the manufacturer currency. If the buyer pays immediately then he has to pay *wr* per unit of the product in his currency.

Suppose there is a time lag (some fixed period) between the order is placed and the amount is paid for the product when it is acquired by the retailer. Thus there exists transaction exposure risk, since the exchange rate (r) may get fluctuate. So the buyer has to pay more or less according to the existing rate at the time of the arrival of the product. Generally the fluctuation in the exchange rate r is very small and random. We model the future exchange rate(FER) as FER= current exchange rate + fluctuation in the exchange rate. The fluctuation in the exchange rate is always some percentage of *r*, hence we can take the FER= $r + r\in_r$ $= r(1+\in_r)$.Where $\in_r$ is also a random variable together with the random variable D. We assume that $\in_r$ lies in [-*a*, *a*]. Here 0<*a*<1.The fluctuation $\in_r$ is unknown but its distribution is known say $\psi(\in_r)$. In this paper we consider uniform distribution for $\in_r$ with support [*a*,*b*] i.e. $\psi(y) = 1/(b-a)$. The expected value of the fluctuation in the exchange rate error is therefore $E(y) = \frac{a+b}{2}$.

If the fluctuation $\in_r$ is positive buyer has to pay more and if it is negative seller will get less. So the question arises here is that who will bare the exchange rate risk? Buyer/retailer OR seller/manufacturer? In this paper we discuss the two situations under the multiplicative demand error. In each case the retailer's optimal policy is to determine the optimum order(q) and selling price(p) of the product so that his expected profit is maximum. At the same time we obtain the manufacturer's optimal policies as well.

The following assumptions are made in the foreign exchange transaction exposure model:

(i)The standard newsvendor problem assumptions apply.

(ii) The global supply chain consists of single retailer- single manufacturer.

(iii) The error in demand is additive.

(iv) Only one of the two-retailer or manufacturer- bears the exchange rate risk.

The following notations are used in the paper:

q= order quantity

p= selling price per unit

D= demand of the product= no. of units required

$\in$= demand error= randomness in the demand.

v= salvage value per unit

s=penalty cost per unit for shortage

c= cost of manufacturing per unit for manufacturer

w_r = purchase cost for retailer

$\in_r$= the exchange rate fluctuation= exchange rate error= randomness in exchange rate

Π= profit function.

IV. THE TWO SCENARIOS

Case-1: Retailer bears the exchange Rate Risk

In the case-1 we assume that the retailer bears the exchange rate risk and manufacturer does not. Thus the manufacturer will get *w* per unit at any point of time and the buyer will have to pay according to the existing exchange rate. So the buyer will be paying $wr(1+\in_r)$ per unit, on the settlement day or when the product is acquired by him. This amount in terms of manufacturer currency is $wr(1+\in_r)/r = w(1+\in_r) = w_r$ (say). Thus w_r is the purchase cost to buyer in seller's currency.

Now the retailer/ buyer will choose the selling price *p* and the order quantity *q* so as to maximize his expected profit. The profit function for the retailer is given by,

$\Pi(p,q)$ =[revenue from *q* items]– [expenses for the *q* items]

$$\Pi(p,q)=\begin{cases}[pD+v(q-D)]-[qw_r] & if\ D\le q\ (overstocking)\\ [pq]-[s(D-q)+qw_r] & if\ D>q\quad (shortage)\end{cases} \quad (1)$$

Note that all the parameters *p*, *v*, *s*, w_r are taken in manufacturer's currency and the salvage value *v* is taken as an income from the disposal of each of the *q*-D leftover.

Since the demand, $D(p,\in)=g(p)\in$, the retailer's profit function (1) for ordering *q* units and keeping selling price *p* is given by,

$$\Pi(p,q)=\begin{cases}[p(g(p)\in)+v\{q-(g(p)\in)\}]-[qw_r] & if\ D\le q\\ [pq]-[s(\{g(p)\in\}-q)+qw_r] & if\ D>q\end{cases}$$

Put *g*(*p*)=*g* and define *z* = *q* / *g*(*p*) = *q* / *g* i.e. *q*=*g z* , for the multiplicative demand error.

Now $D\le q \Leftrightarrow g\in\le q \Leftrightarrow \in\le q/g \Leftrightarrow \in\le z$ and Similarly $D>q \Leftrightarrow \in > z$.

$$\Pi(z,p)=\begin{cases}pg\in+vg(z-\in)-gzw_r & if\ \in\le z\\ pgz-sg(\in-z)-gzw_r & if\ \in>z\end{cases} \quad (2)$$

The equation (2) describes the profit function for the retailer in the manufacturer currency. Note that the parameter *q* is replaced by *z*. Now the retailer wants to find the optimal order quantity q say q* and optimal price p= p* to maximize his expected profit. In order to do this he must find optimal values of the price p and the parameter z, say p* and z* respectively which maximizes his expected profit so that he can determine the optimal order q*=z*g(p*). The profit Π is a function of the random variable $\in$ with support [A, B]. Thus the retailer's expected profit is given by,

$E[\Pi(z,p)]=\int_A^B \Pi(z,p)f(u)du$. (Here we take $\in=u$ for simplicity in (2))Then we get,

$$E[\Pi(z,p)]=\int_A^z[pgu+vg(z-u)-gzw_r]\cdot f(u)du + \int_z^B[pgz-sg(u-z)-gzw_r]\cdot f(u)du \quad (3)$$

Define $\Lambda(z)=\int_A^z(z-u)f(u)du$ [expected leftovers] and

$\Phi(z)=\int_Z^B(u-z)f(u)du$ [expected shortages].

Then the expected profit of the retailer as a function of *z* and *p* is given by

$$E[\Pi(z,p)]=[(g\mu)(p-w_r)]-g[(w_r-v)\Lambda+(p+s-w_r)\Phi] \quad (4)$$

as derived in Sanjay Patel and Ravi Gor[2].

Where $\mu=\int_A^B uf(u)du$ in the equation (4) and it gives the expected value of the randomness *u* in the demand D.

We use Whitin's method [11] to maximize the expected profit function. In this method first we keep *p* fixed in (4) and use the second order optimality conditions $\frac{\partial E}{\partial z}=0$ and $\frac{\partial^2 E}{\partial z^2}<0$ to find the optimum value of z* as a function of *p*. Then we substitute the value of z* in the expected profit (4) so that it becomes a function of *p* and hence the optimal p* can also be obtained. The authors have already derived the optimal policies given below, in Sanjay Patel and Ravi Gor [2].

$$z^* = F^{-1}\left(\frac{p+s-w_r}{p+s-v}\right)$$

(5) Where $F(z)=\int_A^z f(u)\,du$ is the CDF.

This z* gives the optimum solution for maximum profit as a function of *p*. Now substitute this z* in $E[\Pi(z,p)]$ and obtain optimum p* using the second order optimality criteria. Hence the retailer's optimal order q =q* is given by,

$$q^* = g(p^*)z^* = g(p^*)F^{-1}\left(\frac{p^*+s-w_r}{p^*+s-v}\right) \quad (6)$$

Also the manufacturer's profit when the buyer bears the risk is [(selling price of seller)-(cost of purchase to seller)]× no. of units sold, $\Pi_m = (w-c)q^*$.

(7)

Case-2: Manufacturer bears the exchange Rate Risk

In the case-2 we assume that the manufacturer bears the exchange rate risk and retailer does not. Thus the retailer pays w per unit in manufacturer's currency at any point of time and the manufacturer will get according to the existing exchange rate. So the manufacturer will be getting $wr/(r(1+\epsilon_r)) = w_m$ per unit on the settlement day in his currency. Now the retailer's profit function, his expected profit and optimal policies to get maximum expected profit can be obtained by replacing wr by w in case-1. So we get the retailer's profit as,

$$\Pi(p,q)=\begin{cases}[pD+v(q-D)]-[qw] & if\ D\le q\ (overstocking)\\ [pq]-[s(D-q)+qw] & if\ D>q\quad (shortage)\end{cases} \quad (8)$$

And his expected profit as,

$$E[\Pi(z,p)] = [(g\mu)(p-w)] - g[(w-v)\Lambda + (p+s-w)\Phi] \quad (9)$$

The optimal value of z is given by $z^* = F^{-1}\left(\frac{p+s-w}{p+s-v}\right)$

and hence the optimum order quantity is,

$$q^* = g(p^*)z^* = g(p^*)F^{-1}\left(\frac{p^*+s-w}{p^*+s-v}\right).$$

(10)

Also the manufacturer's profit when the buyer bears the risk is [(selling price of seller)-(cost of purchase to seller)]× no. of units sold. $\Pi_m = (w_m - c)q^*$.

(11)

V. SENSITIVITY ANALYSIS

We assume isoelastic demand with multiplicative demand error *u* which follows the uniform distribution $f(u)$ with support [A,B]. Then we obtain the optimum policies and maximum expected profit of the retailer and manufacturer using MAPLE software when anyone of them bears the exchange rate risk. We compute the optimum values by using uniform distribution $\psi(\epsilon_r)$ in the exchange rate error ϵ_r with support

[-0.1,0.1]. In case-1 and case-2 we also compare it with the policies obtained by Arcelus, Gor, Srinivasan [1],for the generalized beta distribution in the exchange rate error for each of the case positive, negative and symmetrical beta distribution.

Recall the uniform probability density function is $f(x,a,,b)=\frac{1}{b-a}$, $a\le x\le b$ with mean $\mu=\frac{a+b}{2}$ and standard deviation $\sigma=\frac{1}{\sqrt{12}}(b-a)$. And the four parameter beta density function is given by $f(y/a,b,\alpha,\beta)=\frac{(y-a)^{\alpha-1}(b-y)^{\beta-1}}{B(\alpha,\beta)(b-a)^{\alpha+\beta-1}}$ where $a\le y\le b, \alpha,\beta>0$ and its transformation in the standard beta distribution by taking $y=a+(b-a)x$ is given by $f(x/0,1,\alpha,\beta)=\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$ where,

$0\le x\le 1, \alpha,\beta>0$ and $B(\alpha,\beta)=\int_0^1 t^{1-\alpha}(1-t)^{1-\beta}\,dt$.

We assume the following parameter values:

Demand support =[A,B]=[0.7,1.1]

Mean Demand = $\mu=\frac{A+B}{2}=0.9$

Linear

demand $g(p)=ap^{-b}, a=500,000,000, b=2.5$

v =Salvage value = 10

s=Penalty cost = 5

c= cost of manufacturing per unit for manufacturer= 20

r=current exchange rate = 45

The computations in case-1 and case-2 of the following tables are done through Maple software.

Case-1 Retailer Bears the risk

Distribution	Parameters of the dist.	p*	q*	Seller's selling price w*	Optimum exp. profit of buyer	Optimum exp.profit of seller
Beta	α=1 , β=3	55.04	20702	32.82	423229	265586
Uniform		58.01	18088	32.87	390501	232953
Variation%		5.3960	12.6267993	0.15234613	7.73292945	12.2871687
		↑	↓	↑	↓	↓
Beta	α=3 , β=1	60.97	15919	32.92	361846	205696
Uniform		58.01	18088	32.87	390501	232953
Variation%		4.8548	13.6252277	0.151883354	7.91911476	13.2511084

		↓	↑	↓	↑	↑
Beta	α=1 , β=1	58.01	18088	32.87	390501	232953
Uniform		58.01	18088	32.87	390501	232953
Variation%		0	0	0	0	0
Beta	α=2 , β=5	55.47	20297	32.83	418165	260544
Uniform		58.01	18088	32.87	390501	232953
Variation%		4.5790	10.8833817	0.121839781	6.61557040	10.5897660
		↑	↓	↑	↓	↓
Beta	α=5 , β=2	60.55	16206	32.91	365718	209306
Uniform		58.01	18088	32.87	390501	232953
Variation%		4.1948	11.6129828	0.121543604	6.77653273	11.2978127
		↓	↑	↓	↑	↑

Case-2 Manufacturer Bears the risk

Distribution	Parameters of the dist.	p*	q*	Buyer's purchase cost w_r*	Optimum exp. profit of buyer	Optimum exp. profit of seller
Beta	α=1 , β=3	55.04	20702	32.82	423299	265586
Uniform		58.01	18088	32.87	390501	232953
Variation%		5.396075	12.6267993	0.15234613	7.74818745	12.2871688
		↑	↓	↑	↓	↓
Beta	α=3 , β=1	60.97	15919	32.92	361846	205696
Uniform		58.01	18088	32.87	390501	232953
Variation%		4.854846	13.6252277	0.15188335	7.91911476	13.2511084
		↓	↑	↓	↑	↑
Beta	α=1 , β=1	58.01	18088	32.87	390501	232953
Uniform		58.01	18088	32.87	390501	232953
Variation%		0	0	0	0	0
Beta	α=2 , β=5	55.47	20297	32.83	418165	260544
Uniform		58.01	18088	32.87	390501	232953
Variation%		4.579051	10.8833818	0.12183978	6.61557041	10.589766
		↑	↓	↑	↓	↓
Beta	α=5 , β=2	60.55	16206	32.91	365718	209306
Uniform		58.01	18088	32.87	390501	232953
Variation%		4.194880	11.6129828	0.1215436	6.77653274	11.2978128
		↓	↑	↓	↑	↑

VI. CONCLUSION

We elaborate uniformly distributed exchange rate fluctuation when the retailer or manufacturer undertakes to share the exchange rate risk and the demand error is modelled in the multiplicative form in the news vendor framework. We also have compared the exchange rate effect with the generalized beta distribution error which is evident from the above tables.

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