

Comparative analysis of pipeline network using flow rate corrections

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Abstract—A comparative evaluation of pipeline network flow analysis using loop equations formulation has been considered. Single loop adjustment and simultaneous loop flow adjustment algorithms were used to determine flow rates in two pipeline network samples. Hazen-Williams and Darcy-Weisbach head loss equations were used to account for the major losses in the networks. The solution algorithms evaluated the convergence of the flow rate correction of the network loops. It was observed from the two cases considered that the solutions converged faster using simultaneous loop flow adjustment and Darcy-Weisbach head loss equation. Nevertheless, single loop adjustment algorithm showed a smoother and better convergence pattern and it is easily amenable to computer programming and takes lesser computer memory. Also, it was observed within the operating parameters considered that both Hazen-Williams and Darcy-Weisbach head loss equations had no significant effect on the rate of flow convergence. Hence, any of the head loss equations can be used in the design of water distribution networks in Nigeria.

Index Terms—Flow rate, flow rate correction, head loss, network loops, pipeline network, water distribution.

I. INTRODUCTION

Pipeline network is an important infrastructure used for transportation purpose. Fluids are moved from one location to another through pipeline network. It is always desirable that these fluids should be moved at a minimum cost. This could be achieved through well-structured design process. Basic to design is analysis, hence the need to develop a procedure of analysis that would be easier to apply and that which will require more routine processes. Most steady-state analysis in pipeline flow has employed the Hardy cross method [1], sparse matrix method [2], and linear theory method [3], [4]. Reference [5] developed a more efficient approach by simultaneously computing corrections for all loops in pipeline network. Reference [6] opined that this approach is useful in designing pipeline network for maximum economy.

Generally, there are four possible solution methods in pipeline analysis. These include loop equations, node-loop equations, node equations and pipe equations. The smallest set of equations

is the loop equations that include one equation for each closed loop and pseudo-loop. The unknowns in the loop equations are the flow rate corrections. In node-loop equations, the total number of independent equations is $n\text{-node} + n\text{-loop}$. A comparison of number of solution algorithms with their modified linear theory (flow adjustment) method and showed that this approach was efficient and robust [7]. Modified linear theory solves directly for the pipe flow rates rather than the loop equations approach of loop flow corrections. The node equations can be written in terms of the nodal heads for each pipe and these equations had been solved using the Newton-Raphson method [8]. It was also suggested to permute the columns of the unknown-head node-arc incidence matrix to make its top $n\text{-node} \times n\text{-node}$ block invertible [9]. Reference [10] suggested using row and column matrix to transform it to trapezoid form, a form in which the top $n\text{-node} \times n\text{-node}$ block is lower triangular. Pipe equations method solve for flow rates and total nodal head simultaneously. Reference [11] devised this method and [12] developed the algorithm. A finite volume procedure was developed to solve the system using Navier-Stoke method [13].

This work will compare two different solution methods to loop equations for pipeline network systems. In order to achieve the desired results, a solution of a single loop adjustment algorithm using Hazen-Williams and Darcy-Weisbach loss equations will be determined. Also a solution of a simultaneous loop flow adjustment algorithm using Hazen-Williams and Darcy Weisbach loss equations will be determined. A comparison of the different results will be done.

II. METHODS

The solution algorithm for steady flow that will be applied in this work will be loop equations formulation. The two algorithms that will be applied are single loop adjustment algorithm and simultaneous loop flow adjustment algorithm. These methods solve the energy equations for loops and pseudo-loops for a loop flow correction. Direct solution of system on non-linear equations is not feasible; hence it is necessary to use iterative solution methods. Generally, these methods start with an estimated solution which is the derivation from the true solution and is reduced to an acceptable tolerance value.

The single loop adjustment algorithm is the most widely used technique for solving for the unknown in water network analysis. In the single loop adjustment algorithm, each loop correction is determined independently of the other loops. In some pipeline network arrangement, several loops may have common pipes so corrections to those loops will impact energy losses around more than one loop. In simultaneous loop flow adjustment algorithm, all the loops are corrected simultaneous and the computational procedure takes into account the iterative influence of flow corrections between loops which have common pipes.

A. FRICTIONAL HEAD LOSSES

Adequate pressure and flow must be maintained in course of design and implementation of a network distribution system. There are basically a quite number of factors that do lead to losses of pressure along distribution networks. Predominant among these losses include frictional losses in pipe, reservoir to pipe connection and vice versa, sudden contractions in pipes and meters. Among all losses aforementioned, review shows that frictional head losses in pipe constitute the largest percentage [14]. Thus, for most practical purposes of analysis, the other losses are usually ignored. There are a number of frictional head loss equations that have been developed to determine the frictional losses along pipeline. The three most common equations are the Manning, Hazen-Williams and Darcy-Weisbach equations. The Manning equation is typically used for open channels. The Hazen-Williams equation and Darcy –Weisbach are used for pipes (closed channels).The Hazen-Williams equation for water distribution networks is [15],

$$h_L = \frac{10.675 L Q^{1.852}}{120^{1.852} D^{4.8704}} \quad (1)$$

The Darcy-Weisbach equation was developed using dimensional analysis. The Darcy-Weisbach equation uses a friction factor, f , instead of a C-factor used in Hazen-Williams. It is given as [16],

$$h_L = \frac{8fLQ^2}{\pi^2 gD^5} \quad (2)$$

The friction factor can be estimated using the Swamee-Jain equation as [16],

$$f = \frac{1.325}{\left[\ln \left(\frac{\varepsilon}{3.7D} + \frac{5.74}{Re^{0.9}} \right) \right]^2} \quad (3)$$

Reference [17] used a fair estimate of friction factor in water distribution network analysis. Therefore, friction factor, f , can be taken as 0.0242 in this work.

B. DETERMINATION OF THE PIPE FLOW RATE CORRECTIONS

The unknowns in the loop equations are the flow rate corrections to the flow rate around each loop. For single loop adjustment algorithm, the flow rate corrections are given as

$$\Delta Q_i = - \left(\frac{\sum h}{\sum n \left| \frac{h}{Q} \right|} \right) \quad (4)$$

In the single loop adjustment algorithm, each loop correction is determined independently of the other loops. In some pipeline

network arrangement, several loops may have common pipes so a correction to those loops other loops. An approach that simultaneously computes corrections for all loops was developed [5]. As in the single loop adjustment algorithm, an initial solution that satisfies continuity at all nodes is required. For a simultaneous loop flow adjustment algorithm, the flow rate corrections can be computed as

$$J_L \Delta Q = -h(Q^{(m-1)}) \quad (5a)$$

In matrix form, equation 5a becomes

$$\begin{bmatrix} \sum n \left| \frac{h_{11}}{Q_{11}} \right| & -\frac{nh_{12}}{Q_{12}} & \dots & -\frac{nh_{1m}}{Q_{1m}} \\ -\frac{nh_{21}}{Q_{21}} & \sum n \left| \frac{h_{22}}{Q_{22}} \right| & \dots & -\frac{nh_{2m}}{Q_{2m}} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{nh_{k1}}{Q_{k1}} & -\frac{nh_{k2}}{Q_{k2}} & \dots & \sum n \left| \frac{h_{km}}{Q_{km}} \right| \end{bmatrix} \begin{pmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \vdots \\ \Delta Q_m \end{pmatrix} = - \begin{pmatrix} \sum h_1 \\ \sum h_2 \\ \vdots \\ \sum h_m \end{pmatrix} \quad (5b)$$

Once the matrices are formed, equation 5b can be solved by any linear equation solver for ΔQ . Scilab which is a high level computer language for scientific computing and data visualization built around the idea of an interactive programming environment [18], is used to solve the system of equations of equation 5b. The left division “\” is such that $X = A \setminus B = A^{-1}B$ is the solution of $AX = B$. Therefore, the solution of equation 5a in scilab becomes

$$\Delta Q_i = J_L \setminus [-h(Q^{(m-1)})] \quad (6)$$

Two cases were used in this work. The first case was a sample network adopted from [15]. The sample network has four loops with twelve different pipe dimensions. The sketch of the network is shown in Figure A1 and the dimensions of the network with initial flow rate are presented in Table B1. The second case was Owerri municipal water reticulation system adopted from [17]. The network system has nine loops with twenty-four different pipe dimensions. Figure A2 and Table B2 show the sketch and the dimensions of the sample network.

III RESULTS DISCUSSION

The solutions of both single loop adjustment algorithm and simultaneous loop flow adjustment algorithm are not exact; since the processes involve iteration. Generally, either a desired tolerance level or numbers of iterations are prescribed at which the process of iteration will be terminated. In this work, the desired tolerances were set in line with network sizes and complexities.

Considering Case I, the solution for single loop adjustment algorithm using Hazen-Williams and Darcy-Weisbach loss equations were gotten at the seventh iterations. However, simultaneous loop flow adjustment converged at third iteration when Darcy-Weisbach loss equation was used and fourth iteration for Hazen-Williams loss equation. For Case II, single loop adjustment algorithm converged at seventeenth iteration for Hazen-Williams loss equation and ninth iteration for Darcy-Weisbach loss equation. Also, the simultaneous loop flow adjustment converged at tenth iteration for

Hazen-Williams loss equation and fifth iteration for Darcy-Weisbach loss equation.

It could be seen from the two cases that the solution converges faster using Darcy-Weisbach loss equation and simultaneous loop flow adjustment algorithm. It is obvious that the solution will converge faster using simultaneous loop flow adjustment algorithm since all the loops are corrected simultaneously at each iteration level. The square index associated with the flow rate for Darcy-Weisbach loss equation is likely contributing to its fast convergence. Single loop adjustment algorithm using Hazen-Williams loss equation takes a longer time to converge though it is more amenable to computer programming. As the complexity of the network system increases, more time and more computer memory are needed for the solution algorithm.

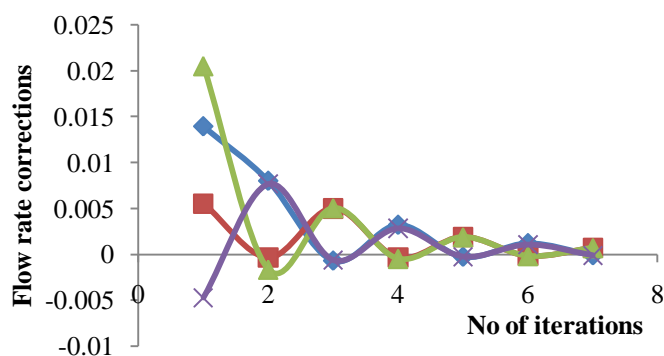


Fig 1: Flow rate correction and number of iteration for Case I using single loop adjustment algorithm and Hazen-Williams loss equation

Figure 1 shows the plot of flow rate correction against the number of iteration for Case I using single loop adjustment algorithm and Hazen-Williams loss equation. Since the initial solutions were by assumption, the values of the flow rate correction were high at the beginning of the iteration process. They converged at the seventh iteration as shown in the plot. At that point, the flow rate obtained can be used for network analysis and design.

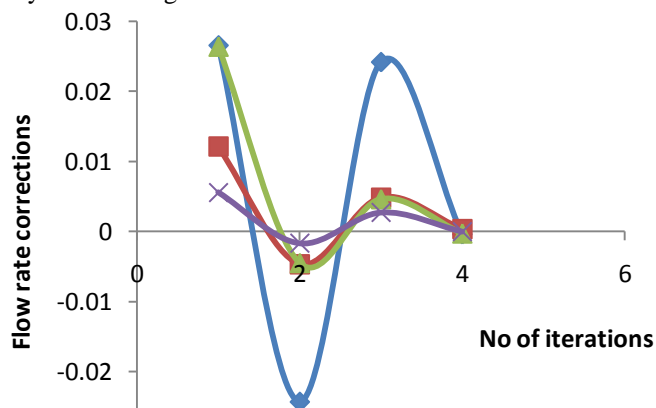


Fig 2: Flow rate correction and number of iteration for Case I using simultaneous loop flow adjustment and Hazen-Williams loss equation

Figure 2 shows the plot of flow rate correction against the number of iteration for Case I using simultaneous loop flow adjustment and Hazen-Williams loss equation. It could be seen that the flow rate corrections converged at fourth iteration. Nevertheless, the values of the flow rate correction were far apart prior to convergence at fourth iteration, though it seemed

that they converged at 2.5, such fraction cannot be putative since iteration process is an integer operation.

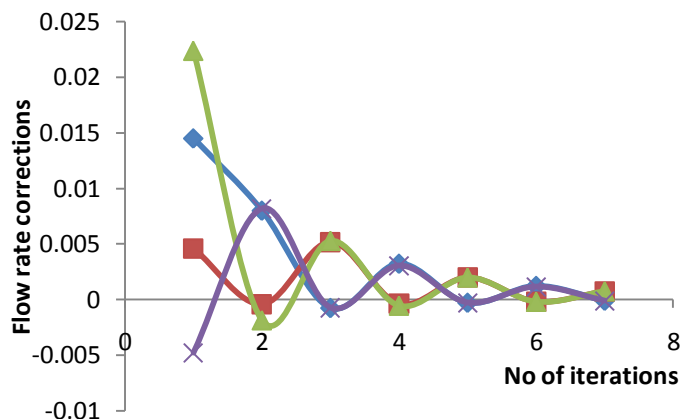


Fig 3: Plot of flow rate correction and number of iteration for Case 1 using single loop adjustment algorithm and Darcy-Weisbach loss equation

Figure 3 shows the plot of flow rate correction against the number of iteration for Case I using single loop adjustment algorithm and Darcy-Weisbach loss equation. It could be seen that the flow rate corrections converged at seventh iteration. Figures 1 and 3 are almost the same, their solution algorithm contributed to their similarity.

Figure 4 shows the plot of flow rate correction against the number of iteration for Case I using simultaneous loop flow adjustment and Darcy-Weisbach loss equation. It could be seen that the flow rate corrections converged at the third iteration. It is important to note that the convergence before the second

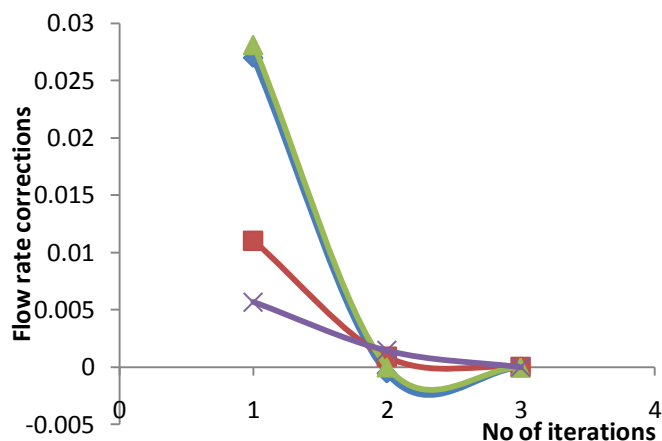


Fig 4: Flow rate correction and number of iteration for Case I using simultaneous loop flow adjustment and Darcy-Weisbach loss equation

iteration is not considered because it does not lie along the zero base line.

Figure 5 shows the plot of flow rate correction against the number of iteration for Case II using single loop adjustment algorithm and Hazen-Williams loss equation. The flow rate correction converged at the seventeenth iteration. The size of the pipeline network led to greater number of iterations. Therefore, larger pipeline network systems will take greater number of iterations before convergence.

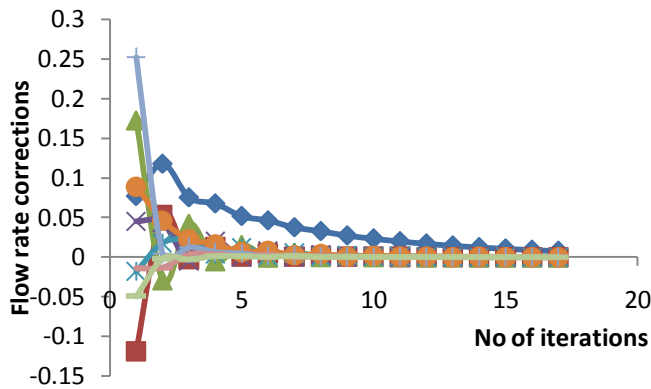


Fig 5: Flow rate correction and number of iteration for Case II using single loop adjustment algorithm and Hazen-Williams loss equation

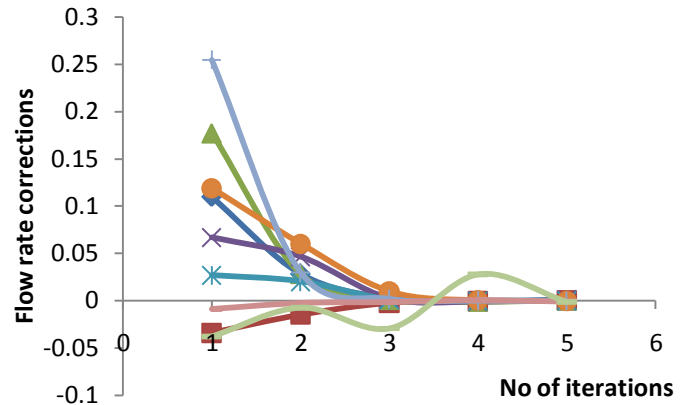


Fig 8: Flow rate correction and number of iteration for Case II using simultaneous loop flow adjustment and Darcy-Weisbach loss equation

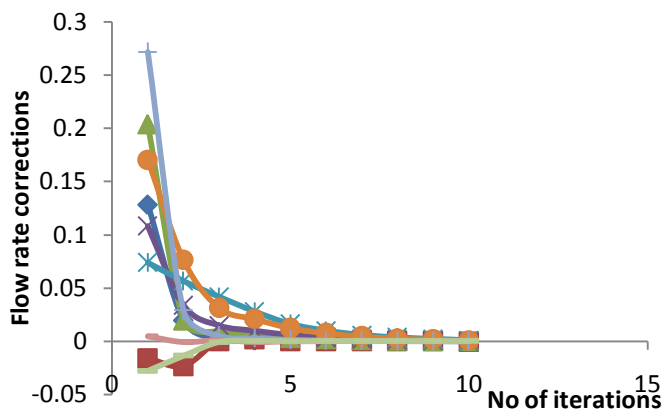


Fig 6: Flow rate correction and number of iteration for Case II using simultaneous loop flow adjustment and Hazen-Williams loss equation

Figures 9 to 16 show the variation of flow rate with number of iteration for all the scenarios. It could be seen that the flow rate became steady as the number of iterations increase. It implies that as the flow rate correction converges, the flow rate gets closer to desired flow condition. The flow rate in most of the cases follows a regular pattern except in Figure 13. In this part, the flow rate in pipes 7 and 8 increased from 0.3m³/s and 0.2m³/s to 0.951523m³/s and 0.649336m³/s respectively. Also in Figure 10, the flow rate in pipe 7 decreased to a minimum value of 0.023848m³/s and sharply increased to 0.262733m³/s. This abrupt change resulted from the influence of the adjoining loops.

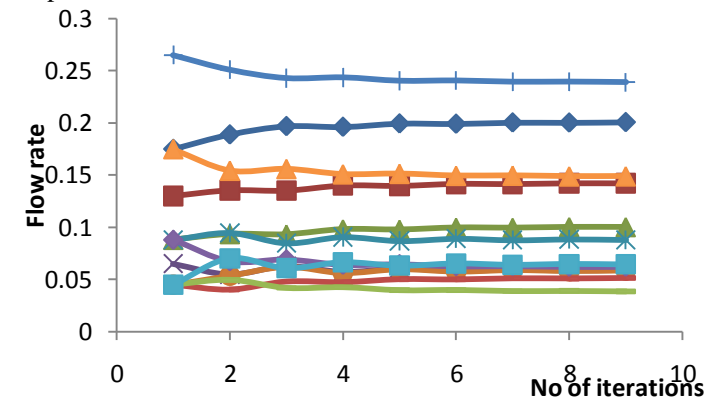


Fig 9: Flow rate and number of iterations for Case 1 using single loop adjustment algorithm and Hazen-Williams loss equation

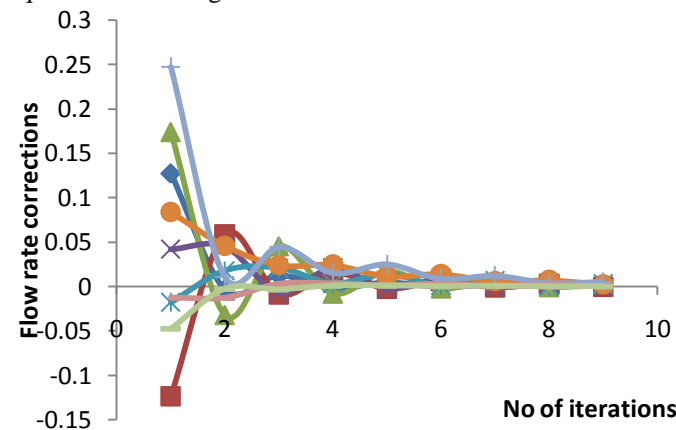


Fig 7: Flow rate correction and number of iteration for Case II using single loop adjustment algorithm and Darcy-Weisbach loss equation

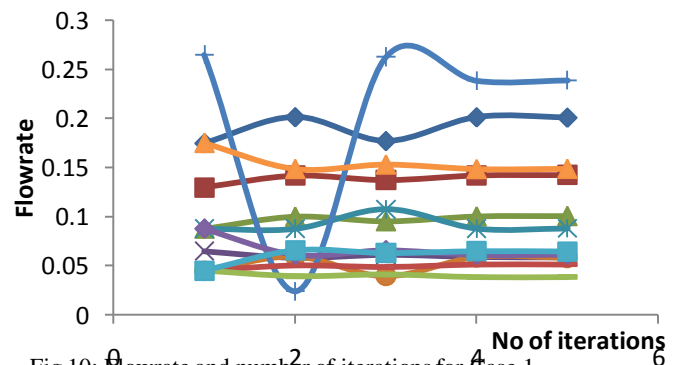


Fig 10: Flowrate and number of iterations for Case 1 using simultaneous loop flow adjustment and Hazen-Williams loss equation

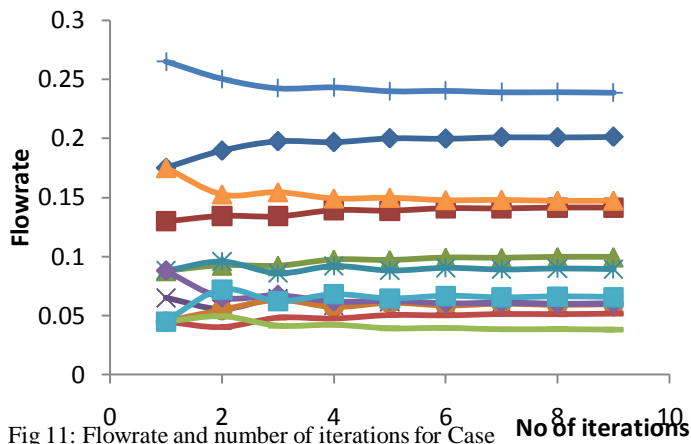


Fig 11: Flowrate and number of iterations for Case I single loop adjustment algorithm and Darcy-Weisbach loss equation

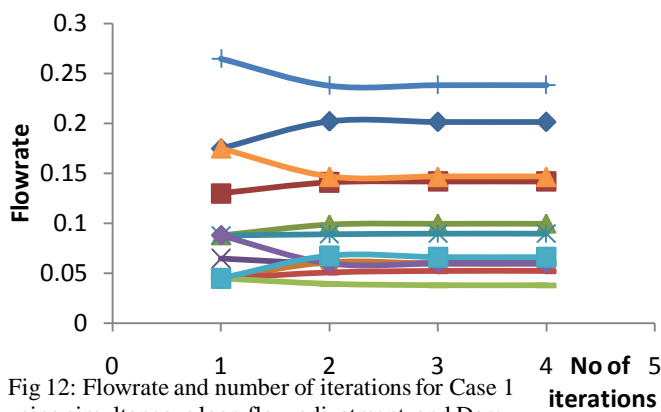


Fig 12: Flowrate and number of iterations for Case I using simultaneous loop flow adjustment and Darcy-Weisbach loss equation

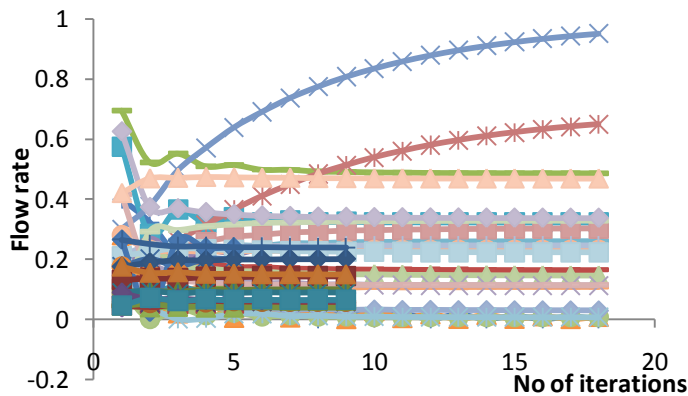


Fig 13: Flow rate and number of iterations for Case II using single loop adjustment algorithm and Hazen-Williams loss equation

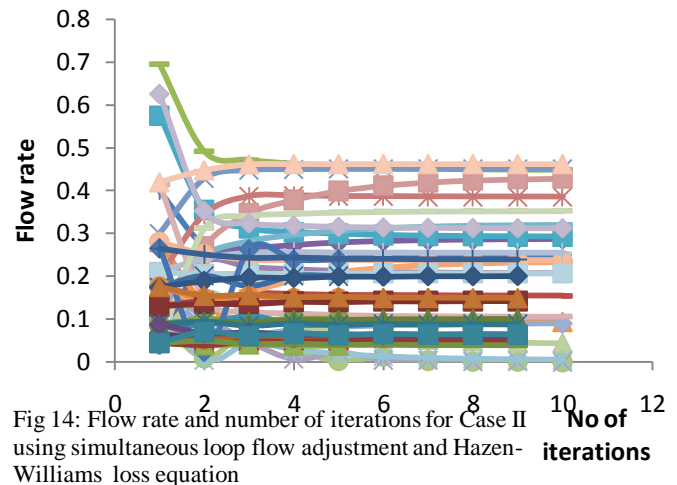


Fig 14: Flow rate and number of iterations for Case II using simultaneous loop flow adjustment and Hazen-Williams loss equation

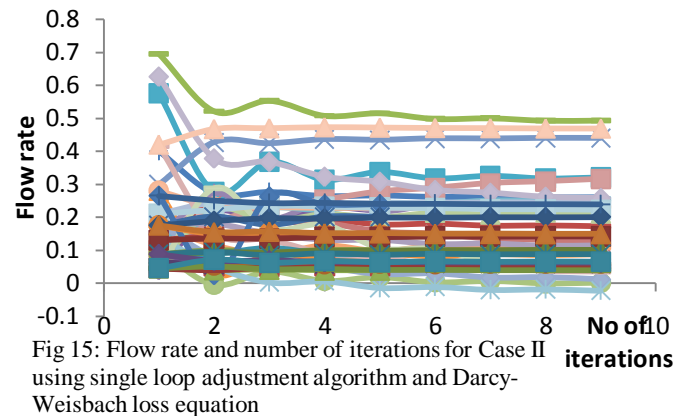


Fig 15: Flow rate and number of iterations for Case II using single loop adjustment algorithm and Darcy-Weisbach loss equation

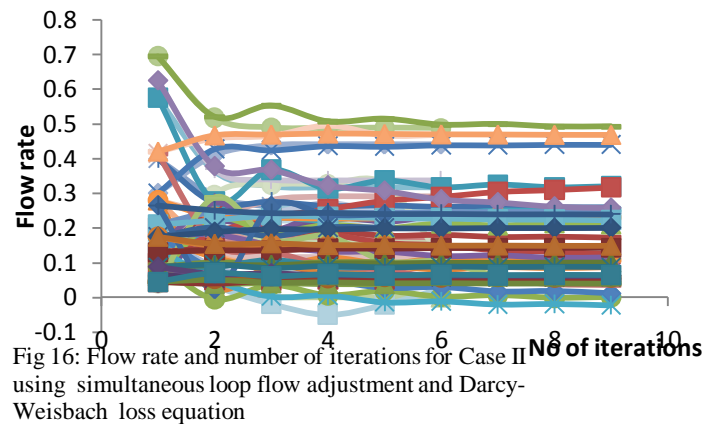


Fig 16: Flow rate and number of iterations for Case II using simultaneous loop flow adjustment and Darcy-Weisbach loss equation

IV CONCLUSION

Two different solution approaches for the pipeline network system have been considered using Hazen-Williams and Darcy-Weisbach loss equations for two case studies. The solution algorithm seeks to evaluate the convergence of the flow rate correction of all the loops in the network. The converged values for the different scenarios were compared. When comparing the solution methods, it is evident that the simultaneous loop flow adjustment algorithm using Darcy-Weisbach loss equation converged solution in fewest number of iterations. The solution showed that at third iterations, the change in the flow vector contains values that were less than the predefined stopping criteria for Case I. Also, the solution converged at the fifth iteration for Case II.

However, single loop adjustment algorithm showed a smoother and better convergence pattern as evident in figures 1, 3, 5 and 7. Single loop adjustment algorithm can easily be

executed in spread sheet like Micro-soft Excel and takes smaller computer memory. Also, it is amenable to hand calculation for small network systems. It was observed that frictional head loss equations did not have significant effect on the rate convergence of the flow within the operating parameters. Therefore, either Hazen-Williams equation or Darcy-Weisbach equation can be applied in water distribution networks in Nigeria.

Nomenclature

- D= pipe diameter (m),
- f = Darcy-Weisbach friction factor,
- g= acceleration due to gravity (m/s²),
- h_L= head loss (m),
- L= pipe length (m),
- Q= pipe flow rate (m³/s),
- Re= Reynolds number,
- ΔQ= corrective discharge (m³/s),
- ε= equivalent roughness.

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Appendix A

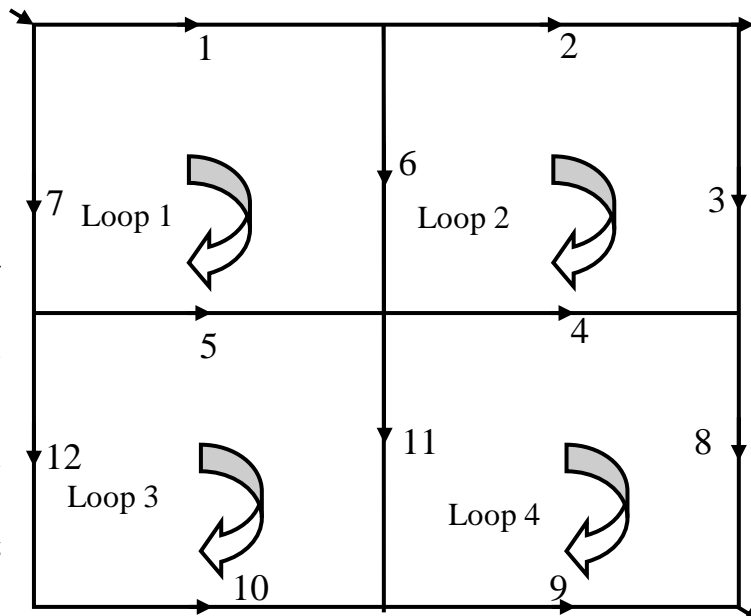


Figure A1: Sample Network (Case I)

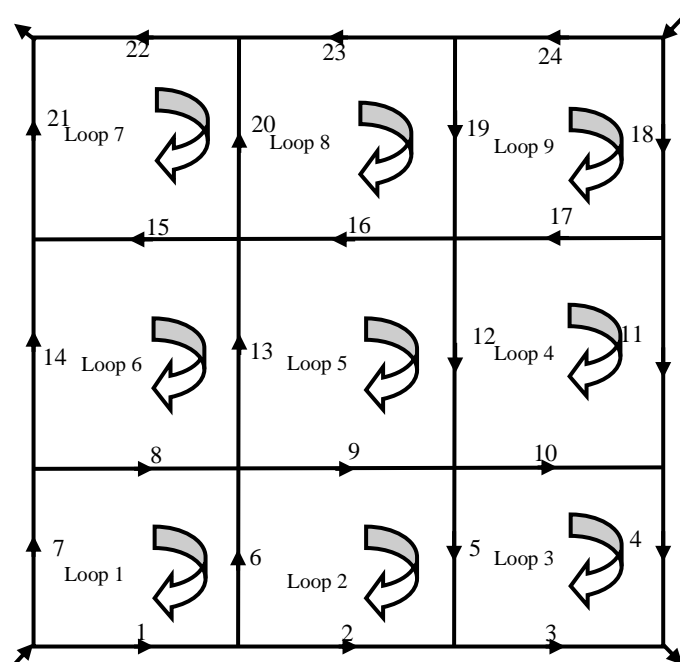


Figure A2: Sample Network (Case II)

APPENDIX B

Table B1: Dimensions of Case I

Pipe	Length (m)	Diameter (m)	Initial Flow Rate (m ³ /s)
1.	915	0.508	0.175
2.	915	0.508	0.130
3.	1220	0.406	0.88
4.	915	0.305	0.065
5.	915	0.406	0.88
6.	1220	0.406	0.045
7.	1220	0.610	0.265
8.	1220	0.305	0.045
9.	915	0.305	0.045
10.	915	0.406	0.88
11.	1220	0.305	0.045
12.	1220	0.406	0.175

23.	300	0.15	0.21
24.	600	0.30	0.42

Table B2: Dimensions of Case II

Pipe	Length (m)	Diameter (m)	Initial Flow Rate (m ³ /s)
1.	700	0.30	0.40
2.	300	0.15	0.12
3.	400	0.30	0.695
4.	350	0.30	0.04
5.	350	0.20	0.575
6.	200	0.20	0.28
7.	300	0.30	0.30
8.	500	0.60	0.30
9.	200	0.15	0.10
10.	200	0.15	0.14
11.	200	0.15	0.14
12.	200	0.15	0.175
13.	200	0.15	0.18
14.	350	0.30	0.10
15.	800	0.30	0.06
16.	200	0.15	0.175
17.	200	0.15	0.14
18.	300	0.15	0.28
19.	200	0.15	0.21
20.	350	0.30	0.415
21.	500	0.30	0.004
22.	600	0.30	0.625