

Position and Velocity control for Two-Wheel Differential Drive Mobile Robot

Cherry Myint^{#1}, Nu Nu Win^{#2}

Abstract— This research presented the position and velocity control for two-wheel differential drive mobile robot (DDMR). In this research, mobile robot motion task is point to point motion task in an obstacle free indoor environment. The aim of this research is to reduce the position error of mobile robot when the robot moves from one point to other setpoints. So, this research mainly focused on the navigation system. This navigation system is done by odometry that mainly used the sensor output as the input command. The distance traveled by the robot was calculated by transforming the number of wheel revolution counted using a magnetic encoder to a linear distance knowing the diameter of the wheel. If the robot astray to user setpoint, kinematics model get the command to fix the position error and then it is transform into analog signal to feed to the motor driving system. On the other hand, proportional integral derivative (PID) controller is used to achieve the desired motors speed for velocity control system. By controlling both velocity and position, the robot can reach to its desired setpoints exactly.

Index Terms— Wheeled Mobile Robot, Geared DC Motor, point to point motion, PID Controller, Odometry

I. INTRODUCTION

Mobile robot is a platform with a large mobility within its environment such as air, land and underwater and then it is not fixed to one physical location. Among many types of mobile robot, differential drive mobile robot (DDMR) is one of simplest and most used structures. It consists of a chassis with two fixed wheels that are driven by each electric motors and then one additional rear wheel can rotate freely in all directions. And then this rear wheel's effect is neglected. Every mobile robots can face systematic error and non-systematic error. Systematic error will be caused by inaccuracies in the system. On the other hand, non-systematic errors will be caused unexpectedly by external disturbance, road and environment condition changing with significant irregularities such as rough surface and slippery floors. This research mainly intended to reduce the systematic error for point to point motion task [1].

In this research, the systematics error may be the orientation error of the mobile robot. The accumulation of orientation errors will cause large position errors which increase proportional with the distance travelled by the robot. So, navigation system is mainly used to detect these errors and that navigation tasks will be simplified if odometry accuracy can be improved.

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Point to point motion task is designed and analyzed by kinematics model in order to achieve its desired positions with required time in an obstacle free indoor environment. Mobile robot kinematics is the dynamic model of how a mobile robot behaves without considering the forces that affect the motion. Mobile robot kinematics is used for position estimation and motion estimation. For the DDMR, the main mobile robot kinematics has been divided into forward kinematics and inverse kinematics. Forward kinematics provides an estimation of the robots geometry given by the robot motion and speed of its wheel. Inverse kinematics provides an estimation of the robot motion and speed of its wheel given by the robots geometry. In this research, forward kinematic model is used to move the robot. In order to complete the control task for mobile, it is needed to control the angular velocity of motor shaft beside the position control. Actual velocity of the motor shaft have to equal the desired ones. So, PID controller is chosen to control the angular velocity of both motor because of its simple theory.

The design process is clearly explained in the next section with detailed information, followed by the results and finally ends with comparison results for error checking and conclusion.

II. DESIGN OF MOBILE ROBOT

In this research, the two-wheel differential drive mobile robot is composed of two DC motors with each optical encoder. In order to control the velocity of mobile robot, robot positioning, mathematical model of kinematic and suitable controller is needed to know as a basic step. So the robot design involves-

- A. Mathematical Modelling of the Mobile Kinematics
- B. PID controller Implementation
- C. Robot positioning
- D. Software Implementation

A. Mathematical Modelling of the Mobile Kinematics

Mathematical modeling of the mobile kinematics is one of the major roles of control system in robot design. The kinematic model takes the velocity of the robot and transforms it into the generalized coordinate vector by the following equation:

$$\begin{cases} x_{t+\Delta} = x_t + V_{lin} \Delta \cos \theta_t \\ y_{t+\Delta} = y_t + V_{lin} \Delta \sin \theta_t \\ \theta_{t+\Delta} = \theta_t + \omega \Delta \end{cases} \quad 1.1$$

And then the velocity of the robot can be taken from each wheel velocity by the following equation:

$$V_{iin} = r \left(\frac{\omega_R + \omega_L}{2} \right) \tag{1.2}$$

$$\omega = r \left(\frac{\omega_R - \omega_L}{L} \right) \tag{1.3}$$

So, for the first step is to consider each wheels velocities from the parameters of the mobile robot such as the axial distance between two wheels, L and the wheel radius, r. For the next step is to describe the velocity of mobile robot from wheel velocity depend on the robot base. In this research, robot base is differential drive system. Robot chassis is made up of compacted plastic sheets in order to reduce the overall weight of the robot because this weight can affect the motor torque in control system.

The mobile robot consists of two DC motors that are fixed with each optical encoder directly. That optical encoder is used to control the motor shaft velocity based on the PID controller. So, PID controller will be discussed in the next sub-section with its basic information. Figure 1 shows the differential drive mobile robot system with required parameters.

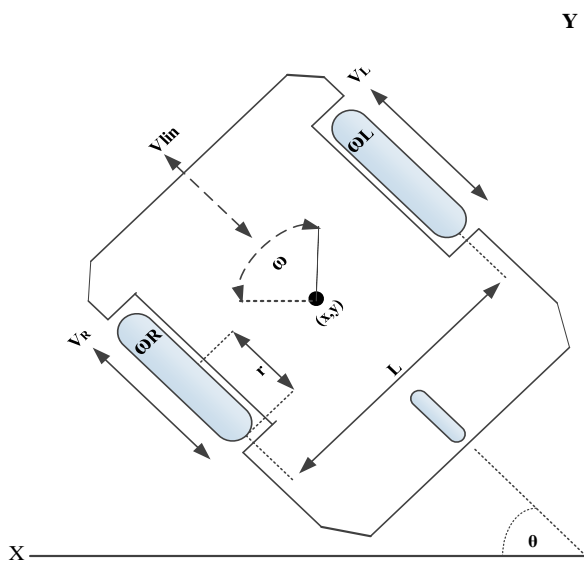


Fig.1. Parameters of mobile robot

- The parameters of the mobile robot based on the kinematic model can be written as follows-
- V_R, V_L –Right wheel and left wheel velocity respectively
- x, y –Relative position of the robot in inertia frame
- θ –Relative heading of the robot in inertia frame
- V_{iin} – Targeted linear velocity of the mobile robot
- ω –Targeted angular velocity of mobile robot
- L – Distance between left and right wheel
- $L=20\text{cm}$
- r – Radius of wheel
- $r=3.3\text{cm}$

Above parameters of differential drive mobile robot are useful as input of basic kinematics model of robot.

Linear velocity of the mobile robot is expressed in equation 1.4.

$$V_{iin} = \frac{r(\omega_R + \omega_L)}{2} \tag{1.4}$$

Angular velocity of the mobile robot is expressed in equation 1.5.

$$\omega = \frac{r(\omega_R - \omega_L)}{L} \tag{1.5}$$

B. PID controller Implementation

Matlab and Simulink software was used to tune and test the PID controller. PID controller is used to control the rotational speed of DC motor with the help of the values of the constants KP, KD and KI. KP depends on present error, KI depends on the accumulation of past errors, and KD is a prediction of future errors. That’s why, the values of the constants KP, KD and KI have to tune exactly and technically based on the DC motor transfer function with its individual parameters. In spite of the simple structure and robustness of this controller, optimally tuning gains of PID controllers in real world have been quite difficult than the simulation in Matlab [2].

The following table shows the effects of increasing each of the controller parameters KP, KI and KD. This table can be used when the values of the constants KP, KD and KI must be tuned jointly on the closed loop performance of stable plants in Matlab simulation and real world certainly.

TABLE I
EFFECTS OF INCREASING EACH OF THE PID PARAMETERS

Response	Rise Time	Overshoot	Settling Time	Steady-state Error
K_P	Decrease	Increase	Small increase	Decrease
K_I	Small decrease	Increase	Increase	Large decrease
K_D	Small decrease	decrease	decrease	Minor change

$$P(s) = \frac{\theta(s)}{V(s)} = \frac{\omega(s)}{V(s)} = \frac{K_t}{(Js+B)(Ls+R)+K_tK_e} \left[\frac{\text{rad}}{V} \right]$$

Where, $P(s)$ is the transfer function of DC motor. The block of DC motor transfer function can be shown as following figure 2.

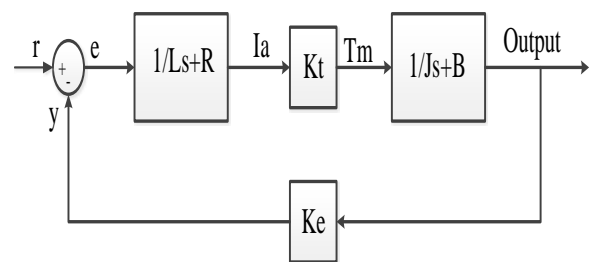


Fig.2. Block diagram of DC motor

Where,

J = system moment of inertia of the rotor

- b = motor viscous friction
- Ke = back-EMF constant
- Kt = motor torque constant
- R = armature winding resistance
- L = armature winding inductance
- V = armature winding input voltage
- I = armature winding current

The DC motor used in the robotic system is 80:1 Gearmotor with 2PPR encoder. This gear motor is a powerful 12V brushed DC motor with an 80:1 gearbox and revolution of 2 pulses per revolution of the motor shaft, which corresponds to 160 pulse per revolution of the gearbox's output shaft. These units have a 19mm-long, 4mm-diameter and O-shaped output shaft. Key specification at 12V are 120PRM and 50mA free-run, 16Nm and 1.8A stall. Table II will show the required parameters of 12V DC motor for motor control.

TABLE II
PARAMETERS FOR 12V GEAR DC MOTOR

Symbol	Parameter
K_e	0.9[v. sec/rad]
K_t	13.8[N-m/A]
R_a	1.6231[Ω]
L_a	50mA
J_m	0.051[kg.m ²]
b_m	12.4[N/rad/sec]
I_a	0.75H

The transfer function for a PID controller is given by:

$$c(s) = K_p + \frac{K_i}{s} + K_D s$$

Where K_p is the proportional gain, K_i is the integral gain and K_D is the derivative gain. And the following block diagram is the control system for DC motor with PID controller.

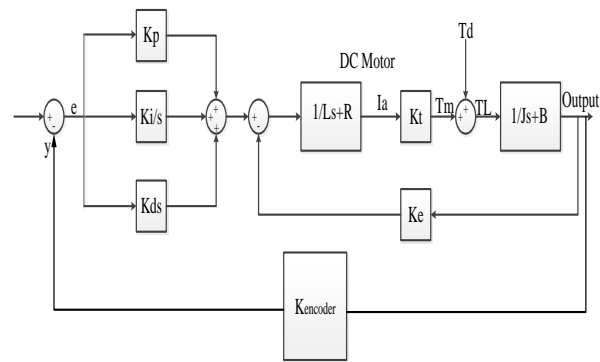


Fig.3. Block diagram of DC motor with PID controller

In figure 3, three variables (r, y and e) are desired input voltage, actual output voltage and voltage error respectively.

So, Simulink model for DC motor can be created in Matlab software by using table II. The mobile platform is arranged to reach the desired output linear speed of 0.3m/s when the motor gets 7.4V input. In figure 5, we can see the linear speed of 0.0003cm/s. So it has the linear speed error in 0.2997cm/s. Therefore, PID controller should be used in this research to reduce this error. And then the error signal is input to the PID controller. It is tuned for the gain parameters K_p , K_i and K_d of the PID controller to get the desired speed effectively using manual tuning method in MATLAB.

Manual tuning method is to first set K_i and K_d values to zero. Increase the K_p until the output of the loop oscillates, then the K_p should be set to approximately half of that value for a "quarter amplitude decay" type response. Then increase K_i until any offset is corrected in sufficient time for the process. However, too much K_i will cause instability. Finally, increase K_d , if required, until the loop is acceptably quick to reach its reference. However, too much K_d will cause excessive response and overshoot [3].

Simulink model of the closed loop DC motor with P controller is shown in figure 4. Running this model will result in response curves of linear speed of motor shaft in figure 5.

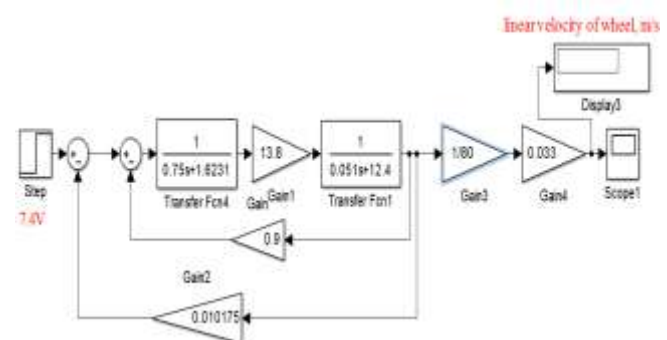


Fig.4. Simulink model of closed loop DC motor with P-controller of $K_p = 1$

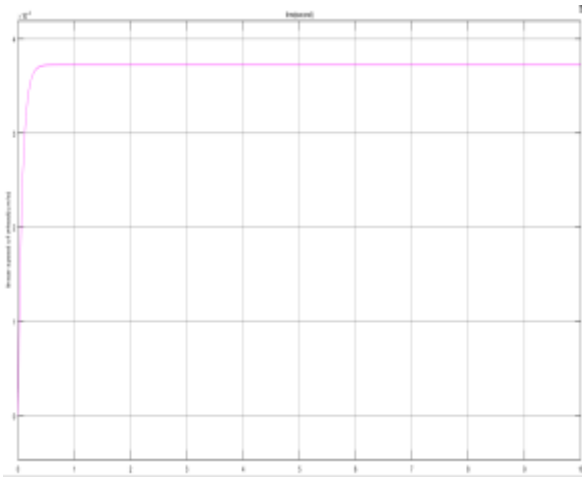


Fig.5. Response curve of the linear speed of close loop DC with P controller of $K_p = 1$

Figure 6 is the Simulink model of the closed loop DC motor, applying PID controller with gains ($K_p = 80, K_i = 3, K_d = 4$). Running this model will result in response curves of linear speed of output shaft of DC motor in figure 7. For 7.4V input, DC motor will smoothly reach desired output linear speed of 0.3cm/s in 1s, without overshoot and any steady-state error.

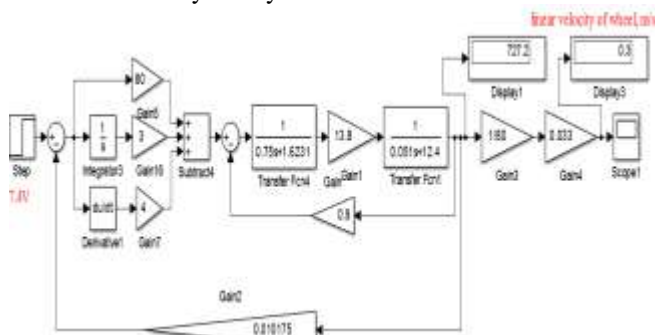


Fig.6. Simulink model of closed loop DC motor with PID-controller

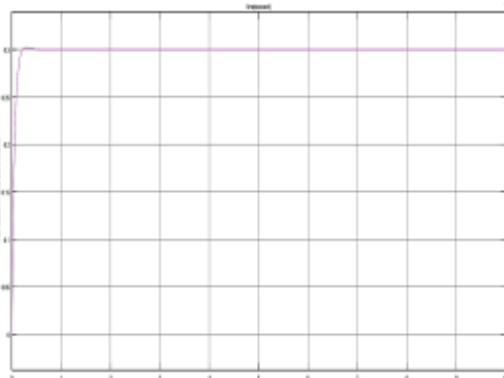


Fig.7. Response curve of the linear speed of close loop DC with PID controller

By using the kinematics model of two-wheel differential drive mobile robot $V_{lin} = \frac{V_R + V_L}{2}$, Simulink model of closed loop mobile robotic platform can be created. This simulink model is shown in figure 8. In this model, the right wheel linear speed, $V_R = 0.3cm/s$ and then the left wheel linear speed, $V_L = \frac{0.3cm}{s}$. So, the linear speed of mobile robotic platform, $V_{lin} = 0.3cm/s$. When both DC motors has the

same speed and the same direction, the robot will move in straight line. The robot will spin at a point when the both DC motor has the same speed and the opposite direction. Running this model will result in response curve of linear speed of the mobile robot in figure 9.

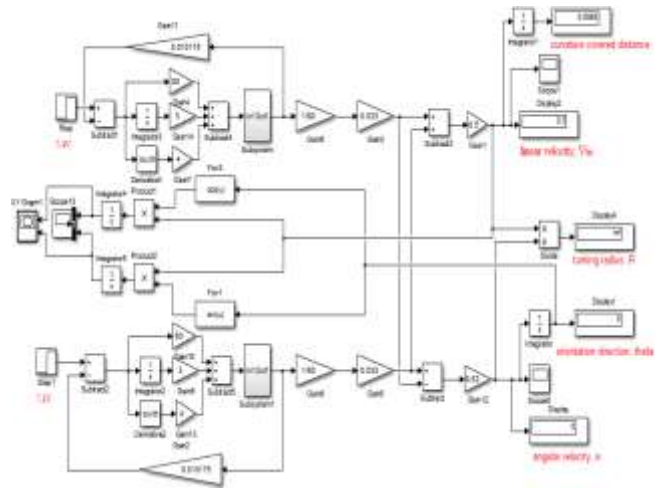


Fig.8. Simulation for the two-wheel differential drive mobile robotic platform based on kinematic model

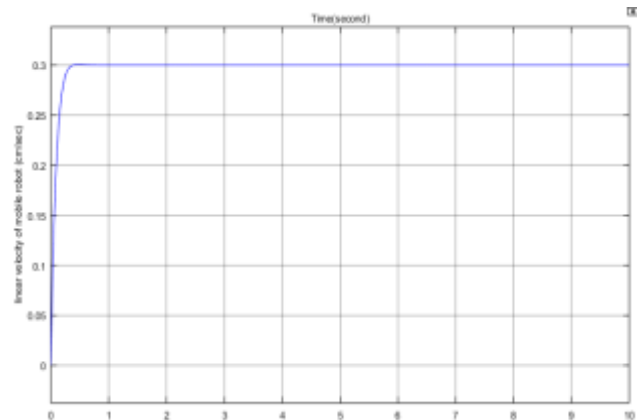


Fig.9. Response curve of the linear speed of mobile robot with PID controller with gains ($K_p = 80, K_i = 3, K_d = 4$)

C. Robot Positioning

Odometry is based on simple equations that are easily implemented and that utilize data from inexpensive wheel encoders. However, odometry is also based on the assumption that wheel revolutions can be translated into linear displacement relative to the floor. This assumption is only of limited validity. One extreme example is wheel slippage: if one wheel was to slip on, say, an oil spill, then the associated encoder would register wheel revolutions even though these revolutions would not correspond to a linear displacement of the wheel.

There are many methods for robot positioning that can roughly be categorized into two groups: relative and absolute position measurements. Odometry is one of the relative position measurement methods [4]. This method uses encoders to measure wheel rotation and/or steering orientation. There are different types of rotational displacement and velocity sensors in use today:

- 1 Brush encoder
- 2 Potentiometer
- 3 Synchro

- 4 Resolver
- 5 Optical encoder
- 6 Magnetic encoder
- 7 Inductive encoder
- 8 Capacitive encoder

In this research, magnetic encoder is used to get the actual distance for position feedback loop. Relative position estimation is extremely dependent on the measurement of the robot's velocity. This encoder can provide 4 pulses per one revolution. If the robot is moving straight ahead, we could simply count encoder pulses to determine its new location.

Distance traveled per pulse = wheel circumference/ no. of pulses per revolution = $\frac{2\pi r}{4} = \frac{2 \times \pi \times 3.3}{4} = 5.184 \text{cm/pulse}$

While turning is centered around a circle whose diameter equals the distance between wheels (20cm), i.e. whose diameter=40cm

Hence each pulse results in a turn = $(5.184/40) \times 360 = 0.576^\circ$ per pulse

D. Software Implementation

The software of the present work includes motor driving system and control system. Motor driving system was done using kinematics model and its differential drive system. The control system is proposed with two separated feedback loops: a velocity feedback loop and a position feedback loop. Figure 10 shows the overall flow chat for the whole system.

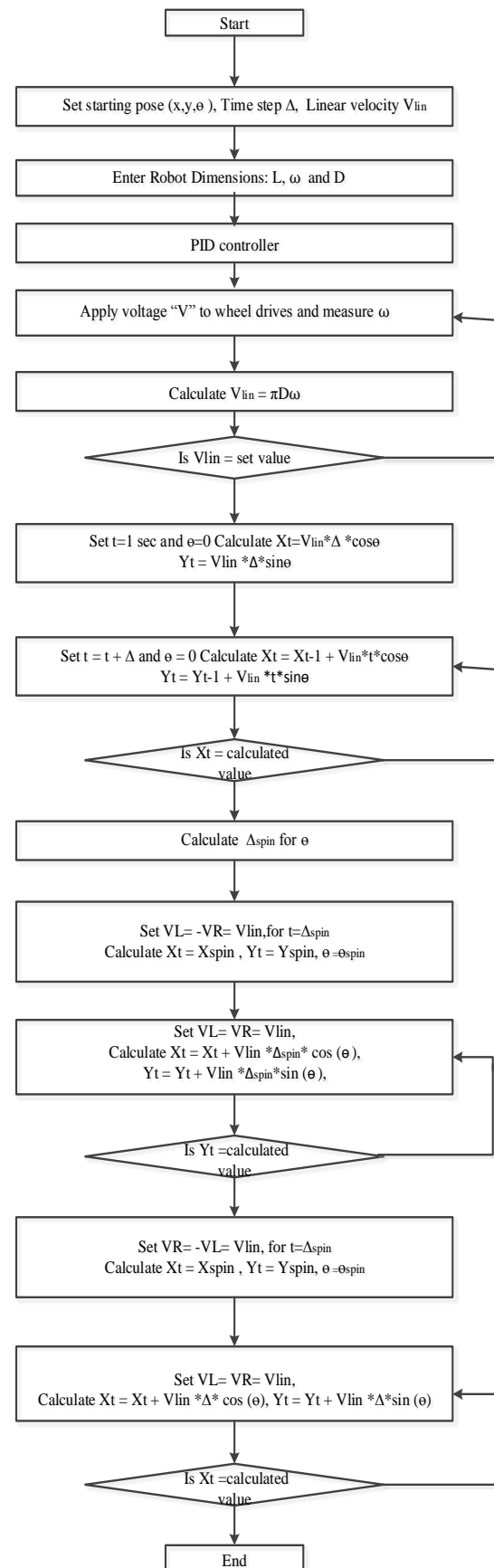


Fig.10. Overall flow chart of the whole mobile robot control system

The first section for software implementation is motor driving system. Firstly the robot needs to fix with its linear velocity, V_{lin} and need to set a starting pose (x, y, θ) and

need to assign a suitable step time Δ . For the next step, the driving system need basically the robot dimensions (L, D, ω) . And then the voltage is applied to robot wheel. Both motor get the same velocity $(V_R = V_L = V_{lin})$ when the robot moves in straight line [5] [6]. Two motors get the same velocity but in opposite direction when the robot spin at a point. So, control system is needed to control the velocity of both motor to be same by checking with the reference speed.

The next section for software implementation is velocity feedback loop. PID controller is the main role in this section. When the gradual voltage is applied to robot wheel, linear velocity of mobile robot is checked with its set value by the help of optical encoder. If the error is detected, PID controller provides both an acceptable stability and damping. This velocity feedback loop can be seen in first loop of overall flow chart of mobile robot.

The final section for software implementation is position feedback loop. Initializing the robot tack the first point. At that time, if robot detect the error in distance or angle, the odometry method provides both an acceptable degree of error reduction. The odometry method mainly uses the output signals from the sensors such as encoder, and magnetic hall sensor by fitting to the wheel of the robot and another sensor is compass. Odometry model estimates the position of the robot with use of the sensor output. The position feedback loop can be seen in second and third loop of overall flow chart of mobile robot.

III. RESULT

The user can see the generalized coordinate vector in the inertial frame from the output results of kinematic models for differential drive mobile robot. The experimental result is the three forms of movement. They are the movement along X-axis, Y-axis and diagonal. Firstly the robot moves from original point (15,15) to the first point (45,15) in zero degree for the first step. This movement can see in figure 11. The robot moves from original point (15,15) to the second point (15,45) in 90 degree for the second step. This movement can be seen in figure 12. The robot moves from original point (15,15) to the third point (75,75) in 45 degree for the third step. This movement can be seen in figure 13. For each step, we can see the error in position and the robot can reach or not to its desired point exactly.

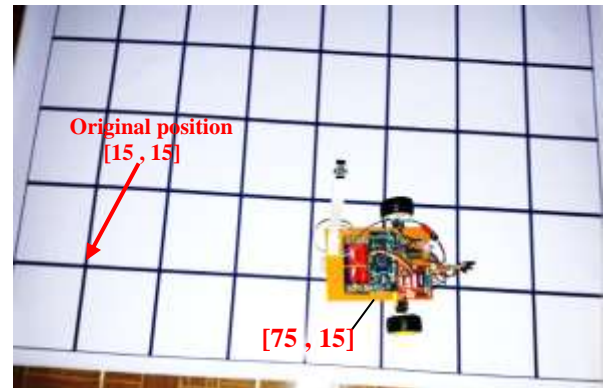


Fig.11. Plot for the mobile robot movement along X-axis

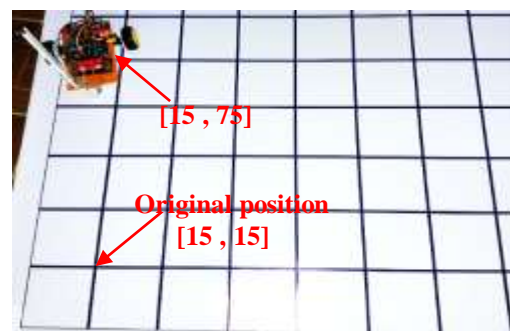


Fig.12. Plot for the mobile robot movement along Y-axis

By using the above three movements, the robot can move from one start point to desired setpoints directly along X-axis, Y-axis and diagonal. The robot moves from start point (15, 15) to the target point (75,75) using three movements. This result for figure 14 will be shown by graph.

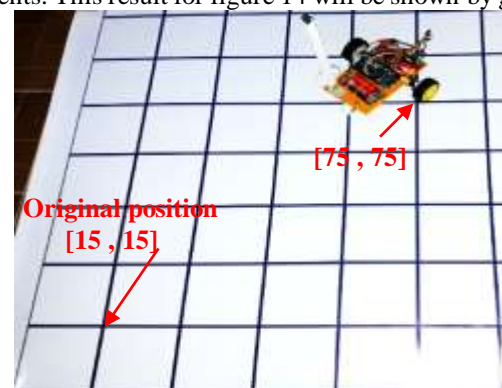


Fig.3. Plot for the mobile robot movement in diagonal

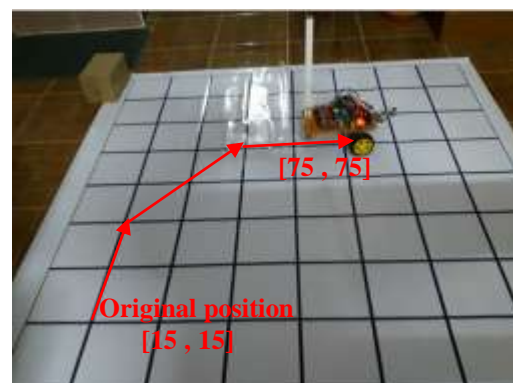


Fig.14. Plot for the mobile robot movement along X-axis, Y-axis and diagonal

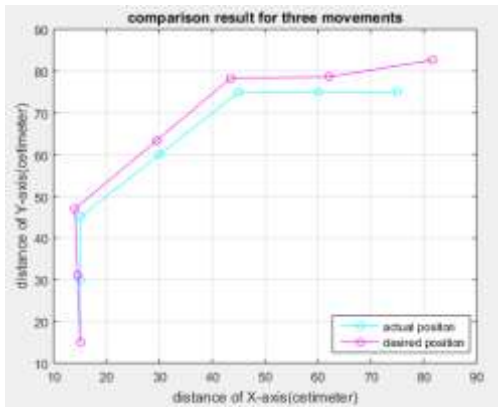


Fig.15. Graph of comparison result for actual position and desired position

IV. COMPARISON RESULT FOR MOBILE ROBOT WITH ERROR CHECKING

In this section, the desired coordinate points of mobile robot are compared with the actual output coordinate points of the experimental result in real world. These actual coordinate points can be derived with the help of proposed Kinematics model and the differential drive equations. The following equations are the basic theories of Kinematics model for two-wheel DDMR.

$$\text{Distance, } D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad 1.6$$

$$\text{Angle, } \theta_t = \arctan\left[\frac{(y_2 - y_1)}{(x_2 - x_1)}\right] \quad 1.7$$

$$x_{t+\Delta} = x_t + V_{lin} \Delta \cos \theta_t \quad 1.8$$

$$y_{t+\Delta} = y_t + V_{lin} \Delta \sin \theta_t$$

$$\theta_{t+\Delta} = \theta_t + \omega \Delta$$

In this research, the linear velocity of the robot is fixed to 30 cm/sec as reference speed. The angular velocity of each motor is then measured at small intervals, checked with the reference speed. If the error exists, the control voltage of DC motor connected to each wheel is modified and applied to the corresponding motor. By using this way, the experimental work will be completed until to reach its final point. So, the comparison results between the desired coordinates see in table III and table IV for figure 11, 12 and 13.

TABLE III
CALCULATION RESULTS FOR $x_{t+1}, y_{t+1}, \theta_{t+1}$

No:	[x1,y1]	[x2,y2]	θ_t	D	x_{t+1}	y_{t+1}	θ_{t+1}
1	[15,15]	[75,15]	0	60	75	15	0
2	[15,15]	[15,75]	90	60	15	75	90
3	[15,15]	[75,75]	45	84.85	75	75	45

TABLE IV

EXPERIMENTAL RESULTS FOR $x_{t+1}, y_{t+1}, \theta_{t+1}$

No:	[x1,y1]	[x2,y2]	θ_t	D	x_{t+1}	y_{t+1}	θ_{t+1}
1	[15,15]	[75,15]	5	64.6	79.6	17	5
2	[15,15]	[15,75]	101	66.67	8	81	101
3	[15,15]	[75,75]	70	86.53	63	87	70

V. CONCLUSION

According to the comparison results of table III, the robot astray to its goal points with the maximum error of $x_{t+\Delta} = 12cm, y_{t+\Delta} = 12cm, \theta_{t+\Delta} = 56.3 \text{ deg ree}$. These conditions are depended on the accuracy of DC motor such as motor torque, load and variable power supply. By satisfying these facts, the robot can reach to its final point without any error in distance and direction. In addition, these mobile robots can be used for mining application by moving from point to other point in desired distance and direction in order to find the dangerous mine if the robot control system is very robust. And then these mobile robot can be also candidates for farming applications, as well as for transportation in nuclear plants and factories.

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