ISSN: 2278 – 7798
International Journal of Science, Engineering and Technology Research (IJSETR)
Volume 5, Issue 10, October 2016

An Integrated Production-Inventory Model with Weibull Deterioration and Stock-Dependent Demand under JIT Production System

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Abstract— In this paper, we present an analytical model for a single-supplier single-buyer integrated inventory system under just-in-time (JIT) multiple deliveries. The model is studied under the assumption that the demand is dependent on stock-level and the units in inventory deteriorate with time according to Weibull rate. We incorporate a single-supplier single-buyer cooperation from the perspectives of both the supplier and the buyer. A solution procedure is provided to obtain the optimal solution. A numerical example is given to illustrate the model. Sensitivity analysis is also performed to explore the effect of key parameters on procurement quantity, number of deliveries and integrated total expected cost.

Keywords— Integrated inventory system, JIT delivery, Weibull deterioration, stock-dependent demand.

I. INTRODUCTION

Inventory theory has been one of the most successful Operational Research techniques to be applied in business, industry and public sector. Inventories are resources of any kind having an economic value and have an extremely important role in the economy of any country. Primarily, inventory is held for transaction purposes. Today's inventory is tomorrow's consumption. The primary function of inventory is to provide customer service considering factors such as availability of consumer goods at the right time, in the right place and at the right cost. Several costs are associated with inventories. These costs are broadly classified into fixed cost and variable cost which together form the total cost of the inventory system. Analysis of an inventory system includes the following steps:

(i) Determination of the properties of the system.
(ii) Formulation of the inventory problem.
(iii) Development of a model of the system.
(iv) Derivation of a solution of the system.

Inventory models are useful in determining the optimal stock of items and deciding the optimality of total cost. The main objective of any production planning and inventory management system is to minimize the total system cost.

Research in the area of inventory theory has made enormous progress in the last two decades. Due to rising costs, shrinking resources, shortened product life cycle, increasing competitive pressure and quicker response, much attention has been paid to the collaboration between the members of the supply chain. The collaboration of the supplier and buyer is one of the key factors for successful supply chain management. This collaboration involves commitment to long term co-operation, shared costs, joint problem solving and even profit sharing. Close co-operation between supplier and buyer can result in more cost effective production and distribution as well as a faster response to customer's demand which creates a beneficial environment for them. Research on the integrated supplier-buyer co-operative inventory problem primarily focuses on the production shipment schedule in terms of the size and frequency of shipments transferred between both parties.

Just-in-time (JIT) plays a crucial role in the present supply chain environments. Companies are using JIT production to gain and maintain a competitive advantage. Successful implementation of JIT requires a new spirit of co-operation between the supplier and buyer. The characteristic features of JIT systems include consistency in quality, small lot sizes, frequent delivery, short lead time and closer supplier ties. Implementation of JIT policies in many companies have also led to the reduction in scrap and storage space requirements. Extensive research has been conducted regarding the producer-buyer relationship in JIT environments. Ben-Daya and Hariga [1] developed a single vendor single buyer integrated production inventory model with stochastic demand and assumed that the lead time varies linearly with the lot size. Yang and Pan [14] have proposed an integrated inventory model involving deterministic variable lead time and quality improvement investment with normally distributed lead time demand under JIT purchasing model without shortages. An extension of Yang and Pan's model has been studied by Ouyang et al. [6]. The model proposed by them allows for shortages and considers reorder point as one of the decision variables which improves the system performance and annual joint total expected cost. For models where shortages are allowed, it is often assumed that the shortages are either backlogged or completely lost. As a physical phenomenon, some customers may like to prefer backlogging during the shortage period, while others would not. So, researchers have turned their attention to models that allow backlogging. Roy et al. [7] have framed an integrated producer-buyer inventory model with constant demand and small lot size in two different production environment: an EMQ (Economic Manufacturing Quantity) and a JIT by allowing shortages for the buyer and backlogged at the beginning of the buyer's cycle.

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While maintaining the inventories, deterioration of items is inherent. Most of the physical goods deteriorate over time. Deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage and loss of entity or loss of marginal value of a commodity that results in decreasing usefulness from the original one. (Chung and Wee [2]). Commodities such as fruits, vegetables, food stuffs are subject to direct spoilage while kept in store. Highly volatile liquids such as gasoline, alcohol, turpentine undergo physical depletion over time through the process of evaporation. Electronic goods, radio-active substances, photographic films, grains deteriorate through a gradual loss of potential or utility with the passage of time. The control and maintenance of inventories for deteriorating items have received great attention in recent years. Several researchers have studied deteriorating inventory in the past with different assumptions on the patterns of deterioration - constant deterioration, exponential deterioration, Weibull deterioration, Gamma deterioration, three parameter Weibull deterioration etc. Many of the researchers in deteriorating inventory have assumed constant rate of deterioration. Yang and Wee [15] have presented a collaborative inventory system with permissible delay in payment for deteriorating items with finite replenishment rate and price-sensitive demand. Chung and Wee [2] have established an integrated two stage production inventory model for the buyer and the supplier with stock dependent selling rates, constant rate of deterioration and imperfect items under JIT multiple deliveries. Jong and Wee [4] have developed an integrated single-buyer single-supplier deteriorating model in a JIT environment and have considered the costs and benefits of implementing JIT delivery. Shah [8] has considered an integrated inventory model for single supplier and single buyer under JIT deliveries when demand is stock dependent and the units in inventory deteriorate at a constant rate. Yang and Wee [13] have developed an economic ordering policy of deteriorating item with a constant production and demand rates.

Weibull distribution has been extensively used in literature to pattern a varying rate of deterioration. Many researchers have incorporated a Weibull deterioration rate. Wu [12] has proposed a deterministic inventory model for items with time varying demand, Weibull distribution deterioration by allowing shortages. Wee at al. [11] have analyzed an integrated two stage inventory model with Weibull distribution deteriorating items by assuming a constant demand rate and limited retailer storage space. Lo et al. [5] have presented an integrated production inventory model by assuming a varying rate of deterioration (Weibull distribution deterioration), partial back ordering, inflation, imperfect production processes and multiple deliveries. They have used the discounted cash flow and classical optimization technique to derive the optimal solution. Chung and Wee [3] have investigated an integrated production inventory deteriorating model considering the pricing policy, imperfect production, inspection planning, warranty period, stock level dependent demand with the Weibull deterioration, partial back order and inflation. Skouri et al. [9] have proposed an inventory model with ramp type demand rate, time dependent (Weibull) deterioration rate and partial backlogging under two different replenishment policies: (a) starting with no shortages and (b) starting with shortages. Sunil and Pravin [10] have developed an Economic Production quantity model without shortages when the deterioration rate follows three parameter Weibull distribution with constant demand and production rates and time varying holding cost.

This paper is organized as follows: In section 2, notations and assumptions are provided. In section 3, a mathematical model taking into account the perspectives of both the buyer and the supplier with the stock dependent demand and the multiple deliveries per order is developed. Research is focused on the production shipment schedule in terms of the size and number of shipments transferred between both parties under perfect quality. The deterioration of items in the inventory follows two parameter Weibull distribution. A solution procedure is provided to illustrate the optimization procedure in section 3.5. In section 4, a numerical example is provided to validate the proposed model. Sensitivity analysis is also carried out and explores the effect of key parameters on procurement quantity, number of deliveries and integrated total expected cost. Conclusions are summarized in section 5.

II. NOTATIONS AND ASSUMPTIONS

The proposed model is derived using following notations and assumptions

A. Notations for the Supplier

$R(I_x(t))$: $(= u + vI_x(t), x = s1, s2, b)$ demand rate where u is constant demand and v is stock dependent parameter. $u > 0$ and $0 \le v < 1$

$P(I_x(t))$	$(= cR(I_x(t))$, $c > 1$, production rate
Q_s	Production lot-size per cycle time
T_1	Production time in years (a decision variable)
T_2	Non-production time in years (a decision variable)
$Is1(t1)$	Inventory level during the production period for the supplier
$Is2(t2)$	Inventory level during the non-production period for the supplier
h_s	Holding cost per unit per unit time for the supplier
A_s	Supplier's set up cost per cycle time
Cds	Unit deteriorating cost for the supplier
OC_s	Set up cost per unit time for the supplier
DC_s	Deterioration cost per unit time for the supplier
HC_s	Inventory holding cost per unit time for the supplier
TC_s	Total cost per unit time for the supplier

B. NOTATIONS FOR THE BUYER

$I_b(t)$	Inventory level per unit time for the buyer
h_b	Holding cost per unit per unit time for the buyer
T_b	Delivery cycle time for the buyer in years(a decision variable)
A_b	Ordering cost per order
N	Number of deliveries per cycle time (a decision variable), integer valued.
Q_b	Procurement quantity per delivery
Cdb	Unit deteriorating cost for the buyer
OC_b	Ordering cost per unit time for the buyer
DC_b	Deterioration cost per unit time for the buyer
HC_b	Inventory holding cost per unit time for the buyer
TC_b	Total cost per unit time for the buyer

C. Assumptions

1. The supply chain comprises of single-supplier, single-buyer and single-item.
2. The demand rate is stock-dependent and the production rate is proportional to the demand rate.
3. Shortages are not allowed for the supplier and the buyer.
4. The lead-time is zero or negligible.
5. The study considers complete cooperation of supplier and buyer.
6. The number of supplier's shipment is an integer.
7. Deterioration of the item follows a two-parameter Weibull distribution $f(t) = abt^{b-1}$ where $0 < a < 1$ is the scale parameter and $b > 0$ is the shape parameter.
8. The deteriorated units can neither be repaired nor replaced during the cycle time.
9. Deterioration of the product is considered only after they have been received into the inventory.
10. Multiple deliveries per order are considered.

Following Chung and Wee [2], we assume that $h_b \ge h_s$ and $C_{db} \ge C_{ds}$. Since production is continuous during the cycle and Q_b units are received by the buyer in each delivery, stock moves up in supplier's inventory system. If $h_b < h_s$ and $C_{db} < C_{ds}$, total cost for the supplier increases and so also the integrated total cost.

Small and medium sized industries do not tolerate waste due to deterioration of the buffer stock. But, they want to improve customer service and so they prefer JIT system. JIT production begins when an order arrives and production takes place according to the buyer's requirements at short notice. Since units in inventory deteriorate with time, the buyers normally select the suppliers who have consistently supplied high quality product with delivery reliability to establish a long-term relationship. So, we assume that the transportation cost per shipment is constant and hence it is not considered in this model.

III. MATHEMATICAL MODEL AND SOLUTION

A. Model Description

The objective of this integrated inventory system is to determine the optimal policy for the single-supplier, single-buyer and single-item with the stock-dependent demand under JIT delivery in order to keep the total relevant cost as low as possible.

B. Model Development

Our model is developed in two stages. The first stage is the manufacturer's inventory system. In this stage, the manufacturer has to take optimal decisions on the quantity to be manufactured to meet buyer's demands taking into account the various costs involved in this inventory system. The second stage is the buyer's inventory system. In this stage, fixed quantities of finished goods with multiple deliveries are delivered to the buyer at a fixed time interval.

The manufacturer's inventory system can be divided into two independent phases depicted by T_1 and T_2. During T_1 time period, there is an inventory buildup and so there is deterioration becomes effective. At $t_1 = T_1$, the production stops and the inventory level increases to its maximum. There is no production during T_2 time period and the inventory level decreases due to deterioration. The inventory level becomes Q_s at $t_2 = T_2$. In buyer's inventory system, there is no production during T_b time period and the inventory level decreases due to demand and deterioration. The buyer has an annual demand of Q_s units for the given product and places regular orders of fixed size Q_s. Each order is received by the buyer in N deliveries, each with a lot of Q_b units at a fixed time interval T_b, that is $Q_s = NQ_b$. The objective of the model is to minimize the total joint annual costs incurred by the supplier and the buyer.

The manufacturer's inventory system at any time t_1 during the production period $[0, T_1]$ can be described by the following differential equation (Jong and Wee [4] and Shah [8]):

$$\frac{dI_{s1}(t_1)}{dt_1} = P(I_{s1}(t_1)) - abt_1^{b-1} I_{s1}(t_1), \quad 0 \le t_1 \le T_1 \tag{1}$$

with initial condition $I_{s1}(0) = 0$.

The manufacturer's inventory system at any time t_2 during the non-production period $[0, T_2]$ can be described by the following differential equation:

$$\frac{dI_{s2}(t_2)}{dt_2} = -abt_2^{b-1} I_{s2}(t_2), \quad 0 \le t_2 \le T_2 \tag{2}$$

with boundary condition $I_{s2}(T_2) = Q_s = NQ_b$.

The buyer's inventory system at any time t during $[0, T_b]$ can be described by the following differential equation:

$$\frac{dI_b(t)}{dt} = -R(I_b(t)) - abt^{b-1}I_b(t), \quad 0 \le t \le T_b \qquad (3)$$

with boundary condition $I_b(T_b) = 0$

The solution of these differential equations are

$$I_{s1}(t_1) = \frac{cu}{e^{at_1^b - cvt_1}} \int_0^{t_1} e^{ax^b - cvx}\, dx, \quad 0 \le t_1 \le T_1 \qquad (4)$$

$$I_{s2}(t_2) = Q_s\, e^{a(T_2^b - t_2^b)}, \quad 0 \le t_2 \le T_2 \qquad (5)$$

$$I_b(t) = \frac{u}{e^{vt+at^b}}\left(\int_0^{T_b} e^{vy+ay^b}\, dy - \int_0^{t} e^{vy+ay^b}\, dy\right), \quad 0 \le t \le T_b \qquad (6)$$

From (6), when $I_b(0) = Q_b$ the procurement quantity Q_b of the buyer is

$$Q_b = u\left(\int_0^{T_b} e^{vy+ay^b}\, dy\right) \qquad (7)$$

There are N deliveries per cycle, each with delivery cycle time T_b. Assuming the cycle time is T, we have $T = NT_b$

C. BUYER'S RELEVANT COSTS

- Ordering cost per unit time

At the start of the delivery, there is an initial ordering cost A_b. The buyer's ordering cost per unit time is

$$OC_b = \frac{NA_b}{T} \qquad (8)$$

- Holding cost per unit time

Inventory is carried during time period T_b. The holding cost is

$$HC_b = \frac{Nh_b}{T}\int_0^{T_b} I_b(t)\, dt = \frac{Nh_b u}{T}\left[\int_0^{T_b} \frac{1}{e^{vt+at^b}}\, dt \int_0^{T_b} e^{vy+ay^b}\, dy - \int_0^{T_b} \frac{1}{e^{vt+at^b}}\left(\int_0^{t} e^{vy+ay^b}\, dy\right) dt\right] \qquad (9)$$

- Deteriorating cost per unit time

The item cost includes loss due to deterioration. The deterioration cost is

$$DC_b = \frac{NC_{db}}{T}(Q_b - R(I_b(T_b))T_b) = \frac{NC_{db}u}{T}\left[\int_0^{T_b} e^{vy+ay^b}\, dy - T_b\right] \qquad (10)$$

The total cost for the buyer is the sum of the ordering cost, the holding cost and the deteriorating cost. Thus, the buyer's total cost per unit time is

$$TC_b(N, T_b) = OC_b + HC_b + DC_b \qquad (11)$$

D. SUPPLIER'S RELEVANT COSTS

- Set-up cost per unit time

At the start of the cycle, there is an initial production set up cost A_s. The set up cost per unit time is

$$OC_s = \frac{A_s}{T} \qquad (12)$$

- Holding cost per unit time

Inventory is carried by the manufacturer during T_1 and T_2 time periods. During these periods, the items are delivered to the buyer in N deliveries, each with a lot of Q_b units at a fixed time interval T_b. Thus, the supplier's holding cost is

$$HC_s = \frac{h_s}{T}\left[\int_0^{T_1} I_{s1}(t_1)\, dt_1 + \int_0^{T_2} I_{s2}(t_2)\, dt_2 - T_b(Q_b + 2Q_b + \dots + (N-1)Q_b)\right] = \frac{h_s}{T}\left[\int_0^{T_1} I_{s1}(t_1)\, dt_1 + \int_0^{T_2} I_{s2}(t_2)\, dt_2 - \frac{N(N-1)Q_bT_b}{2}\right] \qquad (13)$$

- Deteriorating cost per unit time

The item cost includes loss due to deterioration. The deterioration cost is

$$DC_s = \frac{C_{ds}}{T}[P(I_{s1}(T_1))T_1 - Q_s] = \frac{C_{ds}}{T}\left[cuT_1\left(1 + \frac{cv}{e^{aT_1^b - cvT_1}}\int_0^{T_1} e^{ax^b - cvx}\, dx\right) - NQ_b\right] \qquad (14)$$

The total cost for the supplier during the cycle is the sum of the set up cost, the holding cost and the deteriorating cost. Thus, the supplier's total cost per unit time is

$$TC_s(N, T_1, T_2, T_b) = OC_s + HC_s + DC_s \qquad (15)$$

Consequently, the integrated total cost of the supplier and the buyer is

$$TC(N, T_1, T_2, T_b) = TC_s(N, T_1, T_2, T_b) + TC_b(N, T_b) \qquad (16)$$

E. SOLUTION PROCEDURE

The objective of the problem is to determine the optimal values of the number of shipments N, the time periods T_1, T_2 and T_b that minimize the total expected cost (16).

The following optimization technique is used to derive the optimal values.

Step 1: Since the number of deliveries per cycle time N is a discrete variable, Start by choosing N=1.

Step 2: Take the partial derivatives of $TC(N, T_1, T_2, T_b)$ with respect to T_1, T_2, and T_b and equate the results to zero. The necessary conditions for optimality are

$$\frac{\partial TC(N,T_1,T_2,T_b)}{\partial T_1}=0,\quad \frac{\partial TC(N,T_1,T_2,T_b)}{\partial T_2}=0,\quad \frac{\partial TC(N,T_1,T_2,T_b)}{\partial T_b}=0$$

These simultaneous equations can be solved for T_1, T_2, and T_b.

Step 3: Substitute N, T_1, T_2 and T_b found at step 2, into equation (16) and find the $TC(N, T_1, T_2, T_b)$.

Step 4: Let N=N+1, repeat steps (2) and (3) until the minimum $TC(N, T_1(N), T_2(N), T_b(N))$ is found.

The optimal values of N, T_1, T_2, and T_b denoted by N^*, $T_1(N^*)$, $T_2(N^*)$, and $T_b(N^*)$ respectively are obtained such that

$$TC(N^*-1, T_1(N^*-1), T_2(N^*-1), T_b(N^*-1)) \geq TC(N^*, T_1(N^*), T_2(N^*), T_b(N^*))$$

And

$$TC(N^*, T_1(N^*), T_2(N^*), T_b(N^*)) \leq TC(N^*+1, T_1(N^*+1), T_2(N^*+1), T_b(N^*+1))$$

Step 5: From equation (16), we have the minimum value of the average total integrated cost per unit time which is given as

$$TC^*(N^*, T_1(N^*), T_2(N^*), T_b(N^*)) = TC_s^*(N^*, T_1(N^*), T_2(N^*), T_b(N^*)) + TC_b^*(N^*, T_b(N^*)) \quad (17)$$

Step 6: Substituting T_b^* in equation (7), we find the optimal procurement quantity Q_b of the buyer denoted by Q^*_b given by

$$Q_b^* = u\int_0^{T_b^*} e^{vy+ay^b}\,dy \quad (18)$$

Since the integrated total cost function given in equation (16) is a function of four variables, we solve for the optimal values of N, T_1, T_2 and T_b by using pattern search procedure using MATLAB.

IV. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

In order to illustrate the above solution procedure and validate our proposed model, we consider the following numerical example which has been used by Jong and Wee [4] and Shah [8]: c=1.5, u=5000, v=0.05, A_s=\$600, h_s=\$0.9, C_{ds}=\$3.5, A_b=\$100, h_b=\$1.3, C_{db}=\$4, a=0.001, b=2.5. The current study uses the percentage of integrated total cost difference to analyze the proposed model. The percentage of integrated total cost difference is defined as:

$$PICD = \frac{(TC-TC^*)}{TC^*}$$

By applying the solution procedure stated in section 3.5, we obtain the optimal solution as follows: $N^*=2$, T_1^*=0.401 years, T_2^*=0.001 years, T_b^*=0.305 years and the minimum integrated total cost TC^*=\$2849.52. The optimal procurement quantity Q^*_b= 1536.71.

When one subset of the parameters set S = {{u}, {A_s, A_b}, {h_s, h_b}, {C_{ds}, C_{db}}, {a}, {b}, {v}, {c}} decreases by -10% , -20% or increases by +10%, +20%, the relationships between the known parameters, the decision variables and the percentage of integrated total cost difference are given in Tables (1-8) and Figure 1.

Table 1: Sensitivity analysis when constant demand rate (u) changes

u	4000	4500	{5000}	5500	6000
N	3	3	2	2	2
T_1	0.557	0.562	0.401	0.382	0.365
T_2	0.001	0.001	0.001	0.001	0.001
T_b	0.280	0.281	0.305	0.289	0.276
TC	2349.15	2578.18	2849.52*	3021.44	3190.76
Q_b	1127.89	1273.44	1536.71	1601.06	1667.50
$PICD$(%)	-17.56	-9.52	0.00	6.03	11.98

*: the optimal total cost; {}: the base column

Table 2: Sensitivity analysis when supplier's set up cost and buyer's ordering cost (A_s,A_b) change

A_s	480	540	{600}	660	720
A_b	80	90	{100}	110	120
N	2	2	2	3	3
T_1	0.387	0.407	0.401	0.550	0.554
T_2	0.001	0.001	0.001	0.001	0.001
T_b	0.294	0.309	0.305	0.274	0.276
TC	2584.35	2723.54	2849.52*	2938.18	3044.87
Q_b	1480.88	1557.02	1536.71	1379.44	1389.58
$PICD$(%)	-9.31	-4.42	0.00	3.11	6.86

*: the optimal total cost; {}: the base column

Table 3: Sensitivity analysis when holding costs change ($h_b > h_s$)

h_b	1.04	1.17	{1.3}	1.43	1.56
h_s	0.72	0.81	{0.9}	0.99	1.08
N	2	2	2	2	3
T_1	0.365	0.422	0.401	0.408	0.445
T_2	0.001	0.001	0.001	0.001	0.001
T_b	0.277	0.321	0.305	0.310	0.220
TC	2646.90	2727.51	2849.52*	2970.40	3048.85
Q_b	1394.65	1617.98	1536.71	1562.10	1106.08
$PICD$(%)	-7.11	-4.28	0.00	4.24	7.00

*: the optimal total cost; {}: the base column

Table 4: Sensitivity analysis when deteriorating costs change ($C_{db} > C_{ds}$)

Cdb	3.2	3.6	{4}	4.4	4.8
Cds	2.8	3.15	{3.5}	3.85	4.2
N	3	3	2	2	2
T_1	0.544	0.494	0.401	0.400	0.330
T_2	0.001	0.001	0.001	0.001	0.001
T_b	0.273	0.246	0.305	0.303	0.249
TC	2527.70	2713.78	2849.52*	2905.13	3003.59
Q_b	1374.37	1237.61	1536.71	1526.56	1252.79
$PICD$(%)	-11.29	-4.76	0.00	1.95	5.13

*: the optimal total cost; {}: the base column

Table 5: Sensitivity analysis when scale parameter (a) changes

a	0.0008	0.0009	{0.001}	0.0011	0.0012
N	2	2	2	2	2
T_1	0.401	0.401	0.401	0.401	0.401
T_2	0.001	0.001	0.001	0.001	0.001
T_b	0.305	0.305	0.305	0.305	0.305
TC	2849.58	2849.63	2849.52*	2849.68	2849.62
Q_b	1536.71	1536.71	1536.71	1536.71	1536.71
$PICD$(%)	0.00	0.00	0.00	0.01	0.00

*: the optimal total cost; {}: the base column

Table 6: Sensitivity analysis when shape parameter (b) changes

b	2	2.25	{2.5}	2.75	3
N	2	2	2	2	2
T_1	0.401	0.401	0.401	0.401	0.401
T_2	0.001	0.001	0.001	0.001	0.001
T_b	0.305	0.305	0.305	0.305	0.305
TC	2849.88	2850.17	2849.52*	2849.57	2849.76
Q_b	1536.74	1536.72	1536.71	1536.70	1536.70
$PICD$(%)	0.01	0.02	0.00	0.00	0.01

*: the optimal total cost; {}: the base column

Table 7: Sensitivity analysis when stock dependent demand rate (v) changes

v	0.04	0.045	{0.05}	0.055	0.06
N	2	2	2	2	2
T_1	0.401	0.403	0.401	0.400	0.398
T_2	0.001	0.001	0.001	0.001	0.001
T_b	0.305	0.305	0.305	0.304	0.304
TC	2824.97	2837.51	2849.52*	2862.02	2874.45
Q_b	1534.36	1535.54	1536.71	1532.80	1533.97
$PICD$(%)	-0.86	-0.42	0.00	0.44	0.87

*: the optimal total cost; {}: the base column

Table 8: Sensitivity analysis when production rate (c) changes

c	1.2	1.35	{1.5}	1.65	1.8
N	2	2	2	2	2
T_1	0.437	0.406	0.401	0.322	0.354
T_2	0.001	0.001	0.001	0.001	0.001
T_b	0.261	0.275	0.305	0.269	0.326
TC	3369.88	3080.54	2849.52*	2702.45	2537.82
Q_b	1313.57	1384.51	1536.71	1354.10	1643.39
$PICD$(%)	18.26	8.11	0.00	-5.16	-10.94

*: the optimal total cost; {}: the base column

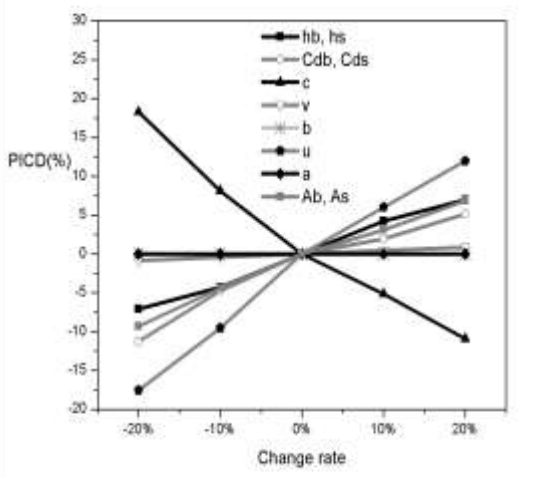

Figure 1: PICD vs. change rate of various parameters

The main observations drawn from the sensitivity analysis are as follows:

(a) The optimal number of deliveries (N) is more sensitive to the parameters {u}, {A_s, A_b}, {h_s, h_b} and {C_{ds}, C_{db}} [Tables 1 to 4]. When the supplier's set up cost and the buyer's ordering cost {A_s, A_b} and the supplier's and buyer's holding costs {h_s, h_b} increase, the optimal delivery number (N) tends to increase [Table 2 and Table 3]. When the constant demand rate {u} and the supplier's and buyer's deteriorating costs {C_{ds}, C_{db}} increase, the optimal delivery number (N) tends to decrease [Table 1 and Table 4].

(b) The procurement quantity (Q_b) is more sensitive to the parameters {u}, {A_s, A_b}, {h_s, h_b}, {C_{ds}, C_{db}} and {c} [Tables 1 to 4 and Table 8]. When the constant demand rate {u} increases, the procurement quantity (Q_b) increases [Table 1].

(c) The integrated total cost (TC), the procurement quantity (Q_b), the production time (T_1) and the delivery time (T_b) are less sensitive to the parameters {a} (scale parameter) and {b} (shape parameter) [Table 5 and Table 6].

(d) The procurement quantity (Q_b), the production time (T_1) and the delivery time (T_b) are less sensitive to the parameter {v}. But, the integrated total cost (TC) increases as {v} increases [Table 7].

(e) The integrated total cost (TC), the production time (T_1) and the delivery time (T_b) are more sensitive to the parameters {u}, {A_s, A_b}, {h_s, h_b}, {C_{ds}, C_{db}}, {c} [Tables 1 to 4 and Table 8]. When the constant demand rate {u}, supplier's set up cost and the buyer's ordering costs {A_s, A_b}, the supplier's and the buyer's holding costs {h_s, h_b} and the supplier's and buyer's deteriorating costs {C_{ds}, C_{db}} increase, the integrated total cost(TC) also increases [Tables 1 to 4]. When the production rate {c} increases the integrated total cost decreases [Table 8].

(f) The non-production period (T_2) is less sensitive for all parameters considered in this model.

(g) The graph in Figure 4.1, shows the percentage change in integrated total cost for percentage changes in the values of the different parameters. Except for changes in the production rate{c}, the percentage change in integrated total cost increases for all other parameters.

V. CONCLUSION

An integrated production-inventory deteriorating model under multiple deliveries is developed by assuming the demand is dependent on stock-level, Weibull deterioration rate and multiple deliveries. The model is derived in two stages to obtain the optimal policy and the procurement quantity. A solution procedure is provided to obtain the optimal solution that minimizes the integrated total expected cost. A numerical example and sensitivity analysis are given to illustrate the model. Further research can be done for imperfect production processes, ramp type demand and a single supplier - multi-buyer systems.

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