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COINCIDENCE FUNCTIONS, VARIOUS STRUCTURES AND THE MINIMIZATION PROBLEM

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Abstract— In a paper [4] David Gauld studied the variation of $\varphi_h(f)$ according as the function $f: X \to Y$ where Y is a Hausdroff. In many familiar cases it is prowed that the upper semi finite topology on the space ζX of all closed sets of a topological space X is the largest one making the coincidence function continuous. Enriching the spaces with the group actions the notion of orbit coincidence sets is introduced and G – version of many results of Gauld is proved in [5].
Exploiting the property of extension of the active homotopy on various other structures like products, twisted products it is found that for those extended maps also the coincidence function becomes continuous the map active structured concerned in the class of PN space the property of lifting of homotopic maps on various structures a small step towards minimization problem is observed.

***Index Terms*— Hausdroff, continuous functions,topological spaces.**

Introduction

Topologising the set of continuous functions from one space X to another Hausdroff space Y with the graph topology and the set ζX of all closed sets of the space X with the upper semi finite topology and for a fixed continuous map $h: X \to Y$, continuity of the function $\varphi_h : \mathcal{F}(X,Y) \to \mathcal{F} X$ taking f in F (X,Y) to the coincidence set of f and h is discussed by David Gauld in [4]. Mainly it is derived that in many familiar cases the upper semi finite topology becomes the largest one making $\varphi_h : \mathcal{F}(X,Y) \to \zeta X$ is Continuous.

By loading the topological spaces with group structure i.e. considering the topological transformation groups the notion of orbits coincidence sets is introduced in [5]. Various new concepts are introduced on way and continuity of the orbit coincidence function is discussed for a fixed mapping $h: X \to Y$

Examples are also formed explaining the difference of the concepts.

While proving results related with the function $\varphi_h : \mathcal{F}(X,Y) \to \zeta X$ and the upper semi finite topology on ζX everywhere the fact is exploited that the initial mapping $h: X \to Y$ becomes the bottom map of an active homotopy [4]. Keeping this idea in mind we observe the results of [4] relating with the extension of maps on various structures like compactification of some particular type or hyper space. We further find that if a certain map $h: X \to Y$ is the bottom map of an active homotopy then the induced maps on various structures obey the same rule. Hence the functions taking φ_{ind} to the coincidence set of induced maps becomes continuous obtaining interesting results. In this direction the study is linked with the minimization problem .the paper is divided into sections.

2. Continuity of the Coincidence Function and the Upper semi Finite Topology.

All the results mentioned in this section are taken from the paper "Variation of fixed point and coincidence sets by David Gauld "[4].

For topological space X and Y and a fixed map $h: X \to Y$ the set

$\varphi_h(f) = \{x \in X : f(x) = h(x)\}$

is called the coincidence set of f and h. It is a closed set of X *when* Y is a Hausdroff space. The basic question considered by Gauld in [4] is how $\varphi_h(f)$ changes when f changes.

When the collection $\zeta(X,Y)$ of all continuous maps is loaded with the graph topology and $\zeta(X)$ the family of closed sets is loaded with the upper semi finite topology, continuing of the function $\varphi_h : \mathcal{F}(X,Y) \to \mathcal{F} X$ is discussed by Gauld. In many familiar cases the upper semifiniite topology becomes the largest one making the function φ_h continuous.

We list below the interesting results by Gauld one by one whose proofs are omitted. Before that some definitions are essential

2.1. Definition. A homotopy $H : X \times I \rightarrow Y$ is called active if $\mathcal{H}(x,t) \neq \mathcal{H}(x,0)$ for $t > 0$ and $for\ all\ x\ \varepsilon X$.

2.2. Definition. A topological space X has property w (strong) if there is an active deformation $H : X \times I \rightarrow Y$

The space R^n has property W (strong). A compact connected topological manifold with Euler characteristic has this property. If X has property W (strong) then so has $X \times Z$ for any space Z.

2.3. Theorem: suppose that X is perfectly normal that $\mathcal{H}: X \times I \rightarrow Y$ is active and that $\mathcal{F}(X,Y)$ is topologised by the graph topology. Define $H: X \rightarrow Y$ by $h(x) = H(x,0)$. Then the upper semi finite topology is the largest topology on ζX for which $\varphi_h : \mathcal{F}(X,Y) \rightarrow \mathcal{F}X$ is continuous.

2.4.Theorm. Suppose that X is perfectly normal that Y has property W (strong) and that $\mathcal{F}(X,Y)$ is topologised by the graph topology. Let $h: X \rightarrow Y$ be continuous. Then the upper semi finite topology is the largest topology on ζX for which $\varphi_h : \mathcal{F}(X,Y) \rightarrow \mathcal{F}X$ is continuous.

2.5Theorem.Suppose that X is perfectly normal that $h: X \rightarrow Y$ is a constant function, that $\pi: [0,1] \rightarrow Y$ is a path with $\pi(0) = h(x)$ and $\pi(t) \neq \pi(0)$ for all $t > 0$ and that $\mathcal{F}(X,Y)$ is topologised by the graph topology. Then the upper semi finite topology is the largest topology on ζX for which $\varphi_h : \mathcal{F}(X,Y) \rightarrow \mathcal{F}X$ is continuous.

3. Variation of Orbit Coincidence Sets

In the present section considering G- spaces and taking the coincidence set to consist of points where orbits coincide, we obtain G-version of many of Gauld's results.

The section begins with a brief description of G-spaces i.e. topological transformation groups. An action on a topological group G on a topological space Y is a continuous map $\theta \quad \pi : Y \rightarrow Y/G$ satisfying $\theta(e,y) = y$ and $(g_1 \ , \ \theta(g_2\,,y) = \theta(g_1 \quad g_2\,,y)$ where g_1, $g_2 \in G$ and e is the identity of G: a topological space together with a given action is called a G-space. Denote $\theta(g,y)$ by$g.y$. For an element y of a $G-$space Y the set $\{g.y\,|g \in G\}$ denoted by $G(y)$ is called the orbit of$y$. The collection Y/G of orbits together with the topology co induced by the map $\pi : Y \rightarrow Y/G$ taking y to $G(y)$ is called the orbit space of y. If Y is Hausdroff and G is compact then Y/G is Hausdroff.

3.1.Definition. A homotopy $H: X \times I \rightarrow Y$ is called G – active if $H(x,t) \notin G(H(x,0)$ for any $t \in (x,0]$.

3.2. Definition. A topological space X is said to have property W (G-strong) if there exists a G – active deformation of X. A path $\alpha: I \rightarrow X$ is called orbit non- overlapping if $\alpha(t) \notin G(\alpha(0)$ for $t \in (x,0]$.

We list below a few results proved in [] which are analogous to results of David Gauld in the G-setting theorem. Let X be a perfectly normal space and $H: X \times I \rightarrow Y$ be a G –active homotopy then the upper semi finite topology is the largest topology on ζX making K_{H_0} continuous.

Theorem: Let that X is perfectly normal space and $h: X \rightarrow Y$ is is a continuous map and Y has an orbit non –overlapping path beginning from the image point of h. Then the upper semi topology on ζX is the largest topology making K_h continuous.

Combining all these facts and results we come to the following theorem whose proof is obvious.

4. Active Homotopy and the Induced Maps

While proving results, Gauld at many places exploited the fact that if a mapping $h: X \rightarrow Y$ becomes the bottom map of an active homotopy then the mapping $\varphi_h : \mathcal{F}(X,Y) \rightarrow \mathcal{F}X$ becomes continuous in the class of perfectly normal spaces.

In this section in the coming results we first observe that if a homotopy $H: X \times I \rightarrow Y$ is active then the induced homotopy on various structures remains active. As a consequence of this result we arrive at a Gauld like theorem that for the induced bottom map, the mapping $\varphi_{indh} : \mathcal{F}(indX, indY) \rightarrow \zeta indX$ becomes continuous provided the spaces are perfectly normal.

4.1. Lemm.

Let $H: X \times I \rightarrow Y$ and $H': X' \times I \rightarrow Y'$ be active homotopies. The product homotopy $H \times H': X \times X' \times I \rightarrow Y \times Y'$ defined as

$(H \times H')(x,x',t) = (H(x,t), H(x',t)$ is active.

Proof. Left to the reader.

Under the above hypothesis, for the mappings $f(x) = H(x,0) and\ f'(x) = H'(x,0)$, the mapping

$\varphi_{hf \times f'} : \mathcal{F}(X \times X', Y \times Y') \rightarrow \zeta(X \times X')$ is continuous.

For continuous map $f : X \rightarrow Y$ there is an induced map $f_H : G \times_H X \rightarrow G \times_H Y$ defined as twisted product.

$f_H([g,x]) = [g, f(x)]$.

Let $f, h: X \rightarrow Y$ be continuous maps. If $H: X \times I \rightarrow Y$ is a homotopy from f to h, then the induced map $\mathcal{H}: G \times_H X \rightarrow G \times_H Y$ defined as $\mathcal{H}([g,x],t) = [g, H(x,t)]$ gives a homotopy from f_H to h_H

It is observed easily that if h is active then Ħ is also active. As a corollary to this observation we note that if

Theorm.4.2. If $H: X \times I \rightarrow Y$ is a homotopy from f to h then the induced map $\mathcal{H}: G \times_H X \times I \rightarrow G \times_H Y$ Defined as $\mathcal{H}([g,x],t) = [g, H(x,t)]$ gives a homotopy from f_H to h_H

Theorm.4.3.. If $H: X \times I \rightarrow Y$ is active and $H(x,0) = h(x)$.

Then $\varphi_{h_H} : \mathcal{F}(G \times_H X, G \times_H Y) \rightarrow \zeta(G \times_H X)$ is continuous in the class of perfectly normal spaces.

5. Various Structures and the Minimization Problem

In this section we relate the problem of extension of various maps on the structures like Cartesian product and composition. By observing properly of extension of homotopies of too on these structures we relate these observations with the minimization problem.

The section begins with the following lemma, the proof of which is easy and left to the reader.

5.1. Lemma: Let $f, g: X \rightarrow Y$ and Let $h, k: Z \rightarrow W$ be two pairs of homotopic maps. Then the product maps $f \times h, g \times k \rightarrow Y \times W$ are homotopic.

Proof. Left to the reader.

By using this lemma we get the following results regarding $M[f,g]$, one step towards the minimization problem.

Theorem.5.2. Let $f, g: X \rightarrow Y$ be continuous maps with $MC[f,g] = 0$ and Let $h, k: Z \rightarrow W$ be continuous map with $MC[f \times h, g \times k] = 0$

Proof. By Definition

$MC[f,g] = inf|\{\chi(f',g'): f' \sim f, g' \sim g\}|$ and

$MC[h,k] = |\{\chi(h',k'): h' \sim h, k' \sim k\}|$

$MC[f,g] = 0$ means there exists $f', g': X \rightarrow Y$ satisfying thATi $\chi(f',g') = \phi$. Similarly $MC[h,k] = 0$ means that there exists maps $h', k': X \rightarrow Y$ satisfying that $\chi(h',k') = \phi$. it is easy to observe that $\chi(f' \times h', g' \times k') = \phi$. Moreover by the above lemma

$f' \times h' \sim f \times h$ and $g' \times k' \sim g \times k$. Combining these we get

$MC[f \times h, g \times k] = 0$

For continuous maps $f, h: X \rightarrow Y$ where X, Y are H-space H being a compact subgroup of G, there is an extension of these maps on the twisted product of X and Y defined as

$f_H: G \times_H X \times I \rightarrow G \times_H Y$

Such that$f_H([g,x]) = [g, f(x)]$.

Observe the following.

Lemma.5.3. Let X, Y be H-spaces where H is a compact subgroup of a group G. Let $f, h: X \rightarrow Y$ be two homotopic maps with $H: X \times I \rightarrow Y$, a homotopy from f *to* h . Then the lifted maps f_H and $h_H : G \times_H X \rightarrow G \times_H Y$ are homotopic.

Proof. The mapping $\mathcal{H}: Gx_H X \times I \rightarrow Gx_H Y$ defined as $\mathcal{H}([g,x],t) = [g, H(x,t)]$ is the required homotopy [8].

By using this lemma we have the following

Theorem.5.4. Let $f, h: X \rightarrow Y$ be continuous maps with $MC[f, h] = 0$ then $MC[f_H, h_H] = 0$.

Proof. By definition

$C[f, h] = inf|\{x(f',h'): f' \sim f, h' \sim h\}|$. Note that $MC[f, h]$ =means that there exits

$f', h': X \rightarrow Y$ satisfying that $\chi\ h') = \emptyset$. . It means that $\chi\ (f_H', h_H') = \emptyset$. Moreover by the above lemma $(f_H' \sim f_H$ and $h_H' \sim h_H$. Combining these two we get $MC[f_H, h_H] = 0$.

Finally for a continuous map $f, g: X \rightarrow Y$ and $h, k: Y \rightarrow Y$.

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