Solving Intuitionistic Fuzzy Linear Programming Problem Based on Ranking Function

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Abstract— **In this paper a systematic process has been proposed to solve a Triangular Intuitionistic Fuzzy Multi Objective Linear Programming Problem (TrIFMOLPP). Using the utility vector the Multi Objective Linear Programming Problem (MOLPP) is transferred to a single objective programming and this single fuzzy object problem is simply solved by one of the fuzzy approaches. The method is illustrated with numerical example.**

Index Terms— Multiple objective linear programming, Triangular intuitionistic fuzzy numbers, Intuitionistic fuzzy linear programming problem.

I. Introduction

The subject of decision making is the study of how decisions are actually made and how they can be made better or more successful. In fuzzy decision making problems, the concept of maximizing decision was first propsed by Bellman and Zadeh [3]. The first formulation of Fuzzy Linear Programming Problem (FLPP) was proposed by Zimmermann[18]. Shaocheng[15] considered the FLPP with fuzzy constraints and defuzzified it by first determining an upper bound for the objective function. Gasimov and Yenilmez [8] solved FLPP with linear membership functions.

Intuitionistic Fuzzy set (IFS) theory was introduced by Atanassov [2] and developed by Angelov [1] and many others addresses this issue of uncertainty. Here the degree of rejection and satisfaction are considered so that the sum of both values is always less than or equal to one.

The concept of IFS, can be viewed as an alternative approach to define a fuzzy set, in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. Lotfi et al [11] proposed a method to obtain the approximate solution of Fuzzy Linear Programming Problems (FLPP). In this study, a method for solving a TrIFMOLPP is presented.

This paper is organized as follows. Section 2, briefly discusses definitions the basic definitions. The proposed method is presented in section 3. An illustrative example is provided in section 4. The last section draws some concluding remarks.

2. Preliminaries

2.1 Intuitionistic Fuzzy set (IFS)

An Intuitionistic Fuzzy Set (IFS) A in X is defined as an object of the form A = $\{\langle x, \mu_A(x), \nu_A(x) : x \epsilon X\rangle\}$ where the functions $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element x ϵ X respectively and for every x ϵ X in A, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ holds.

2.2 Intuitionstic Fuzzy Number (IFN)

An Intuitionstic Fuzzy Number (IFN) $\tilde{A}^I$ is

i) an intuitionistic fuzzy subset of the real line,

ii) normal, (i.e.,) there is some $x_0 \epsilon R$ such that $\mu_{\tilde{A}^I}(x_0) = 1, \nu_{\tilde{A}^I}(x_0) = 0,$

iii) convex for the membership function $\mu_{\tilde{A}^I}$

$\mu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \geq \min\left(\mu_{\tilde{A}^I}(x_1), \mu_{\tilde{A}^I}(x_2)\right),$

for every $x_1, x_2 \epsilon$ R, $\lambda \epsilon [0,1]$.

iv) concave for the non-membership function $\nu_A(x)$, that is,

$\nu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \leq \max\left(\nu_{\tilde{A}^I}(x_1), \nu_{\tilde{A}^I}(x_2)\right),$

for every $x_1, x_2 \epsilon$ R, $\lambda \epsilon [0,1]$.

2.3 Intuitionistic Fuzzy Linear Programming Problem

An IFLPP with intuitionistic fuzzy technological coefficients is defined as

Maximize $z = \sum_{j=1}^{n} c_j x_j$

Subject to $\sum_{j=1}^{n} \tilde{a}_{ij}^I x_j \leq b_j$, $1 \leq i \leq m$,

$x_j \geq 0$, $1 \leq j \leq n$,.............., (1)

Where atleast one $x_j > 0$ and $\tilde{a}_{ij}^I$ is an IFN.

2.4 Intuitionistic Fuzzy Feasible Solution.

Any vector x ϵ R^n which satisfies the constraints and non negative restrictions of (1) is said to be an intuitionistic fuzzy feasible solution.

2.5 Intuitionistic Fuzzy Optimum solution

Let S be the set of all intuitionistic fuzzy feasible solutions of (1). Any vector x_0 ϵ S is said to be an Intuitionistic Fuzzy Optimum solution to (1), if $Cx_0 \geq Cx$ for all xϵS where C= $(c_1, c_2, \dots \dots . c_n)$ and $Cx= c_1x_1 + c_2x_2 +$................$+c_nx_n$.

3. The Proposed method

In this section we describe the proposed method. The steps are as follows:

Step 1

The weighting problem of TrIFMOLPP takes the form:

Maximize (Minimize) Z =

$$\left(\sum_{r=1}^{k} w^r \left((\tilde{B})^{r^T} \otimes \tilde{X}^{IF}\right) = \left(w^1(\tilde{B})^{1^T} \otimes \tilde{X}^{IF}\right) \oplus \left(w^2(\tilde{B})^{2^T} \otimes \tilde{X}^{IF}\right) \oplus \dots\dots\dots\dots \oplus \left(w^k(B)^{k^T} \otimes \tilde{X}^{IF}\right)\right)$$

Subject to

$$\tilde{A}^{IF} \otimes \tilde{X}^{IF} = \tilde{b}^{IF},$$
$$\tilde{X}^{IF} \gtrsim \tilde{0}^{IF}, \; \sum_{r=1}^{k} w^r = 1, w^r \geq 0$$

Step 2

By preference vector approach TrIFMOLPP is transferred to a single fuzzy objective linear problem:

Maximize (minimize) Z = $\left((\tilde{B})^T\right) \otimes \tilde{X}^{IF}$

Subject to $\tilde{A}^{IF} \otimes \tilde{X}^{IF} = \tilde{b}^{IF}$, $\tilde{X}^{IF} \gtrsim \tilde{0}^{IF}$

Step 3

Substituting $(\tilde{B})^T = [\tilde{c}_j]_{1\times n}$, $\tilde{X}^{IF} = [\tilde{x}_j]_{n\times 1}$, $(\tilde{A})^{IF} = [\tilde{a}_{ij}]_{m\times n}$, $(\tilde{b})^{IF} = [\tilde{b}_{ij}]_{m\times 1}$, the above TrIFMOLPP problem may be written as:

Maximize (Minimize) Z = $\sum_{j=1}^{n} \tilde{B}_j^{\,T} \otimes \tilde{x}_j^{IF}$

Subject to $\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j = \tilde{b}_j$, $\tilde{x}_j^{IF} \gtrsim \tilde{0}^{IF}$

Step 4

If all the parameters are represented by triangular inuitionistic fuzzy numbers $\tilde{c}_j^{IF}, \tilde{x}_j^{IF}, \tilde{a}_j^{IF}, \tilde{b}_j^{IF}$ $\langle(\eta_j,\xi_j,\zeta_j),(\eta_j',\xi_j',\zeta_j')\rangle, \langle(x_j,y_j,z_j),(x_j',y_j',z_j')\rangle, \langle(a_{ij},b_{ij},c_{ij}),(a_{ij}',b_{ij}',c_{ij}')\rangle$ $\langle(l_i,m_i,n_i),(l_i',m_i',n_i')\rangle$ respectively then the TrIFLPP problem, obtained in step 3, may be written as:

Maximize(Minimize)

Z= $\langle(\eta_j,\xi_j,\zeta_j),(\eta_j',\xi_j',\zeta_j')\rangle \otimes \langle(x_j,y_j,z_j),(x_j',y_j',z_j')\rangle$ Subject to

$$\sum_{j=1}^{n} \langle(a_{ij},b_{ij},c_{ij}),(a_{ij}',b_{ij}',c_{ij}')\rangle \otimes \langle(x_j,y_j,z_j),(x_j',y_j',z_j')\rangle = \langle(l_i,m_i,n_i),(l_i',m_i',n_i')\rangle$$

$$\left(\tilde{x}_j^{IF} \tilde{y}_j^{IF}, \tilde{z}_j^{IF}\right) \gtrsim \tilde{0}^{IF}$$

Step 5

Assuming $\langle(a_{ij},b_{ij},c_{ij}),(a_{ij}',b_{ij}',c_{ij}')\rangle \otimes \langle(x_j,y_j,z_j),(x_j',y_j',z_j')\rangle = \langle(u_{ij},v,w_{ij}),(u_{ij}',v_{ij}',w_{ij}')\rangle$ the TrIFMOLPP problem, obtained in step 4, may be written as:

Maximize (Minimize) Z = $\mathbb{R}$ $\langle(\eta_j,\xi_j,\zeta_j),(\eta_j',\xi_j',\zeta_j')\rangle \otimes \langle(x_j,y_j,z_j),(x_j',y_j',z_j')\rangle$

Subject to $\sum_{j=1}^{n} \langle(u_{ij},v_{ij},w_{ij}),(u_{ij}',v_{ij}',w_{ij}')\rangle = \langle(l_i,m_i,n_i),(l_i',m_i',n_i')\rangle$

$$\left(\tilde{x}_j^{IF} \tilde{y}_j^{IF}, \tilde{z}_j^{IF}\right) \gtrsim \tilde{0}^{IF}$$

Where $\mathbb{R}(\tilde{A}) = \frac{(x+2y+z)+(a+2b+c)}{8}$ for $\tilde{A} = \langle(x,y,z),(a,b,c)\rangle$

Step 6

The triangular intuitionistic fuzzy linear programming problem, obtained in step 5, is converted into the CLP problem:

Maximize (Minimize) Z = $\mathbb{R}$ $\langle(\eta_j,\xi_j,\zeta_j),(\eta_j',\xi_j',\zeta_j')\rangle \otimes \langle(x_j,y_j,z_j),(x_j',y_j',z_j')\rangle$

Subject to

$$\sum_{j=1}^{n} u_{ij} = l_i,$$
$$\sum_{j=1}^{n} v_{ij} = m_i, \; \sum_{j=1}^{n} w_{ij} = n_i, \forall\, i = 1,2, \dots .m$$
$$\sum_{j=1}^{n} u_{ij}' = l_j',$$
$$\sum_{j=1}^{n} v_{ij}' = m_j', \; \sum_{j=1}^{n} w_{ij}' = n_j', \forall\, i = 1,2, \dots .m,$$
$$y_j - x_j \geq 0, z_j - y_j \geq 0, \forall\, j = 1,2, \dots \dots \dots m$$

Step 7

Find the optimal solution $\tilde{x}_j^{IF} \tilde{y}_j^{IF}, \tilde{z}_j^{IF}, \tilde{a}_j^{IF}, \tilde{b}_j^{IF}, \tilde{c}_j^{IF}$ by solving the CLP problem obtained in step 6.

Step 8

Find the intuitionistic fuzzy optimal solution by putting the values of $\tilde{x}_j^{IF}\tilde{y}_j^{IF},\tilde{z}_j^{IF}$ in $\tilde{x}_j^{IF}$ $= \langle(\tilde{x}_j^{IF}\tilde{y}_j^{IF},\tilde{z}_j^{IF}),(\tilde{a}_j^{IF},\tilde{b}_j^{IF},\tilde{c}_j^{IF})\rangle$.

Step 9

Find the intuitionistic fuzzy value by putting $\tilde{x}_j$ in $\sum_{j=1}^{n}\tilde{B}_j^{r^T} \otimes \tilde{x}_j^{IF}$.

Numerical Example:

In this section a numerical example is given to show applicability of the proposed method. Let us consider the following TrIFMOLPP:

MaximizeZ= $\langle(1,2,4),(6,8,10)\rangle \otimes \langle(x_1,y_1,z_1),(a_1,b_1,c_1)\rangle \oplus \langle(2,4,6),(4,6,8)\rangle \otimes \langle(x_2,y_2,z_2),(a_2,b_2,c_2)\rangle$

Subject to

$\langle(3,7,9),(5,8,10)\rangle \otimes \langle(x_1,y_1,z_1),(a_1,b_1,c_1)\rangle \oplus \langle(6,7,10),(8,9,12)\rangle \otimes \langle(x_2,y_2,z_2),(a_2,b_2,c_2)\rangle = \langle(8,11,20),(9,15,25)\rangle$

$\langle(3,5,8),(4,7,11)\rangle \otimes \langle(x_1,y_1,z_1),(a_1,b_1,c_1)\rangle \oplus \langle(2,3,4),(4,5,6)\rangle \otimes \langle(x_2,y_2,z_2),(a_2,b_2,c_2)\rangle = \langle(11,14,15),(13,16,17)\rangle$

Where $\widetilde{X_1} = \langle(x_1,y_1,z_1),(a_1,b_1,c_1)\rangle$, $\widetilde{X_2} = \langle(x_2,y_2,z_2),(a_2,b_2,c_2)\rangle$

Solution:

Let w= $\frac{z}{2}$, then the given TrIFLPP problem may be written as :

MaximizeZ= $\langle(0.5,1,2),(3,4,5)\rangle \otimes \langle(x_1,y_1,z_1),(a_1,b_1,c_1)\rangle \oplus \langle(1,2,3),(2,3,4)\rangle \otimes \langle(x_2,y_2,z_2),(a_2,b_2,c_2)\rangle$

Subject to

$\langle(3,7,9),(5,8,10)\rangle \otimes \langle(x_1,y_1,z_1),(a_1,b_1,c_1)\rangle \oplus \langle(6,7,10),(8,9,12)\rangle \otimes \langle(x_2,y_2,z_2),(a_2,b_2,c_2)\rangle = \langle(8,11,20),(9,15,25)\rangle$

$\langle(3,5,8),(4,7,11)\rangle \otimes \langle(x_1,y_1,z_1),(a_1,b_1,c_1)\rangle \oplus \langle(2,3,4),(4,5,6)\rangle \otimes \langle(x_2,y_2,z_2),(a_2,b_2,c_2)\rangle = \langle(11,14,15),(13,16,17)\rangle$

Maximize

$Z = \langle(0.5x_1,y_1,2z_1),(3a_1,4b_1,5c_1)\rangle \oplus \langle(x_2,2y_2,3z_2),(2a_2,3b_2,4c_2)\rangle$

Subject to

$\langle(3x_1,7y_1,9z_1),(5a_1,8b_1,10c_1)\rangle \oplus \langle(6x_2,7y_2,10z_2),(8a_2,9b_2,12c_2)\rangle = \langle(8,11,20),(9,15,25)\rangle$

$\langle(3x_1,7y_1,9z_1),(4a_1,7b_1,11c_1)\rangle \oplus \langle(2x_2,3y_2,4z_2),(4a_2,5b_2,12c_2)\rangle = \langle(11,14,15),(13,16,17)\rangle$

Maximize

Z= $\mathbb{R}\langle(0.5x_1+x_2,y_1+2y_2,2z_1+3z_2),(3a_1+2a_2,4b_1+3b_2,\ 5c_1+4c_2)\rangle$

Subject to

$\langle(3x_1+6x_2,7y_1+7y_2,9z_1+10z_2),(5a_1+8a_2,8b_1+9b_2,\ 10c_1+\ 12c_2)\rangle$

$=\langle(8,11,20),(9,15,25)\rangle$

$\langle(3x_1+2x_2,5y_1+3y_2,8z_1+4z_2),(4a_1+4a_2,7b_1+5b_2,\ 11c_1+\ 6c_2)\rangle$

$= \langle(11,14,15),(13,16,17)\rangle$

MaxZ=

$$0.125\left\langle\begin{matrix}(0.5x_1+x_2+2y_1+4y_2+2z_1+3z_2)+\\(3a_1+2a_2+8b_1+6b_2+5c_1+\ 4c_2)\end{matrix}\right\rangle$$

Subject to

$3x_1+6x_2 = 8$, $7y_1+7y_2 = 11, 9z_1+10z_2 = 20$

$3x_1+2x_2 = 11$, $5y_1+3y_2 = 14, 8z_1+4z_2 = 15$

$5a_1+8a_2 = 9$, $8b_1+9b_2 = 15,\ 10c_1+\ 12c_2 = 25$

$4a_1+4a_2 = 13$, $7b_1+5b_2 = 16,\ 11c_1+\ 6c_2 = 17$

The optimal solution of above CLP problems x_1= 4.2, x_2= - 0.75, y_1 = 4.6 , y_2= - 3.1, z_1= 2, z_2= 0.2

a_1= 5.6, a_2= - 2.4, b_1 = 3, b_2= -1, c_1= 0.75, c_2= 1.5

Using step 8, the optimal solution is given by

$\widetilde{X_1} = \langle(x_1,y_1,z_1),(a_1,b_1,c_1)\rangle$, $\widetilde{X_2} = \langle(x_2,y_2,z_2),(a_2,b_2,c_2)\rangle$

$\widetilde{X_1} = \langle(4.2\ ,4.6\ ,2),(5.6,3,0.75)\ \rangle$, $\widetilde{X_2} = \langle(-0.75,-3.1,0.2),(-2.4,-1,1.5)\rangle$

$\widetilde{f_1}$ = (2.7, - 3.2, 9.2) and $\widetilde{f_2}$ = (24, 30, 19.5).

CONCLUSION

A special type of membership and non-membership functions have been proposed to solve the TrIFLPP. Intuitionistic fuzzification of objective function and constraints has also been done in this paper. Also, TrIFLPP is solved using intuitionistic fuzzy decisive set method.

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