g-m_i-closed sets in Supra Topological space

N Nagamani[1] and V. Thiripurasundari[2]

[1]M. Phil Scholar, PG and Research Department of Mathematics,
Sri S.R.N.M.College,
Sattur - 626 203, Tamil Nadu, India.
nagamanin28@gmail.com

[2]Assistant Professor,
PG and Research Department of Mathematics,
Sri S.R.N.M.College,
Sattur - 626 203, Tamil Nadu, India.

ABSTRACT

In this paper, we introduce a new class of closed sets in supra topological spaces, namely supra g-m_i-closed set. We, discuss the relationship between this type of closed sets and other existing closed sets in supra topological spaces. Further, we obtain some basic properties of supra g-m_i-closed sets.

Keywords: supra g-m_i-closed sets, supra m_i-open sets and supra closed.

1 INTRODUCTION

In 1983, Mashhour et al. [3] introduced the supra topological spaces . In 2008, Devi et al. [1] introduced the concept of supra α-open sets and $S\alpha$-continuous functions. In 2011, Ravi et al. [7], [2] introduced and investigated several properties of generalized closed sets, supra g-closed sets, supra sg-closed sets and gs-closed sets. F.Nakaoka and N. Oda some applications of minimal open sets [4]. F.Nakaoka and N. Oda on minimal closed sets. Proceedings of topological spaces theory and its applications [6]. In this paper, we introduced the concept of supra g-m_i-closed sets and study its basic properties. Also we investigated the relationships between this type of closed sets and other existing closed sets in supra topological spaces.

2 PRELIMINARIES

Throughout this paper (X, μ) (or simply X) represent supra topological spaces. For a subset A of X, $cl^{\mu}(A)$and $int^{\mu}(A)$ denote the supra closure of A and supra interior of A respectively. Let us recall the following definitions, which are useful in the sequel.

Definition 2.1. *[3] A subfamily μ of X is said to be a supra topology on X, if*

(i) $X, \phi \in \mu$,

(ii) If $A_i \in \mu$ *for all* $i \in J$, *then* $\cup A_i \in \mu$.

The pair (X, μ) *is called the supra topological space. The elements of* μ *are called supra open sets in* (X, μ) *and the complement of a supra open set is called a supra closed set.*

Definition 2.2. *[3]*

(i) The supra closure of a set A *is denoted by* $cl^{\mu}(A)$ *and is defined as* $cl^{\mu}(A) = \cap\{B : B$ *is supra closed and* $A \subseteq B\}$.

(ii) The supra interior of a set A *is denoted by* $int^{\mu}(A)$ *and is defined as* $int^{\mu}(A) = \cup\{B : B$ *is supra open and* $A \supseteq B\}$.

Definition 2.3. *[3] Let* (X, τ) *be a topological space and* μ *be a supra topology associated with* τ, *if* $\tau \subset \mu$.

Definition 2.4. *A subset* A *of a supra topological space* X *is called a*

(i) supra semi-open set [1] if $A \subseteq cl^{\mu}(int^{\mu}(A))$.

(ii) supra α*-open set [1] if* $A \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(A)))$.

The complement of a supra semi-open (resp. supra α*-open) set is called supra semi-closed (resp. supra* α*-closed).*

Definition 2.5. *A subset* A *of a supra topological space* (X, μ) *is called*

(i) a supra generalized closed set (briefly g^{μ}*-closed) [7] if* $cl^{\mu}(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is supra open in* (X, μ).

(ii) a supra generalized semi-closed set (briefly gs^{μ}*-closed) [2] if* $scl^{\mu}(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is supra open in* (X, μ).

(iii) a supra α*-generalized closed set (briefly* αg^{μ}*-closed) [1] if* $\alpha cl^{\mu}(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is supra open in* (X, μ).

(iv) a supra $\hat{g}$*- closed set (briefly* ω^{μ}*-closed) [8] if* $cl^{\mu}(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is supra semi-open in* (X, μ).

The complement of a g^{μ}*-closed (resp.* gs^{μ}*-closed,* αg^{μ}*-closed, and* ω^{μ}*-closed) set is called* g^{μ}*-open (resp.* gs^{μ}*-open,* αg^{μ}*-open, and* ω^{μ}*-open) set.*

Definition 2.6. *A proper nonempty subset* A *of a topological space* (X, τ) *is called*
(*i*) *A minimal open [4] (minimal closed [6]) set is any open (resp.closed) subset* X *which is contained in* A, *is either* A *or* ϕ.
(*ii*) *A maximal [5] open (maximal closed [6]) set is any open (resp.closed) set which contains* A, *is either* A *or* X.

3 Supra g-m_i-Closed Sets

In this section, we introduce a new class of closed sets in supra topological spaces, namely supra g-m_i-closed set. And, we discuss the relationship between this type of closed sets and other existing closed sets in supra topological spaces.

Definition 3.1. *A subset A of a supra topological space (X, μ) is called supra g-m_i-closed if $cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra m_i-open set in (X, μ).*

proposition 3.2. *Every supra g-m_i-closed set is supra g^μ-closed set.*

Proof. Let $A \subseteq U$, U is a supra m_i-open set. We know that every m_i-open set is an open set. Since, A is g-m_i-closed set, we have $cl^\mu(A) \subseteq U$. Hence A is supra g^μ-closed set. □

Remark 3.3. *Converse of the above proposition need not be true as shown in the following example.*

Example 3.4. *Let $X = \{a, b, c, d\}$, $\mu = \{\phi, \{a\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}$. Then the set $\{a, d\}$ is supra g^μ-closed but not supra g-m_i-closed in (X, μ).*

proposition 3.5. *Every supra g-m_i-closed set is ω^μ-closed set.*

Proof. Let $A \subseteq U$, U is supra m_i-open set in X. We know that every supra m_i-open set is an supra semi open set. Since A is supra g-m_i-closed set, we have $cl^\mu(A) \subseteq U$. Hence A is ω^μ-closed. □

Remark 3.6. *Converse of the above proposition need not be true as shown in the following example.*

Example 3.7. *Let $X = \{a, b, c, d\}$ and $\mu = \{\phi, \{a\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}$. Then the set $\{a, d\}$ is ω^μ-closed but not supra g-m_i-closed set.*

proposition 3.8. *Every supra g-m_i-closed set is gs^μ-closet set.*

Proof. Let $A \subseteq U$ be a supra g-m_i-closed set and U be a supra m_i-open set in (X, μ). We know that every supra m_i-open set is supra open set, then U is supra m_i-open set in (X, μ). Since, A is supra g-m_i-closed set of (X, μ), then $cl^\mu \subseteq U$, whenever $A \subseteq U$ and U is m_i-open, we have $scl^\mu(A) \subseteq cl^\mu(A)$. Then, $scl^\mu()A \subseteq U$ and U is supra open set. Hence A is gs^μ-closed set. □

Remark 3.9. *Converse of the above proposition need not be true as shown in the following example.*

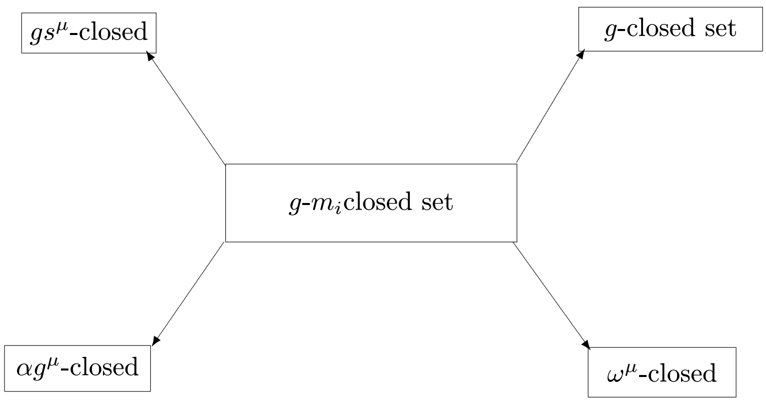
Example 3.10. *Let $X = \{a, b, c, d\}$, $\mu = \{\phi, \{a\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, X\}$. Supra g-m_i-closed $= \{\phi, \{c\}\}$.*
gs^μ-closed set $= \{\phi, \{a\}, \{c\}, \{d\}, \{a, d\}, \{b, c\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}, X\}$. Here $\{a\}$ is gs^μ-closed but not supra g-$m_i$closed set.

proposition 3.11. *Every supra g-m_i-closed set is αg^μ-closed set.*

Proof. Let $A \subseteq U$ be a supra g-m_i-closed set and U be supra m_i-open set in (X, μ). We know that every supra m_i-open set is supra open set, then U is open set in (X, μ). Since, A is supra g-m_i-closed set of (X, μ), then $cl^\mu(A) \subseteq U$, whenever $A \subseteq U$ and U is supra m_i-open set. Also we know that $\alpha cl^\mu(A) \subseteq cl^\mu(A)$. Therefore $\alpha cl^\mu(A) \subseteq U$ and U is supra open. Hence A is αg^μ-closed set □

Remark 3.12. *Converse of the above proposition need not be true as shown in the following example.*

Example 3.13. *Let $X = \{a, b, c, d\}$, $\mu = \{\phi, \{b, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$. Then the set $\{a, b, d\}$ is αg^μ-closed but not g-m_i-closed set.*



4 Basic Properties of Supra g-m_i Closed sets

In this section, we analyse the properties of supra g-m_i-closed set.

proposition 4.1. *The intersection of supra g-m_i-closed set and supra g-closed set is a supra g-m_i-closed set.*

Proof. Let A be supra g-m_i-closed set and F be supra g-closed set. If $U$is any minimal open set with $A \cap F \subseteq U$, then $A \subseteq U \cup F^c$ and so $cl_g^\mu(A) \subseteq U \cup F^c$. Now $cl_g^\mu(A \cap F) \subseteq cl_g^\mu(A \cap F) \subseteq cl_g^\mu(A) \cap F \subseteq U$. Hence $A \cap F$ is supra g-m_i closed set. □

proposition 4.2. *If A is supra minimal open set and g-m_i-closed. Then A is supra closed set.*

Proof. Since, $A \subseteq A$ and A is supra m_i-open and g-m_i closed, we have $cl^\mu(A) \subseteq A$. Therefore we have $cl^\mu = A$ and A is supra closed set. □

proposition 4.3. *A subset A of (X, μ) is g-m_i closed then $cl^\mu(A) \setminus A$ does not contain any non empty supra maximal closed set.*

Proof. Let A be g-m_i closed set of (X, μ). Suppose $F \neq \phi$ is a supra maximal closed set of $cl^\mu(A) \setminus A$. Then, $F \subseteq cl^\mu(A) \setminus A$ implies $F \subseteq cl^\mu(A)$ and F^c. This implies $A \subseteq F^c$. Since, A is g-m_i-closed, $cl^\mu(A) \subseteq U^c$. Consequently, $F \subseteq [cl^\mu(A)]^c$. Hence $F \subseteq cl^\mu(A) \cap [cl^\mu(A)]^c = \phi$. Therefore, F is empty. Which is a contradiction. Hence the proposition follows.
sufficiency, suppose that $A \subseteq U$ and that U is supra m_i-open. If $cl^\mu(A) \nsubseteq U$, then $cl^\mu(A) \cap U^c$ is a non empty supra m_a closed subset of $cl^\mu(A) \setminus A$. Hence $cl^\mu(A) \cap U^c = \phi$ and $cl^\mu(A) \subseteq U$. Therefore A is supra g-m_i-closed set. □

proposition 4.4. *A supra g-m_i closed set A of X is supra m_a-closed if and only if $scl^\mu(A) \setminus A$ is supra m_a-closed set.*

Proof. Necessity If A is g-m_i-closed and supra m_a-closed, then $scl^\mu(A) \setminus A = \phi$ by above theorem . Therefore $scl^\mu(A) \setminus A$ is supra m_a closed set.
sufficiency Suppose that $scl^\mu(A)\setminus A$ is supra m_a-closed. Since, $scl^\mu(A) \subseteq cl^\mu(A)$, $cl^\mu(A)\setminus A$ contains the semi closed set $scl^\mu(A)\setminus A$. Since, A is supra g-m_i-closed, by the above theorem. $scl^\mu(A) \setminus A = \phi$. Hence, $scl^\mu(A) = A$. Therefore A is supra m_a-closed set. □

Remark 4.5. *Union of two supra g-m_i-closed sets need not be an g-m_i-closed set as seen from the following example.*

Example 4.6. *Let $X = \{a, b, c, d\}$ and $\mu = \{\phi, \{a, b\}, \{c, d\}, X\}$. Then the sets $\{a, b\}$ and $\{c\}$ are g-m_i-closed sets but their union $\{a, b, c\}$ is not an g-m_i closed set.*

Remark 4.7. *Intersection of two g-m_i closed sets is ϕ or supra g-m_i closed set as seen from the following example.*

Example 4.8. *Let $X = \{a, b, c, d\}$ and $\mu = \{\phi, \{a, b\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{b, c, d\}, X\}$. Then the sets $\{a, b\}$ and $\{a, c\}$ are supra g-m_i closed sets but their intersection $\{a\}$ is a supra g-m_i closed set. Another one the sets $\{a, b\}$ and $\{c\}$ are supra g-m_i closed sets but their intersection ϕ is a supra g-m_i closed set.*

proposition 4.9. *Let A be a supra g-m_i-closed set in (X, μ). Then $cl^\mu(A) - A$ does not contain a non-empty supra m_a-closed set.*

Proof. Let A be a supra g-m_i-closed set in (X, μ). Suppose that $F \neq \phi$ is a supra m_a closed set contained in $cl^\mu(A) - A$. That is $F \subseteq cl^\mu(A) - A$ which implies $F \subseteq cl^\mu(A)$ and A^c. This implies $A \subset F^c$, F^c is supra m_a-closed in (X, μ). Since, A is supra g-m_i-closed set of (X, μ), then $cl^\mu(A) \subseteq F^c$. Thus $F \subset (cl^\mu(A))^c$. But $F \subseteq cl^\mu(A)$. Therefore $F \subseteq cl^\mu(A) \cap (cl^\mu(A))^c = \phi$. Hence $F = \phi$. □

proposition 4.10. *If A is a supra g-m_i-closed set in a space (X,μ) and $A \subseteq B \subseteq cl^\mu(A)$, then B is also a supra g-m_i-closed set.*

Proof. Let U be a supra m_i-open set of (X,μ) such that $B \subseteq U$. Then, $A \subseteq U$. Since, A is supra g-m_i-closed set, then $cl^\mu(A) \subseteq U$. Also, since, $B \subseteq cl^\mu(A)$, $cl^\mu(B) \subseteq cl^\mu(cl^\mu(A)) = cl^\mu(A)$. Hence, $cl^\mu(B) \subseteq U$. Therefore B is also a supra g-m_i-closed set. □

proposition 4.11. *If A is any supra g-m_i-closed set in (X, μ) then for each $x \in cl^\mu(A)$, $cl^\mu(x) \cap A \neq \phi$.*

Proof. Let A be any g-m_i closed set in (x, μ) such that for each $x \in cl^\mu(A)$, $cl^\mu \cap A = \phi$. Then $A \subseteq [cl^\mu(x)]^c$ where $[cl^\mu(x)]^c$ is an supra open set in X. By theorem 3.2 A is supra g-closed set, we have $A \subseteq [cl^\mu(x)]^c$. Therefore, $cl^\mu(A) \subseteq [cl^\mu(x)]^c$. This is a contradiction to the fact that $x \in cl^\mu(A)$. Therefore $cl^\mu(x) \cap A \neq \phi$.

proposition 4.12. *If A is a supra g-m_i-closed set of (X, μ), then A is supra closed set if and only if $cl^\mu(A) - A$ is m_a-closed.*

Proof. Necessity: Let A be a supra g-m_i-closed and supra closed subset of X. Then $cl^\mu(A) = A$ and so $cl^\mu(A) - A = \phi$ which is a g-m_i-closed. Sufficiency: Let A be a supra g-m_i-closed set and $cl^\mu(A) - A$ be supra m_a-closed. Since, A is supra g-m_i-closed and by proposition 4.9, $cl^\mu(A) - A$ does not contain a non-empty m_a-closed set. But $cl^\mu(A) - A = \phi$. That is $cl^\mu(A) = A$. Hence A is supra closed set. □

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