SYMMETRIC BI-DERIVATION ON MULTIPLICATIVE HYPERRING

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ABSTRACT

In this paper, we introduce the symmetric bi derivation of the multiplicative hyperring and discuss the relationship between the derivation of multiplicative hyperring and multiplicative hyperring. Further we obtain some properties of symmetric bi- derivation on a multiplicative hyperring.

Keywords: multiplicative hyperring, Symmetric bi-derivation multiplicativehyperring.

1 INTRODUCTION

The theory of hyperstructures was introduced in 1934 by marty at the 8th congress Scandinavion Mathematicians. Then several researchers have worked on this new field and developed it. Mittas introduced the notion of canonical hypergroups. Corsini studied the Canonical Hypergroups, Feebly Canonical Hypergroups, Quasi-Canonical Hypergroups, Krasner introduced the notion of hyperrings and hyperfields. G.G. Massouros introduced the theory of hypercompositional structures into the theory of automata. Asokkumar studied the idempotent elements of krasner hyperrings. Babaei et al. studied R parts in hyperrings. The notion of derivations of rings plays a significant role in algebra.The study of derivation in rings got interested after Posner, who gave striking results on derivation of prime rings. Then the notion of derivation has been developed by many authors in various directions like Jordan derivation in rings and near - rings. In 1980, Gy. Maksa introduced the concept of a symmetric bi-derivation on a ring R.

In this paper, we introduce symmetric bi derivation in multiplicative hyperring and investigated some of its properties.

PRELIMINARS

This section explain some basic definition that have been used in the sequel.A hyperoperation * on a non-empty set H is a mapping of H × H into the family of non-

empty subsets of H. In the sense of Marty, a hypergroup (H,*) is a non empty set H equipped with a hyperoperation * which satisfies the following axiom:
(i) x * (y * z) = (x * y) * z for every x, y, z ϵ H
(the associative axiom)
(ii) x * H = H * x = H for every x ϵ H
(the reproductive axiom)
The comprehensive review of the theory of hypergroups appear

A non-empty subset *I* of a canonical hypergroup R is called a canonical subhypergroupof R if *I* itself is a canonical hypergroup under the same hyperoperation as that of R. Equivalently, a non- empty subset *I*of a canonical hypergroup Ris a canonical subhypergroup of R if for every x, y ϵ *I*, x − y ϵ*I*. Here afterwe denote xy instead of x . y. Moreover, for A, B ⊆ R and x ϵ R, by A + Bwe mean the set ∪a2A,b2B (a + b) and $AB = \bigcup_{a\in A\ b\in A}(ab)$, A+ x = A + {x}, x+ B = {x} + B and also −A = {−a: a ϵ A}

Definition 1.1. [7] A multiplicative hyperring is an additive commutative group (R, +) endowed with a hyperoperation ” .”
which satisfies the following condition:
(i) ∀a, b, c ϵ R : a(bc) = (ab)c;
(ii) ∀ a, b, c ϵ R: (a + b)c⊆ ac + bc;
a(b + c) ⊆ ab + ac;
(iii) ∀ a, b ϵ R : (−a)b = a(−b) = −(ab).

If in (iii) we have equalities instead of inclusions then we say that the multiplicativehyperring is strongly distributive. An element e ∈ R is called a weakidentity (identity respectively) if x ∈ ex ∩ xe (ex = xe = x, respectively),for all x ∈ R. Throughout this paper, by a hyperring we mean a multiplicative hyperring.A non-empty subset H of a hyperring (R, +, .) is called subhyperring ofR, if (H, +, .) is itself hyperring. In other words, H is a subhyperring of (R, +, .) if H − H ⊆ H and x . y ⊆ H, for all x, y ⊆ H. A hyperring R is calledan **integral hyperdomain,** if for all x, y ⊆ R, 0 ∈ x. y implies that x = 0 ory = 0

Example 1.2. Let (R, +,.) be a ring, *I*be an ideal of R and ° be the hyperoperation defined on R by x °y = x . y + *I*, for all x, y ∈ R. Then, (R, +, .) is a strongly distributive hyperring. For convenience, the multiplicative hyperring (R, +, °) will be denoted by (R, +, *I*). The ideal *I*is a hyperideal of hyperring (R, +, *I*), since *I*is an additive subgroup of (R, +) and for all x ∈*I*and r ∈R, x °r∪ r °x= (x. r + *I*) ∪ (r. x+*I*) ⊆*I*.

Definition 1.3. [11] Let (R, +, .) be a hyperring. The function d : R→R is called derivation if for all x, y ∈R,

(i)d (x + y) = d(x) + d(y)
(ii)d(x. y) = d(x). y + x. d(y).

Example 1.4. Let R be an abelian group and S be a subgroup of R. For all x, y ∈R. Define x °y = S. Then, (R, +, °) is a hyperring. The function d_1,d_2: R →R defined by d_1(x) = x and d_2(x) = −x. for all x ∈R, are derivations.

Definition 1.5.[11] A hyperring R is called prime if 0 ∈x. r. y, for all r ∈R, implies that either x = 0 or y = 0. R is called semiprime if 0 ∈x. r. x, for all r ∈R, implies that x = 0. It immediate that every prime hyperring is a semiprime hyperring but the converse is not always true.

Example 1.6. Let R = {e, a, b}. Consider the following tables

+	e	a	B
e	e	a	b
a	a	b	e
b	b	e	a

.	e	A	b
e	e	e	e
a	e	{ a, b}	{ a, b}
b	e	{ a, b}	{ a, b}

It is verified that (R, +, .) is prime.

Theorem 1.7.[11] Let *I*be a non-zero hyperideal on a prime hyperring R and x, y ∈R, then
(i) If *I*. x = 0 or x. *I*= 0, then x = 0.
(ii) 0 = x. *I*.y, then x = 0 or y = 0.
(iii) If 0 ∈r. 0 ∩ 0. r, for all r ∈R, x ∈Z and 0 ∈x. y, then x =0 or y = 0.

Definition 1.8.[10] A hyperring R is said to be a reduced hyperring if it has no nilpotent elements. That is, if $x^n = 0$ for all x ∈R and a natural number n, then x = 0.

3. SYMMETRIC BI-DERIVATION OF HYPERRINGS

In this section we define symmetric bi-derivation and Annihilators of hyperring and constructed.

Definition 1.9. Let R be a hyperring. A mapping D : R ×R → R is called symmetric if D(x, y) = D(y, x) for all x, y ∈R.

Definition 1.10. Let R be a multiplicative hyperring. The function D : R × R → R is called symmetric bi-derivation of R if satisfies
(i) D(x + y) = D (x, y) + D(z, y)
(ii)D(xz, y) = D(x, y).z + x. D(z, y).
If the map D is such that
D(xz, y) ⊆D(x, y).z + x. D(z, y) and satisfies the condition (i) then D is called a weak symmetric bi- derivation of R.

Example 1.11. Consider the hyperring R = {e, a, b, c} with the addition and hypermultiplication defined as follows

.	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

.	e	a	b	c
e	e	a	b	c
a	a	{e, c}	{e, c}	{a, b}
b	b	{e,c}	{e, c}	{a, b}
c	c	{a,b}	{a, b}	{e, c}

Define a map D: R × R → R by D(0, 0) = 0,
D(c, e) = D(e, c) = e,
D(a, c) = D (c, a) = a D(c, b) = D(b, c) =c,
D({a, b}, a) = {a, b}= D(a, {a, b} D(a, {a, b})
D({b, c}, b) = {b, c} = D(b, {b, c}).
N ow, D is a symmetric bi-derivation of R. Here D is a strong symmetric bi-derivation of R.

Theorem 1.12. Let D be a bi-derivation on a prime hyperring (R, +,.) and ***I***be a non-zero hyperideal of R. Also, let 0 ∈0.r ∩ r.0 for all r ∈R, then for all x ∈R,
(1) If D(*I*, 0) = 0, then D = 0.
(2)If D(*I*, 0). x = 0 or x. D(*I*, 0) = 0 then
x = 0 or D = 0.
(3)If D(R, 0). x = 0 or x. D(R, 0) = 0 then
x = 0 or D = 0

Proof.(1) For all u ∈*I*and x ∈R, y ∈R,
0 = D(ux, y) = D(u, y).x + u. D(x, y)
=0 + u. D(x, y).
uD(x, y) = 0=*I*.D(x, y). Then D = 0.
(2)If D(I, 0). x = 0. Then
0 = D(uy, z).x = (D(u, z)y + uD(y, z)).x
= D(u, z). y. x + u. D(y, z). x
= u. D(u, z). x
D(u, z). x . u = 0. D(u, z). x. *I*= 0
Hence D = 0 or r =0.
(3) Put R instead of*I* in (2) . Hence give (3).

Definition 1.13. Let S be a non-empty subset of a hyperring R. The set
$Ann_l(s)$ = {x ∈S, xs = 0} is called the left annihilator of s in R.
Similarly, right annihilator $Ann_r(s)$ of in R is defined.

Remark 1.14. In a reduced hyperring R, if
ab = 0for all a, b = 0. Then ba =0 and therefore, there is a no distinction from a left annihilator of S and right annihilator of S in R.

Theorem 1.15. Let D be a symmetric bi-derivation of 2-torsion free prime hyperring R and a ∈R such that ad(x) = 0 (or d(x)a = 0) for all x ∈R. Then either a = 0 or D = 0.

Proof. Suppose ad(x) = 0 for all x ∈R. Then for all y ∈R.
0 = ad(x + y)
= ad(x) + aD(x + y) + aD(x + y) + ad(y).
= aD(x, y) + aD(x, y) For all z ∈R,
0 = aD(xz, y) + aD(xz, y)
= aD(x, y).z + ax.D(z, y) + aD(x, y)Z
+ axD(z,y)
= axD(z, y) + axD(z, y).
Since R is 2-torsion free, axD(z, y) = 0
for all x, y, z ∈R.
Since R is a prime hyperring. a = 0 or
D(z, y) = 0. If a ≠ 0, then D(z, y) = 0.
That is D =0.

Theorem 1.16. Let D be a symmetric bi-derivation of a reduced hyperring R.
Then for any subset s of R.
D(Ann(s), Ann(s)) ⊆Ann(s).

Proof. If x, y ∈Ann(s). Then sx = 0 and
sy = 0. Now for s ∈S,
0 = D(sx, y)
= D (s, y). x + x. D(s , y).
Multiplying by s at right,
0 = D(s, y)xs + sD(x, y)s.sD(x, y)s = 0.

Since R is a reduced hyperring. sD(x, y) = 0. That is D(x, y) ∈Ann(s).
Hence D(Ann(s), Ann(s)) ⊆Ann (s).

Theorem 1.17. Let D be a symmetric bi-derivation of 2 -torsion free reduced hyperring R. If D(D(x, y), y) = 0 for all x, y ∈R, then D = 0.

Proof. Let D(D(x, y), y) = 0 for all x, y ∈R. Replacing x by xz, where z ∈R, it follows that
0 = D(D(xz, y), y)
= D(D(x,y)z+xD(z, y),y)
=D(D(x, y)z, y)+D(xD(z, y),y)
= D(x, y)D(z, y) + D(D(x, y), y)z + xD(D(z, y),y) + D(x, y)D(z, y)
From here, 0 ∈D(x, y)D(z, y) + D(x, y)D(z, y).
Since R is 2 - torsion free hyperring .
D(x, y)D(z, y) = 0 for all x, y, z ∈R. If it is taken x instead of z,
$(D(x, y))^2$= 0 for all x,y ∈ R. Since R is a reduced hyperring, D(x, y) = 0 for all x, y ∈R. That is, D = 0.

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