**prμ-Closed Sets in Supra Topological Spaces**

G.Saroja ¹ and V.Thiripurasundari ²

¹ M.Phil scholar,
PG and Research Department of Mathematics,
Sri S.R.N.M.College,
Sattur - 626 203, Tamil Nadu, India.

² Assistant Professor,
PG and Research Department of Mathematics,
Sri S.R.N.M.College,
Sattur - 626 203, Tamil Nadu, India.

**Abstract**

In this paper, we introduce new class of closed sets in supra topological spaces, namely supra pr - closed set. We, discuss the relationship between this type of closed sets and other existing closed sets in supra topological spaces. Further, we obtain some basic properties of supra pr-closed sets.

**Keywords**: prμ-closed sets and supra regular semi-open.

**1 Introduction**

In 1983, Mashhour et al. [4] introduced the supra topological spaces and studied S-continuous functions and S*-continuous functions. In 2011, Ravi et al. [6] introduced and investigated several properties of supra generalized closed sets, supra sg-closed sets and gs-closed sets in supra topological spaces. In this paper we introduce a new class of closed sets called prμ-closed sets and we obtain some basic properties of prμ-closed sets in supra topological spaces. Further, we investigated the relationship between this type of closed sets and other existing closed sets in supra topological spaces.

**2 Preliminaries**

Throughout this paper (X, μ) (or simply X) represent supra topological spaces. For a subset A of X, clμ (A) and intμ (A) denote the supra closure of A and supra interior of A respectively. Let us recall the following definitions, which are useful in the sequel.

**Definition 2.1.** [4] A subfamily μ of X is said to be a supra topology on X, if

(i) X, φ ∈ μ.
(ii) If Aᵢ ∈ μ for all i ∈ J, then ∪ Aᵢ ∈ μ.

The pair (X, μ) is called the supra topological space. The elements of μ are called supra open sets in (X, μ) and the complement of a supra open set is called a supra closed set.

**Definition 2.2.** [4]
(i) The supra closure of a set A is denoted by cl^μ(A) and is defined as cl^μ(A) = ∩{B : B is supra closed and A ⊆ B}.
(ii) The supra interior of a set A is denoted by int^μ(A) and is defined as int^μ(A) = U{B : B is supra open and A ⊆ B}.

**Definition 2.3.** [4] Let (X, τ) be a topological space and μ be a supra topology associated with τ, if μ ⊆ τ.

**Definition 2.4.** A subset A of a supra topological space X is called a
(i) supra semi-open set [3] if A ⊆ cl^μ(int^μ(A)).
(ii) supra regular open set [1] if A = int^μ(cl^μ(A)).
(iii) supra pre-open set [7] if A ⊆ int^μ(cl^μ(A)).

The complement of a supra open set is called supra semi-closed (resp. supra regular open, supra pre-open) set is called supra semi-closed (resp. supra regular closed, supra pre-closed).

**Definition 2.5.** A subset A of a supra topological space (X, μ) is called
(i) a supra generalized closed set (briefly g^μ-closed) [6] if cl^μ(A) ≤ U whenever A ⊆ U and U is supra open in (X, μ).
(ii) a supra semi-generalized closed set (briefly sg^μ-closed) [3] if scl^μ(A) ≤ U whenever A ⊆ U and U is supra semi-open in (X, μ).
(iii) a supra generalized semi-closed set (briefly gs^μ-closed) [3] if scl^μ(A) ≤ U whenever A ⊆ U and U is supra open in (X, μ).
(iv) a supra generalizer closed set (briefly gpr^μ-closed) [7] if pcl^μ(A) ≤ U whenever A ⊆ U and U is supra open regular set in (X, μ).
(v) a supra regular generalized closed set (briefly rg^μ-closed) [3] if gpr^μ(A) ≤ U whenever A ⊆ U and U is supra open regular set in (X, μ).
(vi) a supra pre-closed set (briefly gp^μ-closed) [7] if pcl^μ(A) ≤ U whenever A ⊆ U and U is supra open in (X, μ).

The complement of a g^μ-closed (resp. sg^μ-closed, gs^μ-closed, gpr^μ-closed, rg^μ-closed, gp^μ-closed) set is called g^μ-open (resp. sg^μ-open, gs^μ-open, gpr^μ-open, rg^μ-open, gp^μ-open) set.

**Definition 2.6.** [7] A subset A of a supra topological space (X, μ) is called supra regular semi-open if for every supra open regular set U such that U ⊆ A ⊆ cl^μ(U).

The complement of a supra regular open set is called supra regular semi-closed.

### 3 pr^μ-Closed set

In this section, we introduce a new class of closed sets in supra topological spaces namely supra pr^μ-closed set. We denote it by pr^μ-closed set.

**Definition 3.1.** A subset A of a supra topological space (X, μ) is called supra pre-regular semi-open closed (briefly pr^μ-closed) if pcl^μ(A) ≤ U whenever A ⊆ U and U is supra regular semi-open in (X, μ).

The complement of pr^μ-closed set is called pr^μ-open. We denote the family of all pr^μ-closed set by pr^μ(X, μ).

**Theorem 3.2.** Every pr^μ-closed set is gpr^μ-closed.

**Proof.** Let A be pr^μ-closed set in (X, μ) and A ⊆ U where U be supra regular open. Since every supra regular open set is supra regular semi-open and A is pr^μ-closed, pcl^μ(A) ≤ U. Hence A is gpr^μ-closed.

**Remark 3.3.** The converse of the above theorem is not true in general as shown in the following example.

**Example 3.4.** Let X = {a, b, c, d} with μ = {φ, {a}, {b}, {a, b}, {a, c}, {a, d}, {a, b, c}, {b, c}, {d}, {a, b, c}, {a, d}, {a, c, d}, {b, c, d}, X}. gpr^μ-closed = {φ, {a}, {b}, {a, b}, {a, c}, {a, d}, {b, c}, X}. pr^μ-closed = {φ, {a}, {b}, {a, b}, {a, c}, {a, d}, {b, c}, {a, c, d}, X}.

**Theorem 3.5.** Every supra closed set is pr^μ-closed.

**Proof.** Let A ⊆ U be a supra closed set and U be any supra regular semi-open in (X, μ). Since A is supra closed, cl^μ(A) = A. Also, we know that pcl^μ(A) ⊆ cl^μ(A) ⊆ U. Hence pcl^μ(A) ⊆ U. Thus A is pr^μ-closed.

**Remark 3.6.** The converse of the above theorem is not true in general as shown in the following example.

**Example 3.7.** Let X = {a, b, c} with μ = {φ, {a}, {b}, {a, b}, X}. pr^μ-closed = {φ, {a}, {b}, {a, b}, {a, c}, {b, c}, X}. supra closed = {φ, {a}, {c}, {b, c}, X}.

Here {b, c} is pr^μ-closed but not supra closed.
**Theorem 3.8.** Every supra pre-closed set is pr\(^a\)-closed.

**Proof.** Let \( A \subseteq U \) be a supra pre closed set and \( U \) be any supra regular semi-open in \((X, \mu)\). Since \( A \) is supra pre-closed, \( \text{cl}^\mu (\text{int}^a (A)) \subseteq A \). Therefore \( \text{pcl}^\mu (A) \subseteq U \). Hence \( A \) is pr\(^a\)-closed.

**Remark 3.9.** The converse of the above theorem is not true in general as shown in the following example.

**Example 3.10.** Let \( X = \{a, b, c\} \) with \( \mu = \{\phi, \{a\}, \{a, b\}, \{b, c\}, X\} \). 

\[ \text{pr}^\mu \text{-closed} = \{\phi, \{a\}, \{b\}, \text{rg}^\mu, \text{pr}^\mu, \{a, b\}, \{a, c\}, \{b, c\}, X\} \]

\[ \text{supra pre-closed} = \{\phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}, X\} \]

Here \( \{a, b\} \) is pr\(^a\)-closed but not supra pre-closed.

**Theorem 3.11.** Every supra regular closed set is pr\(^a\)-closed.

**Proof.** Let \( A \subseteq U \) be a supra regular closed set and \( U \) be any supra regular semi-open in \((X, \mu)\). Since \( A \) is supra regular closed, \( \text{cl}^\mu (\text{int}^a (A)) \subseteq A \). Therefore \( \text{pcl}^\mu (A) \subseteq U \). Hence \( A \) is pr\(^a\)-closed.

**Remark 3.12.** The converse of the above theorem is not true in general as shown in the following example.

**Example 3.13.** Let \( X = \{a, b, c\} \) with \( \mu = \{\phi, \{a\}, \{a, b\}, \{b, c\}, X\} \). 

\[ \text{pr}^\mu \text{-closed} = \{\phi, \{a\}, \{b\}, \text{rg}^\mu, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\} \]

Here \( \{b, c\} \) is rg\(^a\)-closed but not pr\(^a\)-closed.

**Example 3.15.** Let \( X = \{a, b, c, d\} \) with \( \mu = \{\phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\} \).

Here \( \{b, c\} \) is rg\(^a\)-closed but not pr\(^a\)-closed.

**Example 3.16.** Let \( X = \{a, b, c, d\} \) with \( \mu = \{\phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, d, \}, \{a, c, d\}, \{b, c, d\}, X\} \).

Here \( \{a, d\} \) is gp\(^a\)-closed but not pr\(^a\)-closed and \( \{c, d\} \) is pr\(^a\)-closed but not gp\(^a\)-closed.

**Example 3.17.** Let \( X = \{a, b, c, d\} \) with \( \mu = \{\phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\} \).

Here \( \{a, d\} \) is gp\(^a\)-closed but not pr\(^a\)-closed and \( \{c, d\} \) is pr\(^a\)-closed but not gp\(^a\)-closed.

**Example 3.18.** Let \( X = \{a, b, c\} \) with \( \mu = \{\phi, \{a\}, \{b\}, \{a, b\}, X\} \).

Here \( \{a, b\} \) is pr\(^a\)-closed but not gp\(^a\)-closed.

**Example 3.19.** Let \( X = \{a, b, c, d\} \) with \( \mu = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\} \).

Here \( \{a, d\} \) is gp\(^a\)-closed but not pr\(^a\)-closed.

**Remark 3.20.** The following examples show that pr\(^a\)-closeness is independent from sg\(^a\)-closeness.

**Example 3.21.** Let \( X = \{a, b, c\} \) with \( \mu = \{\phi, \{a\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\} \). Then the set \( \{a\} \) is sg\(^a\)-closed and gp\(^a\)-closed but not pr\(^a\)-closed. Also the set \( \{a, b\} \) is pr\(^a\)-closed but neither sg\(^a\)-closed nor gp\(^a\)-closed.

**Remark 3.22.** The following diagram shows the relationships of pr\(^a\)-closed sets with other existing sets.
4 Basic properties of \( pr^u \)-closed set

In this section, we analyse the properties of \( pr^u \)-closed set.

**Remark 4.1.** Union of any two \( pr^u \)-closed set in \((X, \mu)\) need not be \( pr^u \)-closed as shown in the following example.

**Example 4.2.** Let \( X = \{a, b, c, d\} \) with \( \mu = \{\emptyset, \{a\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\} \) Then \( \{\emptyset, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a,c\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\} , X\} \) is \( pr^u \)-closed. Here \{b\} and \{c\} are two \( pr^u \)-closed sets. But the union of \{b\} and \{c\} is \{b, c\} is not \( pr^u \)-closed.

**Remark 4.3.** Intersection of any two \( pr^u \)-closed set in \((X, \mu)\) need not be \( pr^u \)-closed since, in Example 4.2, \{a, b\} and \{a, c\} are \( pr^u \)-closed sets but their intersection \{a\} is not \( pr^u \)-closed.

**Theorem 4.4.** Let \( A \) be \( pr^u \)-closed set in \((X, \mu)\). Then \( pcl^u (A) - A \) does not contain any non empty supra regular semi-closed set.

**Proof.** Let \( A \) be a \( pr^u \)-closed set in \((X, \mu)\). Suppose that \( H \) is a supra regular semi-closed set contained in \( pcl^u (A) - A \). Now \( H^c \) is supra regular semi-open set in \((X, \mu)\) such that \( A \subseteq H^c \). Since \( A \) is \( pr^u \)-closed set of \((X, \mu)\), \( pcl^u (A) \subseteq H^c \). Thus \( H \subseteq (pcl^u (A))^c \). Also since \( H \subseteq pcl^u (A) \) - \( A \). Therefore \( H \subseteq pcl^u (A) \cap (pcl^u (A))^c = \emptyset \). Hence \( H = \emptyset \).

**Theorem 4.5.** If \( A \) is a \( pr^u \)-closed set in a space \((X, \mu)\) and \( A \subseteq B \subseteq pcl^u (A) \), then \( B \) is also a \( pr^u \)-closed set.

**Proof.** Let \( A \) be a \( pr^u \)-closed set and \( U \) be supra semi-open set of \((X, \mu)\) such that \( B \subseteq U \). Then \( A \subseteq U \). Since \( A \) is \( pr^u \)-closed set, \( pcl^u (A) \subseteq U \). Also since \( B \subseteq pcl^u (A) \), \( pcl^u (B) \subseteq pcl^u (pcl^u (A)) = pcl^u (A) \). Hence \( pcl^u (B) \subseteq U \). Hence \( B \) is \( pr^u \)-closed set.

**Theorem 4.6.** The intersection of \( pr^u \)-closed set and a supra pre-closed set is also \( pr^u \)-closed.

**Proof.** Let \( A \) be \( pr^u \)-closed and let \( F \) be supra pre-closed. If \( U \) is any supra regular semi-open set with \( A \cap F \subseteq U \), then \( A \subseteq U \cup F^c \) and so \( pcl^u (A) \subseteq U \cup F^c \). Now, \( pcl^u (A \cap F) \subseteq pcl^u (A) \cap F \subseteq U \). Hence \( A \cap F \) is \( pr^u \)-closed.

**Theorem 4.7.** A subset \( A \subseteq X \) is \( pr^u \)-open if and only if \( F \subseteq pint^u (A) \) whenever \( F \) is supra regular semi closed and \( F \subseteq A \).

**Proof.** Let \( A \) be \( pr^u \)-open set and suppose \( F \subseteq A \), where \( F \) is supra regular semi-closed. Then \( X - A \) is \( pr^u \)-closed set contained in the supra regular semi-open set \( X - F \). Hence \( pcl^u (X - A) \subseteq X - F \). Thus \( F \subseteq pint^u (A) \). Conversely, if \( F \) is supra regular semi-closed set with \( F \subseteq pint^u (A) \) and \( F \subseteq A \), then \( X - pint^u (A) \subseteq X - F \). This implies that \( pcl^u (X - A) \subseteq X - F \). Hence \( X - A \) is \( pr^u \)-closed. Therefore, \( A \) is \( pr^u \)-open set.
Theorem 4.8. If $A$ is a $pr^u$-closed set of $(X, \mu)$, then $A$ is supra pre-closed set if and only if $pcl^u (A) - A$ is supra regular semi-closed.

**Proof. Necessity:** Let $A$ be a $pr^u$-closed and supra pre-closed subset of $X$. Then $pcl^u (A) = A$ and so $pcl^u (A) - A = \emptyset$ which is a supra regular semi-closed.

**Sufficiency:** Since $A$ is $pr^u$-closed and by theorem 4.4, $pcl^u (A) - A$ does not contain any non empty supra regular semi-closed set. But $pcl^u (A) - A = \emptyset$. That is $pcl^u (A) = A$. Hence, $A$ is supra pre-closed.

Theorem 4.9. If $A$ is a supra regular semi-open and $pr^u$-closed subset of $(X, \mu)$, then $A$ is supra pre-closed subset of $(X, \mu)$.

**Proof.** Let $A$ be supra regular semi-open and $pr^u$-closed. Then $pcl^u (A) \subseteq A$. But $A \subseteq pcl^u (A)$. Therefore $A = pcl^u (A)$. Hence $A$ is supra pre-closed.

References


