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pr^{μ}-Closed Sets in Supra Topological Spaces

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Abstract

In this paper, we introduce new class of closed sets in supra topological spaces, namely supra pr - closed set. We, discuss the relationship between this type of closed sets and other existing closed sets in supra topological spaces. Further, we obtain some basic properties of supra pr-closed sets.

Keywords : pr^{μ} -closed sets and supra regular semi-open.

1 Introduction

In 1983, Mashhour et al. [4] introduced the supra topological spaces and studied S-continuous functions and S*-continuous functions. In 2011, Ravi et al. [6] [3] introduced and investigated several properties of supra generalized closed sets, supra sg-closed sets and gs-closed sets in supra topological spaces. In this paper we introduce a new class of closed sets called pr^{μ} -closed sets and we obtain some basic properties of pr^{μ} -closed sets in supra topological spaces. Further, we investigated the relationship between this type of closed sets and other existing closed sets in supra topological spaces.

2 Preliminaries

Throughout this paper (X, μ) (or simply X) represent supra topological spaces. For a subset A of X, $cl^{\mu}(A)$ and $int^{\mu}(A)$ denote the supra closure of A and supra interior of A respectively. Let us recall the following definitions, which are useful in the sequel.

Definition 2.1. [4] A subfamily μ of X is said to be a supra topology on X, if
(i) $X, \varphi \in \mu$,
(ii) If $A_i \in \mu$ for all $i \in J$, then $\cup A_i \in \mu$.

The pair (X, μ) is called the supra topological space. The elements of μ are called supra open sets in (X, μ) and the complement of a supra open set is called a supra closed set.

Definition 2.2. [4]

(i) The supra closure of a set A is denoted by $cl^{\mu}(A)$ and is defined as $cl^{\mu}(A) = \cap\{B : B$ is supra closed and $A \subseteq B\}$.
(ii) The supra interior of a set A is denoted by $int^{\mu}(A)$ and is defined as $int^{\mu}(A) = \cup\{B : B$ is supra open and $A \supseteq B\}$.

Definition 2.3. [4] Let (X, τ) be a topological space and μ be a supra topology associated with τ , if $\tau \subset \mu$.

Definition 2.4. A subset A of a supra topological space X is called a
(i) supra semi-open set [3] if $A \subseteq cl^{\mu}(int^{\mu}(A))$.
(ii) supra regular open set [1] if $A = int^{\mu}(cl^{\mu}(A))$.
(iii) supra pre-open set [7] if $A \subseteq int^{\mu}(cl^{\mu}(A))$.
The complement of a supra semi-open (resp. supra regular open, supra pre-open) set is called supra semi-closed (resp. supra regular closed, supra pre-closed).

Definition 2.5. A subset A of a supra topological space (X, μ) is called
(i) a supra generalized closed set (briefly g^{μ}-closed) [6] if $cl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ) .
(ii) a supra semi-generalized closed set (briefly sg^{μ}-closed) [3] if $scl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra semi-open in (X, μ) .
(iii) a supra generalized semi-closed set (briefly gs^{μ}-closed) [3] if $scl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ) .
(iv) a supra generalized pre-regular closed set (briefly gpr^{μ}-closed) [7] if $pcl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra regular open set in (X, μ) .
(v) a supra regular generalized closed set (briefly rg^{μ}-closed) [2] if $cl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra regular open in (X, μ) .
(vi) a supra generalized pre-closed set (briefly gp^{μ}-closed) [7] if $pcl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ) .
The complement of a g^{μ}-closed (resp. sg^{μ}-closed, gs^{μ}-closed, gpr^{μ}-closed, rg^{μ}-closed, gp^{μ}-closed) set is called g^{μ}-open (resp. sg^{μ}-open, gs^{μ}-open, gpr^{μ}-open, rg^{μ}-open, gp^{μ}-open) set.

Definition 2.6. [7] A subset A of a supra topological space (X, μ) is called supra regular semi-open if for every supra regular open set U such that $U \subseteq A \subseteq cl^{\mu}(U)$.
The complement of a supra regular semi-open set is called supra regular semi-closed.

3 pr^{μ} -Closed set

In this section, we introduce a new class of closed sets in supra topological spaces namely supra pr -closed set. We denote it by pr^{μ}-closed set.

Definition 3.1. A subset A of a supra topological space (X, μ) is called supra pre-regular semi-open closed (briefly pr^{μ}-closed) if $pcl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra regular semi-open in (X, μ).
The complement of pr^{μ}-closed set is called pr^{μ}-open. We denote the family of all pr^{μ}-closed set by $pr^{\mu}(X, \mu)$.

Theorem 3.2. Every pr^{μ}-closed set is gpr^{μ}-closed.
Proof. Let A be pr^{μ}-closed set in (X, μ) and $A \subseteq U$ where U be supra regular open. Since every supra regular open set is supra regular semi-open and A is pr^{μ}-closed, $pcl^{\mu}(A) \subseteq U$. Hence A is gpr^{μ}-closed.

Remark 3.3. The converse of the above theorem is not true in general as shown in the following example.

Example 3.4. Let $X = \{a, b, c, d\}$ with $\mu = \{\varphi, \{c\}, \{d\}, \{c, d\}, X\}$. gpr^{μ}-closed = $\{\varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a,d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\},X\}$; pr^{μ}-closed = $\{\varphi, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a,b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$. Here $\{b, c\}$ is gpr^{μ}-closed but not pr^{μ}-closed.

Theorem 3.5. Every supra closed set is pr^{μ}-closed.
Proof. Let $A \subseteq U$ be a supra closed set and U be any supra regular semi-open in (X, μ). Since A is supra closed, $cl^{\mu}(A) = A$. Also, we know that $pcl^{\mu}(A) \subseteq cl^{\mu}(A) \subseteq U$. Hence $pcl^{\mu}(A) \subseteq U$. Thus A is pr^{μ}-closed.

Remark 3.6. The converse of the above theorem is not true in general as shown in the following example.

Example 3.7. Let $X = \{a, b, c\}$ with $\mu = \{\varphi, \{a\}, \{a, b\}, \{b, c\}, X\}$. pr^{μ}-closed = $\{\varphi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$; supra closed = $\{\varphi, \{a\}, \{c\}, \{b, c\}, X\}$. Here $\{b\}$ is pr^{μ}-closed but not supra closed.

Theorem 3.8. Every supra pre-closed set is pr^{μ}-closed.

Proof. Let $A \subseteq U$ be a supra pre closed set and U be any supra regular semi-open in (X, μ). Since A is supra pre-closed, $cl^{\mu}(int^{\mu}(A)) \subseteq A$. Therefore $pcl^{\mu}(A) \subseteq U$. Hence A is pr^{μ}-closed.

Remark 3.9. The converse of the above theorem is not true in general as shown in the following example.

Example 3.10. Let X = {a, b, c} with μ = {φ, {a}, {b}, {a, b}, {b, c}, X}. pr^{μ}-closed = { φ, {a}, {c}, {a, b}, {a, c}, {b, c}, X} ; supra pre-closed = {φ, {a}, {c}, {a, c}, {b, c}, X} . Here {a, b} is pr^{μ}-closed but not supra pre-closed.

Theorem 3.11. Every supra regular closed set is pr^{μ}-closed.

Proof. Let $A \subseteq U$ be a supra regular closed set and U be any supra regular semi-open in (X, μ). Since A is supra regular closed, $cl^{\mu}(int^{\mu}(A)) \subseteq A$. Therefore $pcl^{\mu}(A) \subseteq U$. Hence A is pr^{μ}-closed.

Remark 3.12. The converse of the above theorem is not true in general as shown in the following example.

Example 3.13. Let X = {a, b, c} with μ = {φ, {a}, {a, b}, {b, c}, X}. pr^{μ}-closed = {φ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, X} ; supra regular closed = {φ, {a}, {b, c}, X}. Here {b} is pr^{μ}-closed but not supra regular closed.

Remark 3.14. The following examples show that pr^{μ}-closeness is independent from rg^{μ}-closeness, gp^{μ}-closeness and g^{μ}-closeness.

Example 3.15. Let X = {a, b, c, d} with μ = {φ, {c}, {d}, {c, d}, X}. rg^{μ}-closed = {φ, {a}, {b}, {a, b}, {a, c}, {a,d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, X} ; pr^{μ}-closed = {φ, {a}, {b}, {a, b}, {c, d}, {a, b, c}, {a,b, d}, {a, c, d}, {b, c, d}, X} . Here {b, c} is rg^{μ}-closed but not pr^{μ}-closed.

Example 3.16. Let X = {a, b, c, d} with μ = {φ, {a}, {b}, {a, b}, {b, d}, {a, b, d}, X}. pr^{μ}-closed = {φ, {c}, {d}, {a,b}, {a, c}, {a, d}, {b, c}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, X} ; rg^{μ}-closed = {φ, {c}, {a, b}, {a, c}, {a,d}, {b, c}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, X}. Here {d} is pr^{μ}–closed but not rg^{μ}-closed.

Example 3.17. Let X = {a, b, c, d} with μ = {φ, {c}, {d}, {c, d}, X}. gp^{μ}-closed = {φ, {a}, {b}, {a, b}, {a, c}, {a,d}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, X} ; pr^{μ}-closed = {φ, {a}, {b}, {a, b}, {c, d}, {a, b, c}, {a,b,d}, {a, c, d}, {b, c, d}, X}. Here {a, d} is gp^{μ}-closed but not pr^{μ}-closed and {c, d} is pr^{μ}-closed but not gp^{μ}-closed.

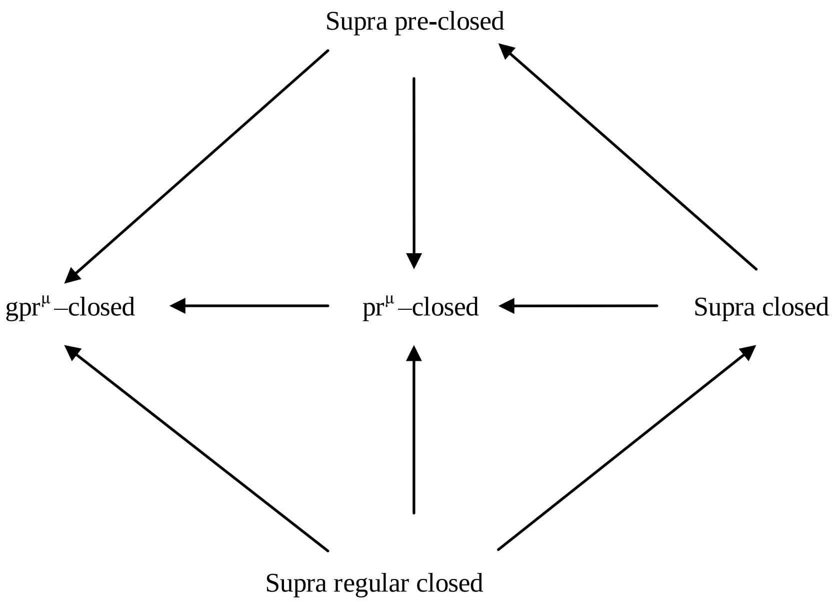
Example 3.18. Let X = {a, b, c} with μ = {φ, {a}, {b}, {a, b}, {b, c}, X}. pr^{μ}-closed = { φ, {a}, {c}, {a, b}, {a, c}, {b, c}, X} ; supra gp^{μ}-closed = {φ, {a}, {c}, {a, c}, {b, c}, X}. Here {a, b} is pr^{μ}-closed but not g^{μ}-closed.

Example 3.19. Let X = {a, b, c, d} with μ = {φ, {c}, {d}, {c, d}, X}. g^{μ}-closed = {φ, {a}, {b}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {a, c, d}{b, c, d}, X} ; pr^{μ}-closed = {φ, {a}, {b}, {a, b}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, X}. Here {a, d} is g^{μ}-closed but not pr^{μ}-closed.

Remark 3.20. The following examples show that pr^{μ}-closeness is independent from sg^{μ}-closeness and gs^{μ}-closeness.

Example 3.21. Let X = {a, b, c, d} with μ = {φ, {a}, {a, c}, {b, c}, {a, b, c}, X}. Then the set {a} is sg^{μ}-closed and gs^{μ}-closed but not pr^{μ}-closed. Also the set {a, b} is pr^{μ}-closed but neither sg^{μ}-closed nor gs^{μ}-closed.

Remark 3.22. The following diagram shows the relationships of pr^{μ}–closed sets with other existing sets.



4 Basic properties of pr^{μ} -closed set

In this section, we analyse the properties of pr^{μ}-closed set.

Remark 4.1. Union of any two pr^{μ}-closed set in (X, μ) need not be pr^{μ}-closed as shown in the following example.

Example 4.2. Let X = {a, b, c, d} with μ = {φ, {a}, {a, c}, {b, c}, {a, b, c}, X} Then {φ, {b}, {c}, {d}, {a, b}, {a,c}, {a, d}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d} , X} is pr^{μ}-closed. Here {b} and {c} are two pr^{μ}-closed sets. But the union of {b} and {c} is {b, c} is not pr^{μ}-closed.

Remark 4.3. Intersection of any two pr^{μ}-closed set in (X, μ) need not be pr^{μ}-closed since, in Example 4.2, {a, b} and {a, c} are pr^{μ}-closed sets but their intersection {a} is not pr^{μ}-closed.

Theorem 4.4. Let A be pr^{μ}-closed set in (X, μ). Then pcl^{μ} (A) − A does not contain any non empty supra regular semi-closed set.

Proof. Let A be a pr^{μ}-closed set in (X, μ). Suppose that H is a supra regular semi-closed set contained in pcl^{μ} (A) − A. Now H^c is supra regular semi-open set in (X, μ) such that A ⊆ H^c. Since A is pr^{μ}-closed set of (X, μ), pcl^{μ} (A) ⊆ H^c. Thus H ⊂ $(pcl^{\mu}(A))^c$. Also H ⊆ pcl^{μ} (A) – A. Therefore H ⊆ pcl^{μ} (A) ∩ $(pcl^{\mu}(A))^c$ = φ. Hence H = φ.

Theorem 4.5. If A is a pr^{μ}-closed set in a space (X, μ) and A ⊆ B ⊆ pcl^{μ} (A), then B is also a pr^{μ}-closed set.

Proof. Let U be a supra regular semi open set of (X, μ) such that B ⊆ U. Then A ⊆ U. Since A is pr^{μ}-closed set, pcl^{μ}(A) ⊆ U . Also since B ⊆ pcl^{μ} (A), pcl^{μ} (B) ⊆ pcl^{μ} (pcl^{μ} (A)) = pcl^{μ} (A). Hence pcl^{μ} (B) ⊆ U. Hence B is pr^{μ}-closed set.

Theorem 4.6. The intersection of pr^{μ}-closed set and a supra pre-closed set is also pr^{μ}-closed.

Proof. Let A be pr^{μ}-closed and let F be supra pre-closed. If U is any supra regular semi-open set with A ∩ F ⊆ U, then A ⊆ U ∪ F^c and so pcl^{μ} (A) ⊆ U ∪F^c. Now, pcl^{μ} (A ∩ F) ⊆ pcl^{μ} (A) ∩ F ⊆ U. Hence A ∩ F is pr^{μ}-closed.

Theorem 4.7. A subset A ⊆ X is pr^{μ}-open if and only if F ⊆ $pint^{\mu}$ (A) whenever F is supra regular semi closed and F ⊆ A.

Proof. Let A be pr^{μ}-open set and suppose F ⊆ A, where F is supra regular semi-closed. Then X −A is pr^{μ}-closed set contained in the supra regular semi-open set X−F. Hence pcl^{μ} (X−A) ⊆ X−F . Thus F ⊆ $pint^{\mu}$ (A). Conversely, if F is supra regular semi-closed set with F ⊆ $pint^{\mu}$ (A) and F ⊆ A, then X−$pint^{\mu}$ (A) ⊆ X−F. This implies that pcl^{μ} (X−A) ⊆ X−F. Hence X−A is pr^{μ}-closed. Therefore, A is pr^{μ}-open set.

Theorem 4.8. If A is a pr^{μ}-closed set of (X, μ), then A is supra pre-closed set if and only if $pcl^{\mu}(A) - A$ is supra regular semi-closed.

Proof. Necessity: Let A be a pr^{μ}-closed and supra pre-closed subset of X. Then $pcl^{\mu}(A) = A$ and so $pcl^{\mu}(A) - A = \varphi$ which is a supra regular semi-closed.

Sufficiency: Since A is pr^{μ}-closed and by theorem 4.4, $pcl^{\mu}(A) - A$ does not contain any non empty supra regular semi-closed set. But $pcl^{\mu}(A) - A = \varphi$. That is $pcl^{\mu}(A) = A$. Hence, A is supra pre closed.

Theorem 4.9. If A is a supra regular semi-open and pr^{μ}-closed subset of (X, μ), then A is supra pre closed subset of (X, μ).

Proof. Let A be supra regular semi-open and pr^{μ}-closed. Then $pcl^{\mu}(A) \subseteq A$. But $A \subseteq pcl^{\mu}(A)$ .Therefore $A = pcl^{\mu}(A)$. Hence A is supra pre-closed.

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