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A new class of generalized delta semiclosed sets using grill delta space

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Abstract

In this paper we apply the notion of generalized semiclosed sets and obtain a new class of generalized delta semiclosed sets using grill delta space. Also we investigate the properties of generalized delta semiclosed sets.

Keywords: Grill, topology τ_G^δ, operator Φ_δ, G-gδs-closed.

Mathematical Subject Classification: 54A10

1 Introduction

The concept of grill topological spaces depended on the two operators are Φ and Ψ. This concept was first introduced by Choquet [2] in 1947. Also, for the investigation of many topological notions similar compactifications, proximity spaces and extension problems of different kinds, Rodyna A. Hosny[6] introduce the concept δ-set using grill and obtain grill delta space. In this paper, we explore the concept of generalized semiclosed sets to define a new class of generalized delta semiclosed sets via grill delta space.

2 Preliminaries

Throughout this paper (X, τ) (or simply X) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively.

Definition 2.1. [1] A collection G of non empty subsets of a space X is called grill on X if
(i) $A \in G$ and $A \subseteq B \subseteq X \Rightarrow B \in G$
(ii) $A, B \subseteq X$ and $A \cup B \in G \Rightarrow A \in G$ or $B \in G$.

Definition 2.2. [4]The δ-interior of a subset A of X is the union of all regular open set of X contained in A and is denoted by $int_\delta(A)$. The subset A is called δ-open if $A = int_\delta(A)$, i.e. a set is δ-open if it is the union of regular open sets. The complement of a δ-open is called δ-closed. Alternatively, a set $A \subseteq (X, \tau)$ is called δ-closed if $A = cl_\delta(A)$, where $cl_\delta(A) = \{x \in X : int(cl(U)) \cap A \neq \varphi, U \in \tau \text{ and } x \in U\}$

Definition 2.3. [6]Let (X, τ^δ, G) be a grill delta space. We define a mapping $\Phi_\delta : P(X) \to P(X)$ denoted by $\Phi_{\delta G}(A, \tau)$ (for $A \in P(X)$) or $\Phi_{\delta G}(A)$ or simply $\Phi_\delta(A)$, called the operator associated with grill G and the topology τ^δ , and is defined by $\Phi_{\delta G} = \{x \in X : A \cap U_x \in G, \forall U_x \in \delta O(X, \tau)\}$

Definition 2.4. [6]Let (X, τ^δ, G) be a grill delta space. We define a map $\Psi_{\delta G} : P(X) \to P(X)$ by $\Psi_{\delta G}(A) = X \setminus (\Phi_\delta(X \setminus A))$ or $\Psi_\delta(A) = A \cup \Phi_\delta(A)$ for all $A \in P(X)$.

Definition 2.5. [6]Corresponding to a grill delta space (X, τ^δ, G) there exists a unique topology τ_G^δ (say) on X given by τ_G^δ = {U ⊆ X : Ψ_δ(X \U) = (X \U)} where for any A ⊆ X, $\Psi_\delta(A) = A \cup \Phi_\delta(A) = \tau_G^\delta - cl_\delta(A)$.

Theorem 2.6. [6]Let (X, τ^δ, G) be a grill delta space. Then
(i) A ⊆ B) $\Phi_\delta(A) \subseteq \Phi_\delta(B)$.
(ii) $\Phi\delta(A \cup B) = \Phi_\delta(A) \cup \Phi_\delta(B)$, for any A ⊆ X.
(iii) $\Phi_\delta(\Phi_\delta(A)) \subseteq \Phi_\delta(A) = cl_\delta(\Phi_\delta(A)) \subseteq cl_\delta(A)$, for any A, B ⊆ X.
(iv) A ⊆ X and A ∉ G ⇒ $\Phi_\delta(A) = \varphi$.

Definition 2.7. [5] A subset A of X is called generalized δ-semiclosed (briefly gδs − closed) if sCl(A) ⊆ U whenever A ⊆ U and U is δ-open in X. The family of all gδs-closed subsets of the space X is denoted by GδSC(X).

3 Generalized delta semiclosed sets with respect to a grill delta space

Definition 3.1. Let (X, τ^δ, G) be a grill delta space. Then a subset A of X is said to be gδs-closed with respect to grill G (G-gδs-closed, for short) if $\Phi_\delta(A) \subseteq U$ and U is δ-open in X.

Definition 3.2. A subset A of X is said to be G-gδs-open if X \ A is G-gδsclosed.

Proposition 3.3. For any (X, τ^δ, G) grill delta space,

(i) Every closed set in X is G-gδs-closed.

(ii) For any set A in X, $\Phi_\delta(A)$ is G-gδs-closed.

(iii) Every τ_G^δ-closed set is G-gδs-closed.

(iv) Any non member of G is G-gδs-closed.

(v) Every gδs-closed is G-gδs-closed.

Proof.
(i)Let A be a closed set. Then cl(A) = A. Let U be a δ-open set in X such that A ⊆ U. Then, $\Phi_\delta(A) = cl_\delta(A) \subseteq cl(A) = A \subseteq U \Rightarrow \Phi_\delta(A) \subseteq U$ ⇒ A is G-gδs-closed.

(ii)Let A be a subset in X. Then $\Phi_\delta(\Phi_\delta(A)) \subseteq \Phi_\delta(A) \subseteq U \Rightarrow \Phi_\delta(A)$ is G-gδsclosed.

(iii)Let A be τ_G^δ -closed set then $\tau_G^\delta - cl_\delta(A) = A \Rightarrow A \cup \Phi_\delta = A \Rightarrow \Phi_\delta(A) \subseteq A$. Therefore, $\Phi_\delta(A) \subseteq U$ whenever A ⊆ U and U is δ-open in X. This implies A is G-gδs-closed.

(iv)Let A ∉ G then $\Phi_\delta(A) = \varphi$ ⇒ A is G-gδs-closed.

(v)Let A be a gδs-closed set and U be a δ-open in X, such that A ⊆ U, then scl(A) ⊆ U, consider $\Phi_\delta(A) \subseteq cl_\delta(A) \subseteq scl(A) \subseteq U$ ⇒ A is G-gδs-closed. Thus every gδs-closed set is G-gδs-closed.

Remark 3.4. The converse of above proposition 3.3(v) need not be true as shown in the following example.

Example 3.5. Let X = {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}}, τ^δ = {φ, X, {a}, {b}, {a, b}} and G = {{a}, {b}, {a, b}, X}, then (X, τ^δ) is a space and G is a grill on X. Let A = {a, b} then $\Phi_\delta(A)$ = {a, b} ⊆ U, where U is δ-open in X. Therefore, A is G-gδs -closed. But scl(A) = X does not subset of {a, b}. Therefore, A is not gδs-closed.

Definition 3.6. Let X be a grill delta space and (φ ≠)A ⊆ X. Then [A] = {B ⊆ X : A ∩ B ≠ φ } is a grill on X, called the principal grill generated by A.

Proposition 3.7. In case of [X] principal grill generated by X, it is known that $\tau = \tau_{[X]}$ so that any [X]-gδs-closed set becomes simply a gδs-closed set and vice-versa.

Theorem 3.8. Let (X, τ^δ, G) be a grill delta space. If a subset A of X is G-gδs-closed then $\tau_G^\delta - cl_\delta(A) \subseteq U$ whenever A ⊆ U and U is δ-open.

Proof.

Let A be a G-gδs-closed set and U be a δ-open in X such that A ⊆ U then $\Phi_\delta(A) \subseteq U \Rightarrow A \cup \Phi_\delta(A) \subseteq U \Rightarrow \tau_G^\delta - cl_\delta(A) \subseteq U$. Thus $\tau_G^\delta - cl_\delta(A) \subseteq U$ whenever A ⊆ U and U is δ-open.

Theorem 3.9. Let (X, τ^δ, G) be a grill delta space. If a subset A of X is G-gδs-closed then for all $x \in \tau_G^\delta - cl_\delta(A)$, $cl_\delta(\{x\}) \cap A \neq \varphi$.

Proof.

Let $x \in \tau_G^\delta - cl_\delta(A)$. If $cl_\delta(\{x\}) \cap A = \varphi \Rightarrow A \subseteq X \setminus cl_\delta(\{x\})$ then by Theorem 3.8, $\tau_G^\delta - cl_\delta(A) \subseteq X \setminus cl_\delta(\{x\})$ which is a contradiction to our assumption that $x \in \tau_G^\delta - cl_\delta(A)$. Therefore, $cl_\delta(\{x\}) \cap A \neq \varphi$.

Theorem 3.10. Let (X, τ^δ, G) be a grill delta space. If a subset A of X is G-gδs-closed then $\tau_G^\delta - cl_\delta(A) \setminus A$ contains no non empty closed set of (X, τ). Moreover $\Phi_\delta(A) \setminus A$ contains no non-empty closed set of (X, τ).

Proof.

Let F be a closed set contained in $\tau_G^\delta - cl_\delta(A) \setminus A$ and let x ∈ F, since F ∩ A = φ we get $cl_\delta(\{x\}) \cap A = \varphi$ which is a contradiction to fact that $cl_\delta(\{x\}) \cap A \neq \varphi$. $\tau_G^\delta - cl_\delta(A) \setminus A$ contains no non-empty closed set of (X, τ). Since $\Phi_\delta(A) \setminus A = \tau_G^\delta - cl_\delta(A) \setminus A$, $\Phi_\delta(A) \setminus A$ contains no non-empty closed set of (X, τ).

Corollary 3.11. Let (X, τ^δ) be a T_1-space and G be a grill on X. Then every G-gδs-closed set is τ_G^δ -closed.

Proof.

Let A be a G-gδs-closed set and $x \in \Phi_\delta(A)$. Then $x \in \tau_G^\delta - cl_\delta(A)$. By Theorem 3.8, $cl_\delta(\{x\}) \cap A \neq \varphi$, $\{x\} \cap A \neq \varphi$, x ∈ A. Therefore, $\Phi_\delta(A) \subseteq A$. Thus A is τ_G^δ -closed.

Corollary 3.12. Let (X, τ^δ) be a T_1-space and G be a grill on X. Then A(⊆ X) is G-gδs-closed set if and only if A is τ_G^δ -closed.

Proposition 3.13. Let G be a grill on a space (X, τ^δ) and A be a G-gδs-closed set. Then the following are equivalent:

(i) A is τ_G^δ -closed.

(ii) $\tau_G^\delta - cl_\delta(A) \setminus A$ is closed in (X, τ).

(iii) $\Phi_\delta(A) \setminus A$ is closed in (X, τ)

Proof.

(i) ⇒ (ii)Let A be τ_G^δ -closed then $\tau_G^\delta - cl_\delta(A) \setminus A = \varphi$ so $\tau_G^\delta - cl_\delta(A) \setminus A$ is a closed set.

(ii) ⇒ (iii)Since $\tau_G^\delta - cl_\delta(A) \setminus A = \Phi_\delta(A) \setminus A$, $\Phi_\delta(A) \setminus A$ is closed in (X, τ^δ).

(iii) ⇒ (i)Let $\Phi_\delta(A) \setminus A$ be closed in (X, τ^δ). Since A is G-gδs-closed by theorem 3.10, $\Phi_\delta(A) \setminus A = \varphi$. So A is τ_G^δ -closed.

Lemma 3.14. Let (X, τ^δ) be a space and G be a grill on X. If A(⊆ X) is τ_G^δ -dense in itself, then $\Phi_\delta(A) = cl_\delta(\Phi_\delta(A)) = \tau_G^\delta - cl_\delta(A) = cl_\delta(A)$.

Proof.

Let A be τ_G^δ -dense in itself $\Rightarrow A \subseteq \Phi_\delta(A) \Rightarrow cl_\delta(A) \subseteq cl_\delta(\Phi_\delta(A)) = \Phi_\delta(A) \subseteq cl_\delta(A) \Rightarrow cl_\delta(A) = \Phi_\delta(A) = cl_\delta(\Phi_\delta(A))$. Now by definition $\tau_G^\delta - cl_\delta(A) = A \cup \Phi_\delta(A) = A \cup cl_\delta(A) = cl_\delta(A)$. Therefore, $\Phi_\delta(A) = cl_\delta(\Phi_\delta(A)) = \tau_G^\delta - cl_\delta(A) = cl_\delta(A)$.

Theorem 3.15. Let G be a grill on a space (X, τ^δ). If A(⊆ X) is τ_G^δ -dense in itself and G-gδs-closed, then A is gδs-closed.

Proof.

Follows from Lemma 3.14.

Corollary 3.16. For a grill G on a space (X, τ^δ). Let A(⊆ X) be τ_G^δ -dense in itself. Then A is G-gδs-closed if and only if A is gδs-closed.

Proof.

Follows from proposition 3.3(v) and theorem 3.15.

Theorem 3.17. For any grill on a space (X, τ^δ) the following are equivalent:

(i) Every subset of X is G-gδs-closed.

(ii) Every δ-open subset of (X, τ^δ) is τ_G^δ -closed.

Proof.
(i) ⇒ (ii)Let A be δ-open in (X, τ^{δ}). Then by (i), A is G-gδs-closed so that $\Phi_\delta(A) \subseteq A \Rightarrow A$ is τ_G^δ -closed.
(ii) ⇒ (i)Let $A \subseteq X$ and U be δ-open in (X, τ) such that $A \subseteq U$. Since U is δ-open by (ii), $\Phi_\delta(U) \subseteq U$. Now $A \subseteq U \Rightarrow \Phi_\delta(A) \subseteq \Phi_\delta(U) \subseteq U \Rightarrow$ A is G-gδs-closed.

Theorem 3.18. For any subset A of a space (X, τ^δ) and a grill G on X. If A is G-gδs-closed then $A \cup (X \setminus \Phi_\delta(A))$ is G-gδs-closed.
Proof.
Let $A \cup (X \setminus \Phi_\delta(A)) \subseteq U$, where U is δ-open in X. Then $X \setminus U \subseteq X \setminus (A \cup (X \setminus \Phi_\delta(A))) = \Phi_\delta(A) \setminus (A)$. Since A is G-gδs-closed, by theorem 3.8, we have $X \setminus U = \varphi$, that is X = U. Since X is the only δ-open set containing $A \cup (X \setminus \Phi_\delta(A))$, $A \cup (X \setminus \Phi_\delta(A))$ is G-gδs-closed.

Proposition 3.19. For any subset A of a space (X, τ^δ) and a grill G on X, the following are equivalent:
(i) $A \cup (X \setminus \Phi_\delta(A))$ is G-gδs-closed.
(ii) $\Phi_\delta(A) \setminus A$ is G-gδs-open.
Proof.
Follows from the fact that $X \setminus (\Phi_\delta(A) \setminus A) = A \cup (X \setminus \Phi_\delta(A))$.

Theorem 3.20. Let (X, τ^δ) be a space, G be a grill on X and A, B be subsets of X such that $A \subseteq B \subseteq \tau_G^\delta - \mathrm{cl}\delta(A)$. If A is G-gδs-closed, then B is G-gδs-closed.
Proof.
Let $B \subseteq U$, where U is δ-open in X. Since A is G-gδs-closed, $\Phi_\delta(A) \subseteq U \Rightarrow \tau_G^\delta - \mathrm{cl}_\delta(A) \subseteq U$. Now, $A \subseteq B \subseteq \tau_G^\delta - \mathrm{cl}_\delta(A) \Rightarrow \tau_G^\delta - \mathrm{cl}_\delta(A) \subseteq \tau_G^\delta - \mathrm{cl}_\delta(B) \subseteq \tau_G^\delta - \mathrm{cl}_\delta(A)$. Thus $\tau_G^\delta - \mathrm{cl}_\delta(B) \subseteq U$ and hence B is G-gδs-closed.

Corollary 3.21. τ_G^δ -closure of every G-gδs-closed set is G-gδs-closed.

Theorem 3.22. Let G be a grill on a space (X, τ^δ) and A, B be subsets of X such that $A \subseteq B \subseteq \Phi_\delta(A)$. If A is G-gδs-closed, Then A and B are gδs-closed.
Proof.
$A \subseteq B \subseteq \Phi_\delta(A) \Rightarrow A \subseteq B \subseteq \tau_G^\delta - \mathrm{cl}_\delta(A)$ and hence by theorem 3.20, B is G-gδs-closed. Again, $A \subseteq B \subseteq \Phi_\delta(A) \Rightarrow \Phi_\delta(A) \subseteq \Phi_\delta(B) \subseteq \Phi_\delta(\Phi_\delta(A)) \subseteq \Phi_\delta(A) \Rightarrow \Phi_\delta(A) = \Phi_\delta(B)$. Thus A and B are τ_G^δ-dense in itself and hence by Theorem 3.15, A and B are gδs-closed.

Theorem 3.23. Let G be a grill on a space (X, τ^δ). Then a subset A of X is G-gδs-closed if and only if $F \subseteq \tau_G^\delta - \mathrm{int}_\delta(A)$ whenever $F \subseteq A$ and F is closed.
Proof.
Let A be G-gδs-open and $F \subseteq A$, where F is closed in (X, τ^δ). Then $X \setminus A \subseteq X \setminus F \Rightarrow \Phi_\delta(X \setminus A) \subseteq X \setminus F \Rightarrow \tau_G^\delta - \mathrm{cl}_\delta(X \setminus A) \subseteq X \setminus F \Rightarrow F \subseteq \tau_G^\delta - \mathrm{int}_\delta(A)$.
Conversely, $X \setminus A \subseteq U$ where U is open in $(X, \tau^\delta) \Rightarrow X \setminus U \subseteq \tau_G^\delta - \mathrm{int}_\delta(A) \Rightarrow \tau_G^\delta - \mathrm{cl}_\delta(X) \subseteq U$. Thus $(X \setminus A)$ is G-gδs-closed and hence A is G-gδs-open.

Conclusion

In this paper, we introduce a new class of generalized delta semiclosed sets using grill delta space and study some of their properties.

References

[1] A. A. Nasefa, A. A. Azzam, *"Some topological operators Via grills"*, Journal of Linear and Topological Algebra, Vol. 05, No. 03, 2016, 199-204.

[2] G. Choquet, *"Sur les notions de filter et grills"*, Comptes Rendus Acad. Sci. Paris, 224(1947), 171-173.

[3] I. Arockiarani and V. Vinodhini, "A new class of generalized semiclosed sets using grills", Scientia Magna, Vol. 8 (2012), No. 2, 8-14.

[4] N. V. Velicko, *"H-closed topological spaces"*, Amer.Math.Soc.Transl., 78(1968), 103-118.

[5] S. S. Benchalli and Umadevi I. Neeli, *"Generalized Delta semiclosed sets in Topological spaces",* Int. Jl. Appl. Mahs, Vol. 24. No. 1(2011), 21-38.

[6] Rodyna A. Hosny, *"δ-Sets with Grill "*, International Mathematical Forum, Vol. 7, 2012, no. 43, 2107 - 2113.