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HOMO-CORDIAL LABELING OF SOME SPECIAL GRAPHS

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ABSTRACT

Let G = (V, E) be a graph with p vertices and q edges. A Homo-Cordial Labeling of a graph G with vertex set V is a bijection from V to {0, 1} such that each uv is assigned the label 1 if f(u) = f(v) or 0 if f(u) ≠ f(v) with the condition that $|v_f(0)-v_f(1)|\leq 1$ and $|e_f(0)-e_f(1)| \leq 1$. The graph that admits a Homo-Cordial Labeling is called Homo-Cordial Graph. In this paper, we prove some graphs such as switching of cycle, middle graph, Pn^2 , pertersen graph and banana tree are homo-cordial graphs.

Keywords: Cordial labeling, Homo-cordial labeling, Homo-cordial graph.
AMS Subject classification(2010): 05C78

1 INTRODUCTION

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G is called edges. The vertex set and edge set of G are denoted by V(G) and E(G) respectively. In this paper, we proved some graphs such as switching of cycle, middle graph, Pn^2 , pertersen graph and banana tree are homo-cordial graphs.

2 PRELIMINARIES

Definition 2.1. Switching on a vertex v of a graph G has the effect of removing all edges incident with the vertex and joining the vertex to all vertices to which it was formerly non-adjacent.

Definition 2.2. The Middle graph M(G) of a graph G is a graph whose vertex set is V(G) ∪ E(G) and in which two vertices are adjacent if either they are adjacent edges in G or one is vertex of G and other is an edge incident with it.

Definition 2.3. Pn^2 is a path of length n − 1 of twice.

Definition 2.4. Let $K_{1,n1}$,$K_{1,n2}$,$K_{1,n3}$, . . . ,$K_{1,nk}$ be a family of stars with the vertex sets

$V(K_{1,n_i}) = \{c_i, a_{i1}, a_{i2}, \ldots, a_{in_i}\}$ and $\deg(c_i) = n_i$, $1 \le i \le k$. A banana tree $BT(n_1, n_2, \ldots, n_k)$ is a tree obtained by adding a new vertex a and joining it to $a_{11}, a_{21}, \ldots, a_{k1}$.

Definition 2.5. A labeling f of G where N = {0, 1} and the induced edge labeling $\bar{f}$ is given by $\bar{f}(u, v) = |f(u) - f(v)|$, $\bar{N} = \{0, 1\}$. We call such a labeling cordial if the following condition is satisfied $|v_f(1) - v_f(0)| \le 1$, $|e_f(1) - e_f(0)| \le 1$, where $v_f(i)$ and $e_f(i)$, i = {0, 1} is the number of vertices and edges of G respectively, with label i. A graph is cordial if it admits a cordial labeling.

Definition 2.6. Let G = (V, E) be a graph with p vertices and q edges. A Homo-Cordial Labeling of a graph G with vertex set V is a bijection from V to {0, 1} such that each uv is assigned the label 1 if f(u) = f(v) or 0 if f(u)≠ f(v) with the condition that $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$. The graph that admits a Homo-Cordial Labeling is called Homo-Cordial Graph.

3 MAIN RESULTS

Theorem 3.1. Switching of Cycle C_n is Homo-Cordial Graph.

Proof:

Let G be the switching of cycle C_n.

Let $V(G) = \{u_i : 1 \le i \le n\}$ and

$E(G) = \{u_iu_{i+1} : 2 \le i \le n - 1\} \cup \{u_1u_i : 3 \le i \le n - 2\}$.

Define $f : V(G) \to \{0, 1\}$.

The vertex labeling are,

$$f(u_i) = \begin{cases} 1 & i \equiv 0,1 \bmod 4 \\ 0 & i \equiv 2,3 \bmod 4 \end{cases} \qquad 1 \le i \le n$$

The induced edge labeling are,

$$f^*[(u_iu_{i+1})] = \begin{cases} 1 & i \equiv 0 \bmod 2 \\ 0 & i \equiv 1 \bmod 2 \end{cases} \qquad 2 \le i \le n-2$$

$$f^*[(u_1u_i)] = \begin{cases} 1 & i \equiv 0,1 \bmod 4 \\ 0 & i \equiv 2,3 \bmod 4 \end{cases} \qquad 3 \le i \le n-1$$

Here, $v_f(1) = v_f(0) + 1$ for $n \equiv 1 \bmod 4$,
$v_f(0) = v_f(1) + 1$ for $n \equiv 3 \bmod 4$,
$v_f(0) = v_f(1)$ for $n \equiv 0,2 \bmod 4$,
$e_f(0) = e_f(1) + 1$ for $n \equiv 0 \bmod 4$ and
$e_f(1) = e_f(0) + 1$ for $n \equiv 1,2,3 \bmod 4$.

Therefore, the switching of cycle C_n satisfies the conditions $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence, the switching of cycle C_n is Homo-Cordial Graph.

Example 3.2. Consider the following switching of cycle graph C_9,

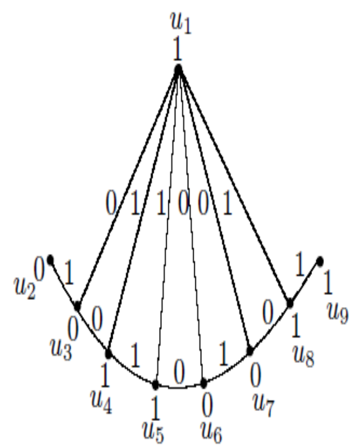

Figure 3.1

Here, $v_f(0) = 4$, $v_f(1) = 5$
$e_f(0) = 6$, $e_f(1) = 7$

Therefore, the switching of cycle graph C_9 satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence , the switching of cycle graph C_9 is Homo-Cordial Graph.

Theorem 3.3. The middle graph $M(P_n)$ is Homo-Cordial Graph.

Proof:

Let $V(M(P_n)) = \{u_i : 1 \leq i \leq n\} \cup \{v_i : 1 \leq i \leq n - 1\}$ and
$E(M(P_n)) = \{u_iu_{i+1} : 1 \leq i \leq n - 1\} \cup \{v_iv_{i+1} : 1 \leq i \leq n - 2\} \cup \{u_iv_i : 1 \leq i \leq n - 1\} \cup \{u_{i+1}v_i : 1 \leq i \leq n - 1\}$.

Define $f : V(M(P_n)) \rightarrow \{0, 1\}$.

The vertex labeling are,

$f(u_i) = 1 \quad 1 \leq i \leq n$
$f(v_i) = 0 \quad 1 \leq i \leq n - 1$

The induced edge labeling are,

$f^*[(u_iu_{i+1})] = 1 \quad 1 \leq i \leq n - 1$
$f^*[(v_iv_{i+1})] = 1 \quad 1 \leq i \leq n - 2$
$f^*[(u_iv_i)] = 0 \quad 1 \leq i \leq n - 1$
$f^*[(u_{i+1}v_i)] = 0 \quad 1 \leq i \leq n - 1$

Here, $v_f(1) = v_f(0) + 1$ for all n and
$e_f(0) = e_f(1) + 1$ for all n.

Therefore, the middle graph $M(P_n)$ satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, the middle graph $M(P_n)$ is Homo-Cordial Graph.

Example 3.4. Consider the following middle graph $M(P_5)$,

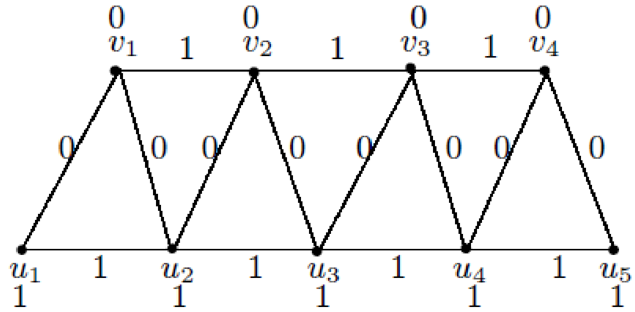

Figure 3.2

Here, $v_f(0) = 4$, $v_f(1) = 5$
$e_f(0) = 8$, $e_f(1) = 7$

Therefore, the middle graph $M(P_5)$ satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, the middle graph $M(P_5)$ is Homo-Cordial Graph.

Theorem 3.5. P_n^2 is Homo-Cordial Graph.

Proof:

Let $V(P_n^2) = \{u_i : 1 \leq i \leq n\}$ and
$E(P_n^2) = \{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iu_{i+2} : 1 \leq i \leq n-2\}$.
Define $f : V(P_n^2) \rightarrow \{0, 1\}$.

The vertex labeling are,

$$f(u_i)=\begin{cases} 1 & i \equiv 1 \bmod 2 \\ 0 & i \equiv 0 \bmod 2 \end{cases} \qquad 1 \leq i \leq n$$

The induced edge labeling are,

$f^*[(u_iu_{i+1})] = 0 \qquad 1 \leq i \leq n-1$
$f^*[(u_iu_{i+2})] = 1 \qquad 1 \leq i \leq n-2$

Here, $v_f(1) = v_f(0) + 1$ for $n \equiv 1 \bmod 2$,
$v_f(1) = v_f(0)$ for $n \equiv 0 \bmod 2$ and
$e_f(0) = e_f(1) + 1$ for all n.

Therefore, the graph P_n^2 satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, the graph P_n^2 is Homo-Cordial Graph.

Example 3.6. Consider the graph P_5^2 ,

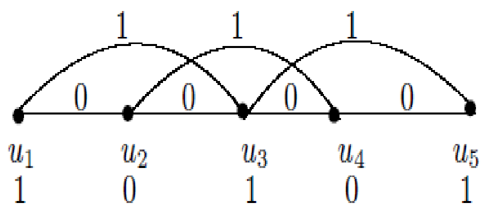

Figure 3.3

Here, $v_f(0) = 2$, $v_f(1) = 3$
$e_f(0) = 4$, $e_f(1) = 3$
Therefore, the graph P_5^2 satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.
Hence, the graph P_5^2 is Homo-Cordial Graph.

Theorem 3.7. Petersen graph is Homo-Cordial Graph.
Proof:
Let G be the petersen graph.
Let $V(G) = \{u_i, v_i : 1 \leq i \leq 5\}$ and
$E(G) = \{(u_iu_{i+2}) : 1 \leq i \leq 3\} \cup \{(u_{i+3}u_i) : 1 \leq i \leq 2\} \cup \{(u_iv_i) : 1 \leq i \leq 5\} \cup \{(v_iv_{i+1}) : 1 \leq i \leq 5\} \cup \{(v_1v_5)\}$.
Define $f : V(G) \rightarrow \{0, 1\}$.
The vertex labeling are,

$$f(u_i)=\begin{cases}1 & i \equiv 0 \bmod 2\\ 0 & i \equiv 1 \bmod 2\end{cases} \qquad 1\leq i \leq 5$$

$$f(v_i) =\begin{cases}1 & i \equiv 1,2 \bmod 4\\ 0 & i \equiv 0,3 \bmod 4\end{cases} \qquad 1\leq i \leq 5$$

The induced edge labeling are,

$f^*[(u_iu_{i+2})] = 1 \qquad 1 \leq i \leq 3$

$f^*[(u_{i+3}u_i)] = 0 \qquad 1 \leq i \leq 2$

$$f^*[(u_iv_i)] =\begin{cases}1 & i \equiv 2,3 \bmod 4\\ 0 & i \equiv 0,1 \bmod 4\end{cases} \qquad 1\leq i \leq 5$$

$$f^*[(v_iv_{i+1})] =\begin{cases}1 & i \equiv 1 \bmod 2\\ 0 & i \equiv 0 \bmod 2\end{cases} \qquad 1\leq i \leq 5$$

$f^*[(v1v5)] = 1$

Here, $v_f(1) = v_f(0)$ and $e_f(1) = e_f(0) + 1$.
Therefore, the petersen graph satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, the petersen graph is Homo-Cordial Graph.

Example 3.8. Consider the following petersen graph,

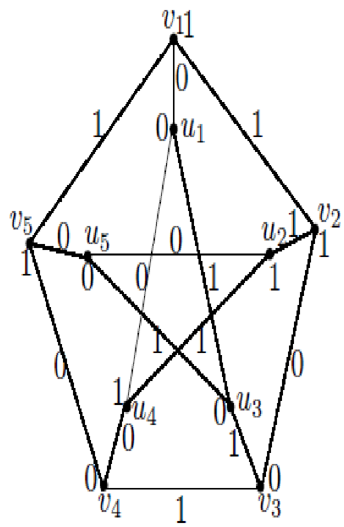

Figure 3.4

Here, $v_f(1) = 5$, $v_f(0) = 5$

$e_f(1) = 8$, $e_f(0) = 7$

Therefore, the petersen graph satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$

Hence, the petersen graph is Homo-Cordial Graph.

Theorem 3.9. A banana tree $BT(n_1, n_2, \ldots, n_k)$ of order n is Homo-Cordial Graph.

Proof:

Let G be the banana tree $BT(n_1, n_2, \ldots, n_k)$.

Let $V(G) = \{u, u_i, v_{ij} : 1 \leq i \leq k, 1 \leq j \leq n - 1\}$ and

$E(G) = \{(u_i v_{ij}) : 1 \leq i \leq k, 1 \leq j \leq n - 1\} \cup \{(uv_{i1}) : 1 \leq i \leq k\}$

Define $f : V(G) \rightarrow \{0, 1\}$.

case 1: When n is odd.

The vertex labeling are,

$f(u) = 0$

$$f(u_i) = \begin{cases} 1 & i \equiv 1 \bmod 2 \\ 0 & i \equiv 0 \bmod 2 \end{cases} \qquad 1 \leq i \leq k$$

$$f(v_{ij})=\begin{cases}1 & j\equiv 1 \bmod 2\\ 0 & j\equiv 0 \bmod 2\end{cases} \quad \text{for i is odd and } 1\le j\le n-1$$

$$f(v_{ij})=\begin{cases}1 & j\equiv 0 \bmod 2\\ 0 & j\equiv 1 \bmod 2\end{cases} \quad \text{for i is even and } 1\le j\le n-1$$

The induced edge labeling are,

$$f^*[(uv_{i1})]=\begin{cases}1 & i\equiv 0 \bmod 2\\ 0 & i\equiv 1 \bmod 2\end{cases} \quad 1\le i\le k$$

$$f^*[(u_iv_{ij})]=\begin{cases}1 & j\equiv 1 \bmod 2\\ 0 & j\equiv 0 \bmod 2\end{cases} \quad \text{for } 1\le i\le k \text{ and } 1\le j\le n-1$$

Here, $v_f(0) = v_f(1) + 1$ for $k\equiv 0 \bmod 2$,
$v_f(0) = v_f(1)$ for $k\equiv 1 \bmod 2$,
$e_f(0) = ef(1)$ for $k\equiv 0 \bmod 2$ and
$e_f(0) = e_f(1) + 1$ for $k\equiv 1 \bmod 2$.

case 2: When n is even.

The vertex labeling are,

$f(u) = 0$

$f(u_i) = 1 \quad 1 \le i \le k$

$$f(v_{ij})=\begin{cases}1 & j\equiv 0 \bmod 2\\ 0 & j\equiv 1 \bmod 2\end{cases} \quad \text{for } 1\le i\le k \text{ and } 1\le j\le n-1$$

The induced edge labeling are,

$f^*[(uv_{i1})]=1 \quad 1 \le i \le k$

$$f^*[(u_iv_{ij})]=\begin{cases}1 & j\equiv 0 \bmod 2\\ 0 & j\equiv 1 \bmod 2\end{cases} \quad \text{for } 1\le i\le k \text{ and } 1\le j\le n-1$$

Here, $v_f(0) = v_f(1) + 1$ for all k and

$e_f(0) = ef(1)$ for all k.

Therefore, the banana tree $BT(n_1, n_2, \ldots, n_k)$ satisfies the conditions

$|v_f(0)-v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$

Hence, the banana tree $BT(n_1, n_2, \ldots, n_k)$ is Homo-Cordial Graph.

Example 3.10. Consider the following the banana tree BT(4, 4, 4, 4),

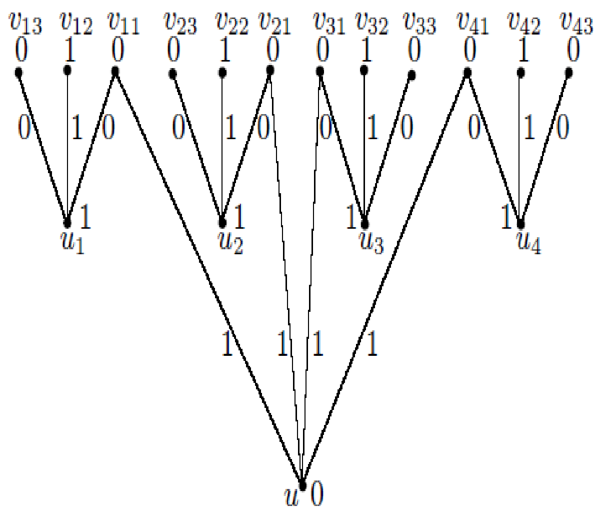

Figure 3.5

Here, $v_f(1) = 8$, $v_f(0) = 9$
$e_f(1) = 8$, $e_f(0) = 8$
Therefore, the banana tree BT(4, 4, 4, 4) satisfies the conditions $|v_f(0)-v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$

Hence, the banana tree BT(4, 4, 4, 4) is Homo-Cordial Graph.

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