ON ANTI FUZZY SUB KS-SEMIGROUPS

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ABSTRACT

In this paper, we fuzzify the new class of algebraic structures bring in by Kim[5]. In this fuzzification , we introduce the notions of anti fuzzy sub KS-semigroups, anti fuzzy KS-ideal, anti fuzzy KS-p-ideal and scrutinize some of their related properties. The idea of this study is to execute the fuzzy set theory and ideal theory in the KS-semigroups. This fuzzification guide to enlargement of new notions over anti fuzzy KS-semigroups.

Keywords: Anti fuzzy sub KS-semigroup, Anti fuzzy KS-ideal, Anti KS-p-ideal.

1 INTRODUCTION

The impression of BCK-semigroups was proposed by Imai and Iseki[2] as a generalization of the concept of set theoretic difference and propositional calculus. Since then a great deal of literature has been produced on the theory of BCK-algebras. Iseki [3] introduced the notion of BCI-algebras, which is a over simplification of BCK-algebras. For the general development of BCK/BCIalgebras, the ideal theory plays an imperative role. Zadeh [8] introduced the notion of fuzzy sets. After the introduction of fuzzy sets by Zadeh [8],almost all of mathematical structures were fuzzified Rosenfeld [7] introduced the notion of fuzzy group.

Recently, the new class of algebraic structures introduced by Kim [5], called KS-semigroups, which is the combination of BCK-algebras and semigroups. In this paper, we fuzzify the new class of algebraic structure introduced by Kim [5]. In this fuzzification , we bring in the notions of anti fuzzy sub KS-semigroups, anti fuzzy KS-ideal, anti fuzzy KS- p-ideal and examine some of their interrelated properties. Using these new views, we claim that some of the results of anti fuzzy KS-semigroups are very bunged related to the results of fuzzy BCK-algebras.

2 PRELIMINARIES

In this section we give the basic definitions and examples of KS-semigroup which are used in subsequal.

Definition 2.1.[5] A BCI-algebra is triple (X,∗, 0) where X is a non-empty set, ”*” is a binary operation on X, 0 ∈X is an element such that the following four axioms are satisfied for every x, y, z ∈X:
(I) ((x *y) *(x *z)) *(z *y) = 0
(II) (x *(x *y)) *y = 0
(III) x *x = 0
(IV) x *y = 0, y *x = 0 implies x = y.
A BCI-algebra X satisfying 0 *x = 0 for all x ∈X is called a BCK-algebra.

Remark 2.2.[5]A BCK-algebra X has the following properties for any
x, y, z ∈X
1. x *0 = x
2. (x *y) *z = (x *z) *y
3. x ≤ y implies that x *z ≤ y *z and z *y ≤ z *x
4. (x *z) *(y *z) ≤ x *y.

Definition 2.3.[5] An KS-semigroup X is a non-empty set X with two binary operations ”*” and ”•” and constant 0 satisfying the axioms:
(V) (X, *, 0) is a BCK-algebra.
(VI)(X, •) is a semigroup.
(VII)The operation ”•” is distributive (on both sides) over the operation ”*” that is,
x • (y *z) = (x • y) *(x • z) and
(x *y) • z = (x • z) *(y • z) for all x, y, z ∈X.
We shall write the multiplication x • y by xy for convenience.

Example 2.4. Let X = {0, a, b, c, d}. Define *-operation and multiplication ”•” by the following tables:

*	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	a	0
b	b	b	0	0	0
c	c	c	c	0	0
d	d	d	d	d	0

•	0	a	b	c	d
0	0	0	0	0	0
a	0	0	0	0	0
b	0	0	0	0	b
c	0	0	0	b	c
d	0	a	b	c	d

Then by routine calculations, we get X is an KS-semigroup.

Definition 2.5.[5] A non-empty subset A of a semigroup (X, •) is said to be left (resp.right) stable if xa ∈A(resp.ax ∈A) whenever x ∈X and a ∈A. Both left and right stable is two-sided stable or simply stable.

Definition 2.6.[5] A non empty subset A of a KS- semigroup X is called a left (resp.right) ideal of X if
(i) A is a left (resp . right) stable subset of (X, •)
(ii) for any x, y ∈X, x *y ∈A and y ∈A imply that x ∈A.

A both of left and right ideal is called a two - sided ideal or simply an ideal.

Definition 2.7. [5] A non-empty subset A of a KS-semigroup X is called a left(resp.right) P-ideal of X if
(i) A is a left (resp.right) stable subset of (X, •).
(ii) For any x, y, z 2 X, (x *y) *z ∈A and y *z ∈A imply that x *z ∈ A.

Definition 2.8.[4] Let X and Y be a KS - semigroup. A mapping f : X −→ Y of KS-semigroup is called a homomorphism if
f(x *y) = f(x) *f(y) and
f(xy) = f(x)f(y), for all x, y ∈X.

Note that if f : X −→ Y is a homomorphism of KS-semigroup then f(0) = 0.

Definition 2.9.[4] Let X and Y be a KS-semigroups and f : X −→ Y . Then f is an epimorphism if f is an onto homomorphism.

Definition 2.10.[6] A non-empty subset S of X with binary operations " *"and "•" is called sub KS-semigroup of X if it satiesfies the following conditions:
KSS1. x *y ∈S and,
KSS2. xy ∈S, for all x, y ∈X.

Example 2.11. Let X = {0, 1, 2} be a KS-semigroup with the following Cayley tables:

*	0	1	2
0	0	0	0
1	1	0	1
2	2	2	0

•	0	1	2
0	0	0	0
1	0	1	0
2	0	0	2

If S = {0, 1}, we get S is a sub KS-semigroup of X.

3 ANTI FUZZY SUB KS-SEMIGROUP

In this section we introduce the notion of anti fuzzy sub KS-semigroup and study their allied belongings.

Definition 3.1. Let X be a non-empty set. A fuzzy subset of X is a function
μ : X −→ [0, 1]. Let μ be the fuzzy subset of X. For a fixed 0 ≤ t ≤ 1, the set
μ_t= {x ∈X | μ(x) ≤ t} is called an lower level set of μ.

Definition 3.2.A fuzzy subset μ in X is called an anti fuzzy sub KS-semigroup of X if it satisfies the following conditions:
AFSKS1. μ(x *y) ≤ max{μ(x), μ(y)},
AFSKS2. μ(xy) ≤ max{μ(x), μ(y)}, for all x,y ∈X.

Example 3.3. Let X = {0, 1, 2, 3} be a KS-semigroup with the following cayley tables:

*	0	1	2	3

0	0	0	0	0
1	1	0	0	3
2	2	1	0	3
3	3	0	1	0

•	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	1	3
3	0	3	2	1

Define a fuzzy subset μ: X → [0, 1] by
μ(0) = 0.5, μ(x) = 0.8, for all x ≠ 0. Then by usual calculations, we get μis an anti fuzzy sub KS-semigroup of X.

Theorem 3.4. A fuzzy set μ of X is a anti fuzzy sub KS-semigroup if and only if for every 0 ≤ t ≤ 1, the lower level set $\mu_t$is either empty or a sub KS-semigroup of X.

Proof. Suppose that μ is a anti fuzzy sub KS-semigroup of X, and $\mu_t \neq 0$, then
for any x, y ∈μ_t, we have
μ(x *y) ≤ max{(μ(x), μ(y)} = t.
Thus, x *y ∈μ_t.
Also, μ(xy) ≤ max{(μ(x), μ(y)} = t.
Thus,xy ∈μ_t.

Hence $\mu_t$is a sub KS-semigroup of X.
Conversely, take t = max{μ(x), μ(y)},
for all x, y ∈X. Since μ_t (≠ 0) is a sub KS-semigroup of X, we have
μ(x *y) ≤ t = max{μ(x), μ(y)} and
μ(xy) ≤ t = max{μ(x), μ(y)}.
Hence μ is a anti fuzzy sub KS-semigroup.

Theorem 3.5. Any sub KS-semigroup of X can be realized as a level sub KS-semigroup of some anti fuzzy sub KS-semigroup.

Proof.Let A be a sub KS-semigroup of X and μ be a fuzzy subset in X defined by

$$\mu(x) = \begin{cases} t & if\ x \in A, \\ 0 & otherwise. \end{cases}$$

where 0 < t <1. It is clear that μ_t= A.
Case :1
If x, y ∈A, then μ(x) = μ(y) = t.
Thus μ(x *y) ≤ max{μ(x), μ(y)} = t and
μ(xy) ≤ max{μ(x), μ(y)} = t.
Hence x *y ∈$\mu_t$and xy ∈μ_t.
Case : 2
If x, y ∉A, then μ(x) = μ(y) = 0.
Thus μ(x *y) ≤ max{μ(x), μ(y)} = 0 and
μ(xy) ≤ max{μ(x), μ(y)} = 0.
Hence x *y ∈$\mu_t$and xy ∈μ_t.
Case : 3
If at most one of x, y ∈A, then at least one of μ(x) and μ(y) is equal to 0.
Thus μ(x *y) ≤ max{μ(x), μ(y)} = t and
μ(xy) ≤ max{μ(x), μ(y)} = t.
Hence x *y ∈$\mu_t$and xy ∈μ_t.

Theorem 3.6. Two level sub KS-semigroups $\mu_s$and $\mu_t$of an anti fuzzy sub KSsemigroups are equal if and only if there is no x ∈X such that s < μ(x) < t.

Proof.Let two level sub KS-semigroup $\mu_s$and μ_t, where
μ_s= {x 2∈X | μ(x) ≤ s} and

$\mu_t = \{x \in x \mid \mu(x) \leq t\}$.
Suppose they are equal that is $\mu_t = \mu_s$.
Assume that without loss of generality us consider s < t.
There exists no x ∈X such that $\mu_s(x) > s$ and $\mu_t(x) < t$.
Hence there is no x ∈X such that
s < μ(x) < t.
Conversely, suppose that there is no x ∈X such that s < μ(x) < t.
To prove $\mu_s = \mu_t$
$x \in \mu_t \Rightarrow \mu(x) \leq t \Rightarrow \mu(x) \leq s \Rightarrow x \in \mu_s$.
Therefore, $\mu_t \subseteq \mu_s$.
Now, $x \in \mu_s \Rightarrow \mu(x) \leq s \Rightarrow \mu(x) \leq t \Rightarrow x \in \mu_t$.
There fore $\mu_s \subseteq \mu_t$.
Hence $\mu_s = \mu t$.

4 ANTI FUZZY KS-IDEAL

In this section, we bring in the notions of anti fuzzy KS-ideal and anti fuzzy KS-p-ideal and prove some relations.

Definition 4.1. A fuzzy subset μ of X is called a left anti fuzzy KS-ideal if
AKSI1. μ(0) ≤ μ(x),
AKSI2. μ(x) ≤ max{μ(x *y), μ(y)},
AKSI3. μ(xa) ≤ max{μ(x), μ(a)}, for all x, y, a ∈X.
A fuzzy subset μ is called a right anti fuzzy KS-ideal if it satisfies KSI1, KSI2 and
AKSI4: μ(ax) ≤ max{μ(x), μ(a)}, for all x, y, a ∈X.
A fuzzy subset μ of X is called a anti fuzzy KS-ideal if it is both left and right fuzzy KS-ideal of X.

Example 4.2. Let X = {0, 1, 2, 3, 4} be a KS-semigroup defined by the following Cayley tables:

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	1	0
2	2	2	0	0	0
3	3	3	3	0	0
4	4	4	4	4	0

•	0	1	2	3	4
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	2
3	0	0	0	2	3
4	0	1	2	3	4

Define a fuzzy subset μ : X −→ [0.1] by μ(0) = 0.6 and μ(x) = 0.9 for all x ≠ 0. Then by usual calculations, we get μ is an left anti fuzzy KS-ideal of X.

Definition 4.3.A fuzzy subset μ of X is called a left anti fuzzy KS-p-ideal if
AKSP1. μ(0) ≤ μ(x),
AKSP2. μ(x *z) ≤ max{μ(x *y) * z), μ(y * z)},
AKSP3. μ(xa) ≤ max{μ(x), μ(a)} for all x, y, z, a ∈X.
A fuzzy subset μ is called a right anti fuzzy KS-p-ideal if it satisfies KSP1, KSP2 and
AKSP4: μ(ax) ≤ max{μ(x), μ(a)}, for all

x, y, a ∈X.
A fuzzy subset μ of X is called a anti fuzzy KS-p–ideal if it is both left and right anti fuzzy KS-p-ideal of X.

Example 4.4. Let X = {0, 1, 2, 3} be a KS-semigroup with the following Cayley tables:

*	0	1	2	3
0	0	0	0	0
1	1	0	1	1
2	2	2	0	2
3	3	3	3	0

•	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	1	1
3	0	3	2	2

Define a fuzzy subset μ : X −→ [0, 1] by μ(0) = 0.4 and μ(x) = 0.7 for all x≠ 0. Then by usual calculations we get μ is a left anti fuzzy KS-p-ideal of X.

Theorem 4.5. Every left (resp.right) anti fuzzy KS-p-ideal of X is a left (resp.right) anti fuzzy KS-ideal of X.

Proof. Let μ be a left anti fuzzy KS- p- ideal of X.

Then μ satisfies the conditions of KSI1 and KSI3. We have
μ(x * z) ≤ max{μ((x * y) * z), μ(y * z)},
put z = 0, we get
μ(x * 0) ≤ max{μ((x * y) * 0), μ(y * 0)}.
Thus μ(x) ≤ max{μ(x * y), μ(y)}.
Hence μ is a left anti fuzzy KS-ideal of X.
However the converse of the theorem is not true.

Counter example:
Let X = {0, 1, 2} be a KS-semigroup with the Cayley tables:

*	0	1	2
0	0	0	0
1	2	0	0
2	2	1	0

•	0	1	2
0	0	0	0
1	0	1	0
2	0	0	2

Define a fuzzy subset μ : X −→ [0, 1] by μ(0) = 0.2 and μ(x) = 0.6 for all x ≠ 0. Then by usual calculations, we get μ is a left anti fuzzy KS-ideal of X.
But it is not left fuzzy KS-p-ideal of X, since μ(2*1) < max{μ(2*1)*1), μ(1*1)}.

Remark 4.6. Let μ be a left (resp.right) anti fuzzy KS-ideal of X. Then the non-empty level set μ_t is may not be KS-ideal of X.
Counter example: Let X = {0, 1, 2, 3} be a KS-semigroup defined by the following tables:

*	0	1	2	3
0	0	0	0	0
1	1	0	2	3
2	2	2	0	1
3	3	2	1	0

•	0	1	2	3
0	0	0	0	0
1	0	1	0	2
2	0	2	0	3
3	0	3	0	1

Define a fuzzy subset μ : X −→ [0, 1] by μ(0) = 0.3, μ(1) = 0.2, μ(2) = 0.4, μ(3) = 0.6 and t = 0.5. Then by usual calculations, we get μ is a left anti fuzzy KS-ideal of X. But the non-empty level set μ_t is not be KS-ideal of X.
Indeed ,$\mu_{t=0.5}$ = {0, 1, 2}, if 3 ∈ X and 1 ∈ μ_t, we have 31 = 3 ∉ μ_t.

Remark 4.7. Let μ be a left (resp. right) anti fuzzy KS-ideal of X. Then the non-empty level set μ_t is may be KS-ideal of X.

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	2	1	3
2	2	4	0	2	1
3	3	3	2	0	2
4	4	4	4	3	0

•	0	1	2	3	4
0	0	0	0	0	0
1	0	0	1	2	3
2	0	2	1	3	2
3	0	2	0	3	4
4	0	1	2	3	4

Define a fuzzy subset μ : X −→ [0, 1] by μ(0) = 0.2, μ(1) = 0.4, μ(2) = 0.5, μ(3) = 0.7, μ(4) = 0.8, t = 0.5. Then by usual calculations, we get μ is a left anti fuzzy KS-ideal of X.
Now, μ_t = {0, 1, 2} is also left KS-ideal of X.

Definition 4.8. Let λ and μ be the fuzzy subsets in a set X. The upper Cartesian product $\lambda \overline{x} \mu$: X × X −→ [0, 1] is defined by $(\lambda \overline{x} \mu)(x, y) = \max\{\lambda(x), \mu(y)\}$, for all x, y ∈ X.

Theorem 4.9. Let λ and μ be a left (resp. right) anti fuzzy KS-ideal of X. Then $\lambda \overline{x} \mu$ is also a left (resp. right) anti fuzzy KS-ideal of X.

Proof. For any (x, y) ∈ X × X, we have

(i) $(\lambda \overline{x} \mu)(0, 0) = \max\{\lambda(0), \mu(0)\}$
$\leq \max\{\lambda(x), \mu(x)\}$
$= (\lambda \overline{x} \mu)(x, y)$.

(ii) For any (x_1, x_2), $(y_1, y_2) \in X \times X$, we have
$(\lambda \overline{x} \mu)(x_1, x_2) = \max\{\lambda(x_1), \mu(x_2)\}$
$\leq \max\{\max\{\lambda(x_1 * y_2), \lambda(y_1),$
$\max\{\mu(x_2 * y_2), \mu(y2)\}\}$
$= \max\{\max\{\lambda((x_1 * y_1),$
$\mu(x_2 * y_2))\}, \max\{\lambda(y_1), \mu(y_2)\}\}$
$= \max\{(\lambda \overline{x} \mu)((x_1, x_2) * (y_1,y_2)),$
$(\lambda \overline{x} \mu)(y_1, y_2)\}$

(iii) For any x, a ∈ X then xa ∈ X and (x, y), $(a_1, a_2) \in X \times X$, we have
$(\lambda \overline{x} \mu)(x, y)(a_1, a_2) = (\lambda \overline{x} \mu)(xa_1, ya_2)$
$= \max\{\lambda(xa_1), \mu(ya_2)\}$
$\leq \max\{\max\{\lambda(x), \lambda(a_1)\},$
$\max\{\mu(y), \mu(a_2)\}\}$
$= \max\{\max\{\lambda(x), \mu(y)\},$
$\max\{\lambda(a_1), \mu(a_2)\}\}$
$= \max\{(\lambda \overline{x} \mu)(x, y),$
$(\lambda \overline{x} \mu)(a_1, a_2)\}$.

Theorem 4.10. Let λ and μ be fuzzy subsets of X such that $\lambda \overline{x} \mu$ is a left (resp.right) anti fuzzy KS-ideal of X × X. Then
(i) either $\lambda(0) \leq \lambda(x)$ or $\mu(0) \leq \mu(x)$, for all x ∈ X.
(ii) if $\lambda(0) \leq \lambda(x)$, then either $\mu(0) \leq \lambda(x)$or $\mu(0) \geq \mu(x)$, for all x ∈ X.
(iii) if $\mu(0) \leq \mu(x)$, then either $\lambda(0) \leq \lambda(x)$ or $\lambda(0) \leq \mu(x)$, for all x ∈ X.

Proof. (i)Suppose $\lambda(0) > \lambda(x)$ and $\mu(0) > \mu(x)$. Then $\max\{\lambda(0), \mu(0)\} > \max\{\lambda(x), \mu(x)\}$ So, $(\lambda \overline{x} \mu)(0) > (\lambda \overline{x} \mu)(x)$, which is a contradiction to fact that λ¯×μ is a left anti fuzzy KS-ideal of X × X. Hence (i) follows.
(ii) Assume that $\lambda(0) \leq \lambda(x)$. By the definition of anti fuzzy KS-ideal in KSI2, we have
$\mu(x) \leq \max\{\mu(x * y), \mu(y)\}$.
Then, $(\lambda \overline{x} \mu)(0) \leq \max\{(\lambda \overline{x} \mu)(0 * x),$
$(\lambda \overline{x} \mu)(x)\}$
$(\lambda \overline{x} \mu)(0) \leq \max\{(\lambda \overline{x} \mu)(0), (\lambda \overline{x} \mu)(x)\}$
$\max\{\lambda(0), \mu(0)\} \leq \max\{\max\{\lambda(0), \mu(0)\},$
$\max\{\lambda(x), \mu(x)\}\}$
Hence either $\mu(0) \leq \lambda(x)$ or $\mu(0) \leq \mu(x)$.
(iii) Put μ instead of λ in (ii). Hence give (iii).

Theorem 4.11. Let λ and μ be a fuzzy subsets of X. If $\lambda \overline{x} \mu$ is a left (resp. right) anti fuzzy KS-ideal of X ×X then either λ or μ is a left (resp. right) anti fuzzy KS-ideal of X.

Proof. By theorem 4.9 of (i), without lose of generality, we assume that
$\mu(0) \leq \mu(x)$, for all x ∈ X.
By Theorem 4.9 of (iii) that either $\lambda(0) \leq \lambda(x)$ or $\lambda(0) \leq \mu(x)$, for all x ∈ X.
If $\lambda(0) \leq \mu(x)$,
then $(\lambda \overline{x} \mu)(0, x) = \max\{\lambda(0), \mu(x)\}$
$= \mu(x)$...........(A)
Since $\lambda \overline{x} \mu$ is a left anti fuzzy KS-ideal of X×X, therefore for all (x_1, x_2), (y_1, y_2) and $(a_1, a_2) \in X \times X$, then
$(\lambda \overline{x} \mu)((x_1, x_2)) \leq \max\{(\lambda \overline{x} \mu)((x_1, x_2) *$
$(y_1,y_2)), (\lambda \overline{x} \mu((y_1, y_2))\}$
and
$((\lambda \overline{x} \mu))((x_1, x_2)(a_1, a_2) \leq \max\{(\lambda \overline{x} \mu)((x_1,$
$x_2)), (\lambda \overline{x} \mu)(a_1, a_2)\}$.
$(\lambda \overline{x} \mu)((x_1, x_2)) \leq \max\{(\lambda \overline{x} \mu)((x_1 * y_1), (x_2 *$
$y_2)), (\lambda \overline{x} \mu)((y_1, y_2))\}$ and
$(\lambda \overline{x} \mu)((x_1, x_2)(a_1, a_2)) \leq \max\{(\lambda \overline{x} \mu)((x_1, x_2)),$
$(\lambda \overline{x} \mu)((a_1, a_2))\}$.
If $x_1 = y_1 = a_1 = 0$
$(\lambda \overline{x} \mu)((0, x_2)) \leq \max\{(\lambda \overline{x} \mu)((0 * 0), (x_2*y_2)),$
$(\lambda \overline{x} \mu)(0, y_2)\}$
and
$((\lambda \overline{x} \mu)((0, x_2)(0, a_2)) \leq \max\{(\lambda \overline{x} \mu)((0, x_2)),$
$(\lambda \overline{x} \mu)(0, a_2)\}$.
Using (A) we get
$\mu(x_2) \leq \max\{\mu(x_2 * y_2), \mu(y_2)\}$ and
$\mu(x_2a_2) \leq \max\{\mu(x_2 * a_2), \mu(a_2)\}$.
This complete the proof.

Definition 4.12. Let A be a fuzzy subset in S, the weakest fuzzy relation on S, that is fuzzy relation on A is μ_A given by
$\mu_A(x, y) = \max\{A(x),A(y)\}$.

Theorem 4.13. Let A be a fuzzy subset in X and μ_A be a weakest fuzzy relation on X and xx = x for all $x \in X$. Then A is a left (resp.right) anti fuzzy KS-ideal of X if and only if μ_A is a left (resp. right) anti fuzzy KS-ideal of X × X.

Proof. Suppose that A is a left anti fuzzy KS-ideal of X.
(i) For all $x, y \in X$, we have
$\mu_A(0, 0) = \max\{A(0),A(0)\}$
$\geq \max\{A(x),A(y)\}$
$= \mu_A(x, y)$.
(ii)For all (x_1, x_2) and (y_1, y_2) in X × X, then we have
$\mu_A(x_1, x_2) = \max\{A(x_1),A(x_2)\}$
$\leq \max\{\max\{A(x_1 * y_1),A(y1)\}, \max\{A((x_2 * y_2)),A(y_2)\}\}$
$= \max\{\max\{(A(x_1 * y_1),A(x_2\ \ y2))\}, \max\{A(y_1),A(y_2)\}\}$
$= \max\{\mu_A((x_1 * y_1), (x_2 * y_2)), \mu_A(y_1, y_2)\}$
$= \max\{\mu_A((x_1, x_2) * (y_1, y_2)), \mu_A(y_1, y_2)\}$
(iii) For all (x_1, x_2) and (a_1, a_2) in X × X, then we have
$\mu_A((x_1, x_2)(a_1,a_2)) = \mu_A((x_1\ a_1), (x_2\ a_2))$
$= \max\{A(x_1\ a_1),A(x_2\ a_2)\}$
$\leq \max\{\max\{A(x_1),A(a_1)\}, \max\{A(x_2),A(a_2)\}\}$
$= \max\{\max\{A(x_1),A(x_2)\}, \max\{A(a_1),A(a_2)\}\}$
$=\max\{\mu_A(x_1, x_2), \mu_A(a_1, a_2)\}$
Hence μA is a left anti fuzzy KS-ideal of X × X.
Conversely, μ_A is a left anti fuzzy KS-ideal of X × X. Then for all $(x, y) \in X$, we have
(i) $\max\{A(0),A(0)\} = \mu_A(0, 0)$
$\leq \mu_A(x, y)$
$= \max\{A(x),A(y)\}$
It follows that $A(0) \leq A(x)$.
(ii)For all (x_1, x_2) and (y_1, y_2) in X × X
$\max\{A(x1),A(x2)\} = \mu_A(x1, x2)$
$\leq \max\{\mu_A((x_1, x_2) * (y_1, y_2)), \mu_A(y_1, y_2)\}$
$= \max\{\mu_A((x_1 * y_1), (x_2*y_2)), \mu_A(y_1, y_2)\}$
$= \max\{\max\{A(x_1 * y_1),A(x_2* y_2)\},\max\{A(y_1),A(y_2)\}\}$
$= \max\{\max\{A(x_1*y_1),A(y_1)\}, \max\{A(x_2 * y_2),A(y_2)\}\}$
Put $x_2 = y_2 = 0$, then
$A(x_1) = \max\{A(x_1 * y_1),A(y_1)\}$.
(iii)For all (x_1, x_2) and (a_1, a_2) in X × X, then we have
$\max\{A(x_1\ a_1),A(x_2\ a_2)\} = \mu_A((x_1\ a_1), (x_2\ a_2))$
$= \mu_A((x_1, x_2)(a_1, a_2))$
$\leq\max\{\max\{A(x_1),A(x_2)\}, \max\{A(a_1),A(a_2)\}\}$
$= \max\{\max A(x_1),A(a_1)\}, \max\{A(x_2),A(a_2)\}\}$
Put $x_2 = a_2 = 0$, then
$A(x_1\ a_1) \leq \max\{A(x_1),A(a_1)\}$
Hence A is a left anti fuzzy KS-ideal of X.

5 Some Results On Homomorphism Of KS-semigroup

In this section, we scrutinize some effects on homomorphism of KS-semigroup.

Definition 5.1. Let f : X −→ Y be a mapping of KS-semigroup and μ be a fuzzy subset of Y . The map μ^f is the pre-image of μ under f if $\mu^f = \mu(f(x))$, for all $x \in X$.

Theorem 5.2. Let f : X −→ Y be a homomorphism. If μ is a left (resp.right) anti fuzzy KS-ideal of Y then μ^f is a left (resp.right) anti fuzzy KS-ideal of X.

Proof. (i)For any $x \in X$, then
$\mu^f(x) = \mu(f(x)) \geq \mu(0) = \mu(f(0)) = \mu^f(0)$.
(ii) For any $x, y \in X$, then
$\mu^f(x) = \mu(f(x)) \leq \max\{\mu(f(x) * f(y), \mu(f(y))\}$
$= \max\{\mu(f(x * y)\mu(f(y))\}$
$= \max\{\mu f(x * y)\mu f(y)\}$
(iii) For any $x, a \in X$, then
$\mu^f(xa) = \mu(f(xa))$
$= \mu(f(x)f(a))$
$\leq \max\{\mu(f(x))\mu(f(a))\}$
$= \max\{\mu^f(x)\mu^f(a)\}$.
Hence μ^f is a left anti fuzzy KS-ideal of X.

Theorem 5.3. Let $f : X \longrightarrow Y$ be an epimorphism. If μf is a left (resp.right) anti fuzzy KS-ideal of X then μ is a left (resp.right) anti fuzzy KS-ideal of Y .

Proof. (i) Let $y \in Y$, then there exists $x \in X$ such that $f(x) = y$. Then
$\mu(y) = \mu(f(x)) = \mu^f(x) \geq \mu^f(0) = \mu(f(0)) = \mu(0)$.
(ii) Let $x, y \in Y$, then there exists $a, b \in X$ such that $f(a) = x$, $f(b) = y$.
Then $\mu(x) = \mu(f(a)) = \mu^f(a)$
$\leq \max\{\mu^f(a * b), \mu^f(b)\}$
$= \max\{\mu(f(a * b), \mu(f(b))\}$
$= \max\{\mu(f(a) * f(b)), \mu(f(b))\}$
$= \max\{\mu(x * y), \mu(y)\}$
(iii) Let $x, a \in Y$, then there exists $l,m \in X$ such that $f(l) = x$, $f(m) = a$.
Then $\mu(xa) = \mu(f(l)f(m))$
$= \mu^f(lm)$
$\leq \max\{\mu^f(l)\mu^f(m)\}$
$= \max\{\mu(f(l)), \mu(f(m))\}$
$= \max\{\mu(x), \mu(a)\}$.
Hence μ is a left anti fuzzy KS-ideal of Y .

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