On Multi Granular Nano Topology using Three Equivalence Relation

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Abstract— **In this paper, we introduce new type of multi granular nano topology using three equivalence relation. There is an endeavour to study its properties based on their approximations. Here, we define and study the multi granular nano forms of open sets and closed sets namely multi granular nano α-open (closed) set, multi granular nano semi-open (closed) set, multi granular nano pre-open (closed) set, multi granular nano regular-open (closed) set.**

Index Terms— **Multi granular nano topology using three equivalence relation, Multi-lower approximation, Multi-upper approximation, Multi-boundary, Multi granular nano forms of open sets, Multi granular nano forms of closed sets.**

INTRODUCTION

Lellis Thivagar et al [2] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower and upper approximations of X. The elements of a nano topological space are called the nano-open sets. He has also studied nano closure and nano interior of a set.

In view of granular computing [1], [3] nano topological space is based on a single granulation (only one indiscreniblity relation). But this concept of Multi granular nano topological model[5], where the set approximations are defined by Multiple indiscreniblity relations on the universe.

In this paper, we establish a new nano topology called Multi granular nano topology using three equivalence relations on U in terms of the lower and upper approximations of a set and its boundary region and explore some results using multi granular nano forms of open sets and closed sets.

Preliminaries

Definition 2.1. Let U be the universe. Let R be an equivalence relations on U named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. Let $X \subseteq U$.

(i)The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \cup_{x \in U} \{[x] : R(x) \subseteq X\}$ where R(x) denote the equivalence class determined by x.

(ii)The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \cup_{x \in U} \{[x] : R(x) \cap X \neq \emptyset\}$, where R(x) denote the equivalence class determined by x.

(iii)The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is,
$B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then, by the properties of approximation spaces, $\tau_R(X)$ satisfies the following axioms:

(i)U and $\emptyset \in \tau_R(X)$.

(ii)The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii)The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano-open sets. The elements of $[\tau_R(X)]^c$ are called as nano-closed sets.

Definition 2.3. If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and

(i)if $A \subseteq U$, then the nano interior of A is defined as the union of all nano-open subsets of A and it is

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denoted by N Int(A). That is, N Int(A) is the largest nano-open subset of A.

(ii)The nano closure of A is defined as the intersection of all nano-closed sets containing A and it is denoted by N Cl(A). That is, N Cl(X) is the smallest nano-closed set containing A.

MULTI GRANULAR NANO TOPOLOGY USING THREE EQUIVALENCE RELATION

Definition 3.1. Let U be the universe. P , Q and R be any three equivalence relations on U named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. Let $X \subseteq U$.

(i)The Multi-lower approximation of X with respect to P , Q and R is the set of all objects which can be for certain classified as X with respect to P or Q or R and it is denoted by $L_{P+Q+R}(X)$. That is,

$L_{P+Q+R}(X) = U_{x \in U} \{[x] : P(x) \subseteq X \text{ or } Q(x) \subseteq X \text{ or } R(x) \subseteq X\}$, where P (x), Q(x) and R(x) denote the equivalence class determined by x.

(ii)The Multi-upper approximation of X with respect to P , Q and R is the set of all objects which can be possibly classified as X with respect to P and Q and R and it is denoted by $U_{P+Q+R}(X)$. That is,

$U_{P+Q+R}(X) = U_{x \in U} \{[x] : P(x) \cap X \neq \emptyset \text{ and } Q(x) \cap X \neq \emptyset \text{ and } R(x) \cap X \neq \emptyset\}$, where P (x), Q(x) and R(x) denote the equivalence class determined by x.

(iii)The Multi-boundary region of X with respect to P , Q and R is the set of all objects which can be classified neither as X nor as not X with respect to P and Q and R and it is denoted by $B_{P+Q+R}(X)$. That is,

$B_{P+Q+R}(X) = U_{P+Q+R}(X) - L_{P+Q+R}(X)$.

Definition 3.2. Let U be the universe and P , Q and R any three equivalence relations on U and $\tau_{P+Q+R}(X) = \{U, \emptyset, L_{P+Q+R}(X), U_{P+Q+R}(X), B_{P+Q+R}(X)\}$, where $X \subseteq U$. Then $\tau_{P+Q+R}(X)$ satisfies the following axioms:

(i)U and $\emptyset \in \tau_{P+Q+R}(X)$.

(ii)The union of elements of any subcollection of $\tau_{P+Q+R}(X)$ is in $\tau_{P+Q+R}(X)$.

(iii)The intersection of the elements of any finite subcollection of $\tau_{P+Q+R}(X)$ is in $\tau_{P+Q+R}(X)$.

That is, $\tau_{P+Q+R}(X)$ forms a topology on U called as the Multi granular nano topology on U with respect to X. We call $(U, \tau_{P+Q+R}(X))$ as the Multi granular nano topological space.

Example 3.3. Let U = {a, b, c, d, e}, U/P = {{a}, {b, c}, {d, e}}, U/Q = {{a, b, c}, {d, e}} and U/R = {{a}, {b, c, d}, {e}} be three equivalence relations on U and let $X = \{a,b\} \subseteq U$. Now, $L_{P+Q+R}(X)= \{a\}$, $U_{P+Q+R}(X)= \{a, b, c, d\}$, and $B_{P+Q+R}(X)= \{b,c,d\}$. Hence, the multi granular nano topology, $\tau_{P+Q+R}(X) = \{U, \emptyset, \{a\}, \{a, b, c, d\}, \{b, c, d\}\}$.

Theorem 3.4. Let (U, R) be the approximation space U/P , U/Q, U/R $\in$ R be three equivalence relations defined on U respectively. Then for $X \subseteq U$, the following properties hold:

(i) $L_{P+Q+R}(X) \subseteq X$
(ii) $U_{P+Q+R}(X) \supseteq X$,
(iii)$L_{P+Q+R}(X^c) = [U_{P+Q+R}(X)]^c$,
(iv)$U_{P+Q+R}(X^c) = [L_{P+Q+R}(X)]^c$,
(v)$L_{P+Q+R}(\emptyset) = \emptyset$,
(vi)$U_{P+Q+R}(\emptyset) = \emptyset$,
(vii)$L_{P+Q+R}(U) = U$,
(viii)$U_{P+Q+R}(U) = U$,
(ix)$L_{P+Q+R}(X) = L_{P+R+Q}(X) = L_{Q+P+R}(X) = L_{Q+R+P}(X) = L_{R+P+Q}(X) = L_{R+Q+P}(X)$.
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Theorem 3.5. Let (U, R) be the approximation space, U/P, U/Q, U/R $\in$ R be two equivalence relations defined on U respectively, and let $X \subseteq U$. Then, the following properties hold:

(i) $L_{P+Q+R}(L_{P+Q+R}(X)) = L_{P+Q+R}(X)$,
(ii) $U_{P+Q+R}(U_{P+Q+R}(X)) = U_{P+Q+R}(X)$,
(iii) $L_{P+Q+R}(X) = L_P(X) \cup L_Q(X) \cup L_R(X)$,
(iv)$U_{P+Q+R}(X) = U_P(X) \cap U_Q(X) \cap U_R(X)$.

Remark 3.6. The elements of $\tau_{P+Q+R}(X)$ are called as multi granular nano-open sets. The elements of $[\tau_{P+Q+R}(X)]^c$ are called as multi granular nano-closed sets.

Definition 3.7. If $(U, \tau_{P+Q+R}(X))$ is a multi granular nano topological space with respect to X where $X \subseteq U$ and

(i) if $A \subseteq U$, then the multi granular nano interior of A is defined as the union of all multi granular nano-open subsets of A and it is denoted by M GN Int(A). That is, MGNInt(A) is the largest multi granular nano-open subset of A.

(ii)The multi granular nano closure of A is defined as the intersection of all multi granular nano-closed sets containing A and it is denoted by M GN Cl(A). That is, M GN Cl(X) is the smallest multi granular nano-closed set containing A.

MULTI GRANULAR NANO FORMS OF OPEN SETS

Definition 4.1. Let $(U, \tau_{P+Q+R}(X))$ is a multi granular nano topological space and $A \subseteq U$. Then, A is said to be (i)multi granular nano semi-open if
$A \subseteq$ MGNCl(MGNInt(A)),
(ii)multi granular nano pre-open if
$A \subseteq$ MGNInt(MGNCl(A)),

(iii)multi granular nano α-open if
A ⊆ MGNInt(MGNCl(MGNInt(A))).

MGNSO(U,X), MGNPO(U,X), $\tau^{\alpha}_{P+Q+R}(X)$ respectively denote the families of all multi granular nano semi-open, multi granular nano pre-open and multi granular nano α-open subsets of U.

Example 4.2. Let U = {a,b,c,d,e}, U/P ={{a}, {b,c}, {d,e}}, U/Q = {{a,b,c}, {d,e}}, U/R = {{a}, {b,c,d}, {e}} be three equivalence relation on U and let X = {a,b} ⊆ U.Hence, the Multi granular nano topology

$\tau_{P+Q+R}(X)$ = {U, Ø, {a}, {a,b,c,d}, {b,c,d}},
$[\tau_{P+Q+R}(X)]^c$ = {U, Ø, {b,c,d,e}, {e}, {a,e}}.

Then, MGNSO(U,X) = {U, Ø, {a}, {a,e}, {b,c,d}, {a,b,c,d}, {b,c,d,e}}, MGNPO(U,X) = {U, Ø, {a}, {b}, {c},{d}, {b,c}, {b,d}, {c,d}, {b,c,d}, {a,b,c,d}}, $\tau^{\alpha}_{P+Q+R}(X)$ = {U, Ø, {a}, {b,c,d}, {a,b,c,d}}.

Remark 4.3. Classes of nano α-open, nano semi-open and nano pre-open sets are independent with respect to various nano topologies. This is shown by the following examples.

Example 4.4. Let U = {a,b,c,d,e}, U/P ={{a}, {b,c}, {d,e}}, U/Q = {{a,b,c}, {d,e}}, U/R = {{a}, {b,c,d}, {e}} be three equivalence relation on U and let X = {a,b} ⊆ U.

Nano forms of open sets for the equivalence relation P on U: $L_P(X)$ = {a}, $U_P(X)$ = {a,b,c}, $B_P(X)$ = {b,c}.
Then, $\tau_P(X)$ = {U, Ø, {a}, {a,b,c}, {b,c}} and $[\tau_P(X)]^c$ = {U, Ø, {b,c,d,e}, {d,e}, {a,d,e}}. Nano semi-open sets are U, Ø, {a}, {a,d}, {a,e}, {b,c},{a,b,c}, {a,d,e}, {b,c,d}, {b,c,e}, {a,b,c,d}, {a,b,c,e}, {b,c,d,e}. Nano pre-open sets are U, Ø, {a}, {b}, {c}, {b,c}. Nano α-open sets are U, Ø, {a}, {a,b,c}, {b,c}.
Nano forms of open sets for the equivalence relation Q on U: $L_Q(X)$ = {a}, $U_Q(X)$ = {a,b,d}, $B_Q(X)$ = {b,d}.
Then, $\tau_Q(X)$ = {U, Ø, {a}, {a,b,d}, {b,d}} and $[\tau_Q(X)]^c$ = {U, Ø, {b,c,d,e}, {c,e}, {a,c,e}}. Nano semi-open sets are U, Ø, {a}, {a,c}, {a,e}, {b,d},{a,b,d}, {a,c,e}, {b,c,d}, {b,d,e}, {a,b,c,d}, {a,b,d,e}, {b,c,d,e}. Nano pre-open sets are U, Ø, {a}, {b}, {d}, {b,d}. Nano α-open sets are U, Ø, {a}, {b,d}.
Nano forms of open sets for the equivalence relation R on U: $L_R(X)$ = {a}, $U_R(X)$ = {a,b,c,d}, $B_R(X)$ = {b,c,d}.
Then, $\tau_R(X)$ = {U, Ø, {a}, {a,b,c,d}, {b,c,d}} and $[\tau_R(X)]^c$ = {U, Ø, {b,c,d,e}, {e}, {a,e}}. Nano semi-open sets are U, Ø, {a}, {a,e}, {b,c,d}, {a,b,c,d}, {b,c,d,e}. Nano pre-open sets are U, Ø, {a}, {b}, {c},{d}, {b,c}, {b,d}, {c,d}, {b,c,d}, {a,b,c,d}. Nano α-open sets are U, Ø, {a}, {b,c,d}, {a,b,c,d}.

Theorem 4.5. If A is multi granular nano-open in (U, $\tau_{P+Q+R}(X)$), then it is multi granular nano α-open in U.

Proof. Since A is multi granular nano-open in U, MGNIntA = A. Then,
MGNCl(MGNInt(A)) = MGNCl(A) ⊆ A.
That is, A ⊆ MGNCl(MGNIntA). This implies that,
MGNInt(A) ⊆ MGNInt(MGNCl(MGNInt(A))). That is, A ⊆ MGNInt(MGNCl(MGNInt(A))).
Thus, A is multi granular nano α-open.

Theorem 4.6. $\tau^{\alpha}_{P+Q+R}(X)$ ⊆ MGNSO(U,X) in a multi granular nano topological space (U, $\tau_{P+Q+R}(X)$).
Proof. If A ϵ $\tau^{\alpha}_{P+Q+R}(X)$,
A⊆ MGNInt(MGNCl(MGNInt(A)))
⊆ MGNCl(MGNInt(A)) and hence AϵMGNSO(U,X).

Remark 4.7. The converse of the above theorem is not true. In example (4.2), {a, e}, {b, c, d, e} are multi granular nano semi-open but are not in multi granular nano α-open in U.

Theorem 4.8. $\tau^{\alpha}_{P+Q+R}(X)$ ⊆ MGNPO(U,X) in a multi granular nano topological space (U, $\tau_{P+Q+R}(X)$).
Proof. If A ϵ $\tau^{\alpha}_{P+Q+R}(X)$,
A ⊆ MGNInt(MGNCl(MGNInt(A))).
Since, MGNInt(A) ⊆ A,
MGNInt(MGNCl(MGNInt(A)))
⊆ MGNInt(MGNCl(A)).
That is, A ⊆ MGNInt(MGNCl(A)). This implies that,
A ϵ MGNPO(U,X).
Thus, $\tau^{\alpha}_{P+Q+R}(X)$ ⊆ MGNPO(U,X).

Remark 4.9. The converse of the above theorem is not true. In example (4.2), {b}, {c}, {d}, {b, c}, {b,d} and {c d} are multi granular nano pre-open but are not in multi granular nano α-open in U.

Theorem 4.10.
$\tau^{\alpha}_{P+Q+R}(X)$=MGNSO(U,X)∩MGNPO(U,X).
Proof. If A ϵ $\tau^{\alpha}_{P+Q+R}(X)$, then A ϵ MGNSO(U,X) and A ϵ MGNPO(U,X) by theorems 4.4 and 4.6 and hence A ϵ MGNSO(U,X) ∩ MGNPO(U,X).
That is, $\tau^{\alpha}_{P+Q+R}(X)$ ⊆ MGNSO(U,X) ∩ MGNPO(U,X).
Conversely,
if A ϵ MGNSO(U,X) ∩ MGNPO(U,X), then
A ⊆ MGNCl(MGNInt(A)) and
A⊆ MGNInt(MGNCl(A)).
Therefore,
MGNInt(MGNCl(A))
⊆ MGNInt(MGNCl(MGNCl(MGNInt(A))))
= MGNInt(MGNCl(MGNInt(A))).

That is, MGNInt(MGNCl(A))
⊆ MGNInt(MGNCl(MGNInt(A))).
Also, A ⊆ MGNInt(MGNCl(A))
⊆ MGNInt(MGNCl(MGNInt(A))) implies that
A ⊆ MGNInt(MGNCl(MGNInt(A))).
That is, A ϵ $\tau^{\alpha}_{P+Q+R}(X)$
Thus, MGNSO(U,X) ∩ MGNPO(U,X) ⊆ $\tau^{\alpha}_{P+Q+R}(X)$
Hence, $\tau^{\alpha}_{P+Q+R}(X)$ = MGNSO(U,X) ∩MGNPO(U,X).

Theorem 4.11. If A and B are multi granular nano semi open in U, then A U B is also multi granular nano semi open in U.
Proof. If A and B are multi granular nano semi-open in U, then A ⊆ MGNCl(MGNInt(A)) and
B ⊆ MGNCl(MGNInt(B)).
Consider
AUB
⊆ MGNCl(MGNInt(A))UMGNCl(MGNInt(B))
= MGNCl(MGNInt(A) U MGNInt(B))
⊆ MGNCl(MGNInt(A U B))
and hence A U B is multi granular nano semi-open.

Remark 4.12. If A and B are multi granular nano semi open in U, then A ∩ B is not multi granular nano semi open in U.

Definition 4.13. A subset A of a multi granular nano topological space (U, $\tau_{P+Q+R}(X)$) is multi granular nano regular open in U, if MGNInt(MGNCl(A)) = A.

Example 4.14. Let U = {x, y, z}, U/P = {{x}, {y}, {z}}, U/Q = {{x}, {y,z}} and U/R = {{x}, {y, z}} be three equivalence relation on U. Let X = {x, z}. Then, the multi granular nano topology on U with respect to X is given by $\tau_{P+Q+R}(X)$ = {U, Ø, {y}, {x, z}}. The multi granular nano-closed sets are U, Ø, {x, z}, {y}. Also, MGNInt(MGNCl(A)) = A for A = U, Ø, {y} and {x,z} and hence these sets are multi granular nano-regular open in U.

Theorem 4.15. Any multi granular nano regular open set is multi granular nano-open.
Proof. If A is multi granular nano regular open in (U,$\tau_{P+Q+R}(X)$), A = MGNInt(MGNCl(A)).
Then, MGNInt(A) = MGNInt(MGNInt(MGNCl(A)))
= MGNInt(MGNCl(A)) = A.
That is, A multi granular nano-open in U.

Remark 4.16. The converse of the above theorem is not true. For example, let U = {a, b, c, d} with U/P = {{a}, {b}, {c}, {d}}, U/Q = {{c}, {d}, {a, b}} and U/R = {{a}, {b}, {c, d}} be any three equivalence relation on U. Let X = {a, c}. Then $\tau_{P+Q+R}(X)$ = {U, Ø, {d}, {a, c, d}, {a, c}} and the multi granular nano-closed sets are U, Ø, {a, b, c}, {b, d}, {b}. The multi granular nano regular open sets are U, Ø, {d} and {a, c}. Thus, {a, c, d} is multi granular nano-open but is not multi granular nano regular open. Also, we note that the multi granular nano regular open sets do not form a topology, since {d} U {a, c} = {a, c, d} is not multi granular nano regular open, even though {d} and {a, c} are multi granular nano regular.

MULTI GRANULAR NANO FORMS OF CLOSED SETS

Definition 5.1. Let (U, $\tau_{P+Q+R}(X)$) be a multi granular nano topological space and A ⊆ U. Then A is said to be
(i)Multi granular Nano semi-closed if MGNInt(MGNCl(A)) ⊆ A,
(ii)Multi granular Nano pre-closed if MGNCl(MGNInt(A)) ⊆ A,
(iii)Multi granular Nano α-closed if MGNCl(MGNInt(MGNCl(A))) ⊆ A.
MGNSC(U,X), MGNPC(U,X) and MGNα-CS respectively denote the families of all multi granular nano semi-closed, multi granular nano pre-closed and multi granular nano α-closed subsets of U.

Example 5.2. Let us consider U = {a, b, c, d, e}, U/P = {{a}, {b, c}, {d, e}}, U/Q = {{a, b, c}, {d, e}}, U/R = {{a}, {b, c, d}, {e}}. Let X = {a, b}. Then,
$\tau_{P+Q+R}(X)$ = {U, ∅, {a}, {a,b, c, d}, {b, c, d}},
$[\tau_{P+Q+R}(X)]^c$ = {Ø, U, {b, c, d, e}, {e}, {a, e}}.
Multi granular Nano semi-closed sets:
MGNSC(U,X) = {Ø , U, {a}, {a, b}, {b, c, d}, {a,b,c,d}, {b, c, d, e}}.
Multi granular Nano pre-closed sets:
MGNPC(U,X) = {Ø , U, {a, e}, {b, c, d, e}}.
Multi granular Nano α-closed sets:
MGNα-CS = {Ø , U, {a, e}, {b, c, d, e}}.

Theorem 5.3.
(i)Every multi granular nano-closed set is a multi granular nano semi-closed set,
(ii)Every multi granular nano-closed set is a multi granular nano pre-closed set,
(iii)Every multi granular nano-closed set is a multi granular nano α-closed set.
Proof. Let A be a closed set. Then, we have MGNCl(A) = A.
(i)We have to prove that MGNInt(MGNCl(A)) ⊆ A which implies that A is a multi granular nano semi-closed set.
MGNInt(MGNCl(A)) = MGNInt(A) ⊆ A. Hence, A is a multi granular nano semi-closed set.
(ii)We have to prove that MGNCl(MGNInt(A)) ⊆ A which implies that A is a multi granular nano pre-closed set.
MGNCl(MGNInt(A)) = MGNInt(A) ⊆ A.
Hence, A is a multi granular nano pre-closed set.

(iii)We have to prove that MGNCl(MGNInt(MGNCl(A))) ⊆ A which implies that A is a multi granular nano α-closed set.
MGNCl(MGNInt(MGNCl(A)))
= MGNCl(MGNInt(A)) = MGNInt(A) ⊆ A.
Hence, A is a multi granular nano α-closed set.

Remark 5.4. The converse of the above theorem (i) and (ii) are need not be true.

Theorem 5.5. MGNSC(U, X) ⊆ MGNα-CS(U, X) in a multi granular nano topological space (U, τ_{P+Q+R}(X)).
Proof. Let A ∈ MGNSC(U, X). This implies that MGNInt(MGNCl(A)) ⊆ A.
Then, MGNCl(MGNInt(MGNCl(A)))
⊆ MGNCl(A) = A, since MGNCl(A) = A.
Thus, A ∈ MGNα-CS.

Theorem 5.6. MGNPC(U, X) ⊆ MGNα-CS(U, X) in a multi granular nano topological space (U, τ_{P+Q+R}(X)).
Proof. Let A ϵ MGNPC(U,X). This implies that MGNCl(MGNInt(A)) ⊆ A. We know that MGNCl(A) = A. Then, MGNCl(MGNInt(MGNCl(A)))
= MGNCl(MGNInt(A))⊆ A. Thus, A ϵ MGNα-CS.

Theorem 5.7.
MGNα-CS= MGNSC(U,X)∩MGNPC(U,X).
Proof. If A ϵ MGNSC(U,X) and A ϵ MGNPC(U,X), then
A ϵ MGNα-CS, since by theorem (5.5) and (5.6).
This implies that A ϵ MGNSC(U,X)∩MGNPC(U,X).
That is, MGNSC(U,X)∩MGNPC(U,X) ⊆ MGNα-CS.
Conversely,
if A ϵ MGNα-CS, then
MGNCl(MGNInt(MGNCl(A))) ⊆ A.
This implies that MGNCl(MGNInt(A)) ⊆ A, since MGNCl(A) = A.
Thus, A ϵ MGNPC(U,X).
Also, MGNCl(MGNInt(MGNCl(A))) ⊆ A.
This implies that
MGNCl(MGNInt(MGNCl(A)))
= MGNInt(MGNCl(A)) ⊆A,
since MGNCl(A) = A. Thus, A ϵ MGNSC(U,X).
That is, A ϵ MGNSC(U,X)∩MGNPC(U,X).
Thus, MGNα-CS⊆MGNSC(U,X)∩MGNPC(U,X).
Hence, MGNα-CS = MGNSC(U,X)∩MGNPC(U,X).

Theorem 5.8. If A and B are multi granular nano semi closed in U, then A∩B is also multi granular nano semi closed in U.
Proof. If A and B are multi granular nano semi-closed in U, then MGNInt(MGNCl(A)) ⊆ A and MGNInt(MGNCl(B)) ⊆ B. Consider,
MGNInt(MGNCl(A∩B))
⊆MGNInt(MGNCl(A)∩MGNCl(B))
= MGNInt(MGNCl(A))∩MGNInt(MGNCl(B)
⊆A∩B.
Hence, A∩B is multi granular nano semi-closed.

Remark 5.9. If A and B are multi granular nano semi closed in U, then AUB is not multi granular nano semi closed in U.

Definition 5.10. Let (U, τ_{P+Q+R}(X)) be a multi granular nano topological space and A⊆ U. Then A is said to be multi granular nano regular closed, if MGNCl(MGNInt(A)) ⊆ A.

Example 5.11. Let U = {a, b, c, d} with U/P = {{a}, {b}, {c}, {d}}, U/Q = {{c}, {d}, {a, b}} and U/R = {{a}, {b},{c, d}} be any three equivalence relation on U. Let X={a,c}. Then, τ_{P+Q+R}(X) = {U, Ø, {d}, {a, c, d}, {a, c}} and the multi granular nano-closed sets are U, Ø, {a, b, c}, {b, d}, {b}. The multi granular nano regular closed sets are U, Ø, {b, d}, {a, b, c}, {a, b, d}, and {b, c, d}.

Theorem 5.12. Every multi granular nano closed set is a multi granular nano regular closed.
Proof. Let A be a multi granular nano closed set in X such that A ⊆U. Then, MGNCl(A) = A.
We have to prove that MGNCl(MGNInt(A)) ⊆ A.
Since A is multi granular nano closed in U, MGNCl(A)=A⇒MGNCl(MGNInt(A)) = MGNInt(A) ⊆ A.
Hence, every multi granular nano-closed set is multi granular nano regular closed set.

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