International Journal of Science, Engineering and Technology Research (IJSETR), Volume 6, Issue 6, May 2017, ISSN: 2278 – 7798

STATIC FLEXURAL ANALYSIS OF THICK BEAM BY HYPERBOLIC SHEAR DEFORMATION THEORY

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Abstract: **Since the revolution of different theories in the shear deformation theories, a Hyperbolic Shear Deformation Theory (HPSDT) taking into account transverse shear deformation effect has been evolved, which will be used for the static flexure analysis of thick isotropic beams. The theory assumes a parabolic variation of transverse shear stress across the thickness of the beams. The displacement field of the theory contains two variables, hyperbolic sine and cosine function will be used in the displacement field in terms of thickness coordinate to represent shear deformation. The transverse shear stress will be obtained directly from the use of constitutive relations, satisfying the shear stress-free boundary conditions at top and bottom of the beam. Governing differential equations and boundary conditions of the theory will be obtained using the principle of virtual work. General solutions of thick isotropic simply supported and cantilever beam subjected to various loading conditions will be analysed for axial displacement, transverse displacement, axial stress and transverse shear stress. The results of the present theory will be compared with those of other refined shear deformation theories of beam to verify the accuracy of the theory.**

Keywords:Thick beam, shear deformation, isotropic beam, transverse shear stress, static flexure, hyperbolic shear deformation theory, principle of virtual work.

1. INTRODUCTION

The shear flexible materials in aircraft, automotive, shipbuilding and other industries are extensively used worldwide and they have shown interest in the accurate prediction of structural behaviour of beams. Beam theories involve the reduction of a three dimensional problems of elasticity theory to a one-dimensional problem. Since the thickness dimension is much smaller than the longitudinal dimension, it is possible to approximate the distribution of the displacement, strain and stress components in the thickness dimension. The various methods of development of refined theories based on the reduction of the three dimensional problems of mechanics of elastic bodies are discussed by Gol denveizer [1], Kil chevskiy [2], Donnell [3], Vlasov and Leontev [4], Sayir and Mitropoulos [5].

It is well-known that elementary theory of bending of beam based on Euler-Bernoulli theorem that the plane sections which are perpendicular to the neutral layer before bending remain plane and perpendicular to the neutral layer after bending, implying that the transverse shear and transverse normal strains are zero. Thus, the theory disregards the effects of the shear deformation. It is also known as classical beam theory. The theory is applicable to slender beams and should not be applied to thick or deep beams. For the thick beams analysis elementary theory of beam (ETB) is used, deflections are underestimated and buckling loads are overestimated. This is the outcome of neglecting transverse shear deformations in ETB. Rankine [6], Bresse [7] were the first to include both the rotatory inertia and shear flexibility effects as refined dynamical effects in beam theory. This theory is, referred as Timoshenko beam theory as mentioned in the literature by Rebello, et.al. [8] and based upon kinematics it is known as first-order shear deformation theory (FSDT).

Ghugal and Dahake [9] have developed a trigonometric shear deformation theory for flexure of thick or deep beams, taking into account transverse shear deformation effect. First order shear deformation theory contains the same number of variables in the present theory. Displacement field contains the sinusoidal function in terms of thickness coordinate to represent the shear deformation effects. This theory prevents the need of shear correction factor. A variationally consistent refined hyperbolic shear deformation theory for flexure and free vibration of thick isotropic beam is developed by Ghugal and Sharma, Sayyad and Ghugal. In this theory transverse shear deformations effects are considered. In this paper, a hyperbolic shear deformation theory is developed for static flexural analysis of thick isotropic beams. Hyperbolic shear deformation theory (HPSDT) is applied to a Simply supported thick isotropic beams for analysing the axial displacement, Transvers displacement, Axial bending stress and transverse shear stress. The numerical results obtained for various lengths to thickness ratios of the beams and the results obtained are compared with those of Elementary, Timoshenko, Trigonometric and other higher order refined theories and with the available solution in the literature.

2. FORMULATION OF PROBLEM

Consider a thick isotropic simply supported beam of length l in x direction, width b in y direction and depth h as shown in figure-1. Where x, y, z are Cartesian co-ordinates. The beam is subjected to transverse load of intensity q(x) per unit length of beam. Under this condition, the axialdisplacement, transverse displacement, axial bending stress and transverse shear stress are required to be determined.

2.1 Assumptions made in the theoretical formulations

1. The axial displacement (u) consist of two parts:

a. Displacement given by elementary theory of bending.

b. Displacement due to shear deformation, which is assume to be hyperbolic in nature with respect to thickness coordinate.

2. The transverse displacement (w) in z direction is assumed to be function of x co-ordinate.

3. One-dimensional constitutive laws are used.

4. The beam is subjected to lateral load only.

2.2 The Displacement Field

Based on the above mentioned assumptions, the displacement field of the present beam theory can be expressed as follows. The hyperbolic function is assigned according to the shearing stress distribution through the thickness of beam.

$$u(x,z) = -z\frac{\partial w}{\partial x}(x) + \left[h\sinh\left(\frac{z}{h}\right) - \frac{4}{3}\frac{z^2}{h^2}\cosh\left(\frac{1}{2}\right)\right]\phi(x) \quad (1)$$

$$w(x) = w(x) \quad (2)$$

Where,

u = Axial displacement in x direction which is a function of x, z.

w = Transverse displacement in z direction which is function of x.

Φ = Rotation of cross section of beam at neutral axis due to shear which is an unknown function to be determined and it is function of x.

Normal strain:

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

$$\varepsilon_x = -z\frac{\partial^2 u}{\partial x^2} + \left[h\sinh\left(\frac{z}{h}\right) - \frac{4}{3}\frac{z^3}{h^2}\cosh\left(\frac{1}{2}\right)\right]\frac{\partial\phi}{\partial x} \quad (3)$$

Shear strain:

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

$$\gamma_{xz} = \left[\cosh\left(\frac{z}{h}\right) - 4\frac{z^3}{h^2}\cosh\left(\frac{1}{2}\right)\right]\phi \quad (4)$$

Stresses: The one-dimensional Hooke's law is applied for isotropic material, stress x is related to strain x and shear stress is related to shear strain by the following constitutive relations.

$$\sigma_x = E\varepsilon_x = -zE\frac{\partial^2 u}{\partial x^2} + E\left[h\sinh\left(\frac{z}{h}\right) - \frac{4}{3}\frac{z^3}{h^2}\cosh\left(\frac{1}{2}\right)\right]\frac{\partial\phi}{\partial x} \quad (5)$$

$$\tau_{xz} = G\gamma_{xz} = G\left[\cosh\left(\frac{z}{h}\right) - 4\frac{z^3}{h^2}\cosh\left(\frac{1}{2}\right)\right]\phi \quad (6)$$

Where, E and G are the elastic constants of the beam material.

2.3 Governing Differential Equations

Governing differential equations and boundary conditions are obtained from Principle of virtual work. Using equations for stresses, strains and principle of virtual work. Using equations for stresses, strains and principle of virtual work, variationally consistent differential equations for beam under consideration are obtained. The principle of virtual work when applied to beam leads to:

$$b\int_{x=0}^{x=L}\int_{z=-h/2}^{z=h/2}\left(\sigma_x.\delta\varepsilon_x + \tau_{xz}.\delta\gamma_{xz}\right)dxdz + \rho b\int_{x=0}^{x=L}\int_{z=-h/2}^{z=h/2}\left(\frac{\partial^2 u}{\partial t^2}.\delta u + \frac{\partial^2 w}{\partial t^2}.\delta w\right)dxdz - \int_{x=0}^{x=L} q\delta w dx = 0 \quad (7)$$

Where δ = variational operator.

Employing Greens theorem in above equation successively, we obtained the coupled Euler-Langrange equations, which are the governing differential equations and associated boundary conditions of the beam. The governing differential equations obtained are as follows:

$$EI\left[\frac{\partial^4 w}{\partial x^4} - A_0\frac{\partial^3\phi}{\partial t^3}\right] = q(x) \quad (8)$$

$$EI\left[A_0\frac{\partial^3 w}{\partial x^3} - B_0\frac{\partial^2\phi}{\partial x^2}\right] + GAC_0\phi = 0 \quad (9)$$

Where, A_0, B_0 and C_0 are the stiffness coefficients in governing equations. The associated consistent natural boundary conditions obtained are of following form along the edges x = 0 and x = L.

$$EI\left[\frac{d^3 w}{dx^3} - A_0\frac{d^2\phi}{dx^2}\right] = 0 \quad (10)$$

Where w is prescribed

$$EI\left[\frac{d^2 w}{dx^2} - A_0\frac{d\phi}{dx}\right] = 0 \quad (11)$$

Where dw/dx is prescribed.

$$EI\left[A_0\frac{d^2w}{dx^2}-B_0\frac{d\phi}{dx}\right]=0 \qquad (12)$$

Where ϕ is prescribed.

The flexural behaviour of beam is given by solution of above equations 8 and 9 by discarding all terms containing time derivatives and satisfying the associate boundary conditions. The stiffness coefficient used in governing equations 8, 9, 10, 11 and 12 are described as below:

$$A_0=\left[12\cosh\left(\frac{1}{2}\right)-24\sinh\left(\frac{1}{2}\right)-\frac{1}{5}\cosh\left(\frac{1}{2}\right)\right] \qquad (13)$$

$$B_0=\begin{Bmatrix}6[\sinh(1)-1]-200\cosh^2\left(\frac{1}{2}\right)\\ +432\sinh\left(\frac{1}{2}\right)\cosh\left(\frac{1}{2}\right)+\left(\frac{1}{21}\right)\cosh^2\left(\frac{1}{2}\right)\end{Bmatrix} \qquad (14)$$

$$C_0=\begin{Bmatrix}\left(\frac{1}{2}\right)[\sinh(1)-1]+16\cosh^2\left(\frac{1}{2}\right)\\ -36\sinh\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)\cosh\left(\frac{1}{2}\right)+\left(\frac{1}{5}\right)\cosh^2\left(\frac{1}{2}\right)\end{Bmatrix} \qquad (15)$$

2.4 The General solution of governing equilibrium equations of beam

The general solution for transverse displacement w(x) and φ(x) can be obtained from equation 8 and 9 by discarding the terms containing time (t) derivatives. Integrating and rearranging the equation 8, we obtained the following equation

$$\frac{d^3w}{dx^3}=A_0\frac{d^2\phi}{dx^2}+\frac{Q(x)}{D} \qquad (16)$$

Where, Q(x) is generalized shear force for beam.

$$Q(x)=\int_0^x q\,dx+k_1 \qquad (17)$$

The second governing equation 9 can be written as:

$$\frac{d^3w}{dx^3}=\frac{B_0}{A_0}\frac{d^2\phi}{dx^2}-\beta\phi \qquad (18)$$

Now using equations 16 and 18 a single equation in terms of φ is obtained as:

$$\frac{d^2\phi}{dx^2}-\lambda^2\phi=\frac{Q(x)}{D} \qquad (19)$$

The general solution of equation 19 is as follows:

$$\phi=k_2\cosh\lambda x+k_3\sinh\lambda x-\frac{Q(x)}{\beta D} \qquad (20)$$

Where the constants α, β, λ and D used in above equations are given below,

$$\alpha=\left(\frac{B_0}{A_0}-A_0\right),\quad \beta=\left(\frac{GAC_0}{DA_0}\right)\quad \lambda^2=\frac{\beta}{\alpha},\quad D=EI$$

The equation of transverse displacement w(x) is obtained by substituting the expression of φ(x) in equation 18 and integrating it thrice with respect to x. The general solution for w(x) is obtained as follows:

$$EIw(x)=\iiiint q\,dxdxdxdx+\frac{D}{\lambda^3}\left(\frac{B_0}{A_0}\lambda^2-\beta\right)(k_2\sinh\lambda x+k_3\cosh\lambda x)\frac{k_1x^3}{6}+k_4\frac{x^2}{6}+k_5x+k_6 \qquad (21)$$

Where k_1, k_2, k_3, k_4, k_5 and k_6 are the constants of integration and can be obtained by applying the boundary conditions of the beams.

3. ILLUSTRATIVE EXAMPLE

In order to prove the efficiency of the present theory, the following numerical examples are considered. The following material properties for beam are used.

Material properties:

1. Modulus of Elasticity E = 210 GPa

2. Poisson's ratio = 0.30

3. Density =7800 Kg/m^3

Example 1: Simply supported beam with varying load $q(x)=q_0\left(\frac{x}{L}\right)$

A Simply supported beam with the origin of beam on left end support at x = 0. The beam is subjected to varying load of q(x) over the span L on surface z = h/2 acting in the z direction.

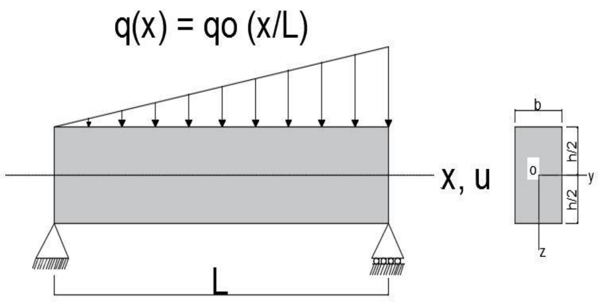

Figure-1: Simply Supported Beam Under x-z Plane

The boundary conditions at Simple Support (x = 0 and x = L) associated with this problem are as follows:

$$EI\frac{d^2w}{dx^2}=EI\frac{d\phi}{dx}=EIw=0 \tag{22}$$

General expressions obtained for w(x) and φ(x) are as follows:

$$\varphi(x)=\frac{q_0\cosh\lambda x}{BD\lambda\cosh\lambda L}-\frac{q_0}{BD}\frac{x^2}{2L}-\frac{C_1}{BD} \tag{23}$$

$$\bar{w}(x)=\left(2\frac{x^5}{L^5}-\frac{10}{3}\frac{x^3}{L^3}+\frac{7}{3}\frac{x}{L}\right)$$
$$+10\frac{E}{G}\frac{A_0^2}{C_0}\frac{h^2}{L^2}\left(\frac{\sinh\lambda x}{\lambda^2L^2\cosh\lambda L}-\frac{1}{6}\frac{x^3}{L^3}-\frac{1}{\lambda^2L^2}\frac{x}{L}+\frac{1}{6}\frac{x}{L}\right) \tag{24}$$

The axial displacement, stresses and transverse shear stress obtained based on above solutions are as follows:

$$\bar{u}=-\frac{z}{h}\frac{1}{10}\frac{L^3}{h^3}\left[\begin{array}{l}\left(5\frac{x^4}{L^4}-10\frac{x^2}{L^2}+\frac{7}{3}\right)\\+10\frac{E}{G}\frac{A_0^2}{C_0}\frac{h^2}{L^2}\left(\frac{\cosh\lambda x}{\lambda L\cosh\lambda L}-\frac{1}{2}\frac{x^2}{L^2}-\frac{1}{\lambda^2L^2}+\frac{1}{6}\right)\end{array}\right]$$
$$+\frac{A_0}{C_0}\frac{E}{G}\frac{L}{h}\left[\sinh\left(\frac{z}{h}\right)-\frac{4}{3}\frac{z^3}{h^2}\cosh\left(\frac{1}{2}\right)\right]$$
$$\left(\frac{\cosh\lambda x}{\lambda L\cosh\lambda L}-\frac{1}{2}\frac{x^2}{L^2}-\frac{1}{\lambda^2L^2}+\frac{1}{6}\right) \tag{25}$$

$$\bar{\sigma}_x=-\frac{z}{h}\frac{1}{10}\frac{L^2}{h^2}\left[\begin{array}{l}\left(20\frac{x^3}{L^3}-20\frac{x}{L}\right)\\+10\frac{E}{G}\frac{A_0^2}{C_0}\frac{h^2}{L^2}\left(\frac{\sinh\lambda x}{\cosh\lambda x}-\frac{x}{L}\right)\end{array}\right]$$
$$+\frac{A_0}{C_0}\frac{E}{G}\left[\sinh\left(\frac{z}{h}\right)-\frac{z^3}{h^3}\frac{4}{3}\cosh\left(\frac{1}{2}\right)\right]\left(\frac{\sinh\lambda x}{\cosh\lambda x}-\frac{x}{L}\right) \tag{26}$$

$$\bar{\tau}_{zx}^{EE}=\left(\frac{1}{80}\right)\left(\frac{L}{h}\right)\left(4\frac{z^2}{h^2}-1\right)\left(60\frac{x^2}{L^2}-20\right)$$
$$+10\left(\frac{E}{G}\right)\left(\frac{h^2}{L^2}\right)\left(\frac{A_0^2}{C_0}\right)\left(\frac{\lambda L\cosh\lambda x}{\cosh\lambda L}-1\right)$$
$$\left(\frac{1}{48}\cosh\left(\frac{1}{2}\right)\left(16\frac{z^4}{h^4}-1\right)-\cosh\left(\frac{1}{2}\right)-\cosh\left(\frac{z}{h}\right)\right) \tag{27}$$

$$\bar{\tau}_{zx}^{CR}=\frac{A_0}{C_0}\frac{h}{L}\left[\cosh\left(\frac{z}{h}\right)-4\frac{z^2}{h^2}\cosh\left(\frac{1}{2}\right)\right]$$
$$\left(\frac{\cosh\lambda x}{\lambda L\cosh\lambda L}-\frac{1}{2}\frac{x^2}{L^2}-\frac{1}{\lambda^2L^2}+\frac{1}{6}\right) \tag{28}$$

4. NUMERICAL RESULTS

The numerical results for axial displacement, transverse displacement, bending stress and transverse shear stress are presented in following non-dimensional form and the values are presented in Table-1 and Table -2

Table 1: Non-Dimensional Axial Displacement ($\bar{u}$) at (x = 0.75L, z = h/2), Transverse Displacement ($\bar{w}$) at (x = 0.75L, z = 0.0), Axial Stress ($\bar{\sigma}_x$) at (x = 0.75L, z = h/2), Maximum Transverse Shear Stresses $\bar{\tau}_{zx}^{CR}$ and $\bar{\tau}_{zx}^{EE}$ at (x = 0.0, z = 0.0) of the Simply Supported Beam subjected to varying load for Aspect Ratio 4. (Example 1)

Source	Model	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
Present	HPSDT	5.5903	0.6874	5.4451	0.9959	0.9992
Ghugal	TSDT	5.1100	0.6872	7.6927	1.2007	1.0320
Krishnamurthy	HSDT	5.5902	0.6874	5.4450	0.9988	1.1991
Timoshenko	FSDT	5.9375	0.6877	5.2500	0.8000	1
Bernoulli	ETB	5.4708	0.5811	5.2500	-	2

Table 2: Non-Dimensional Axial Displacement ($\bar{u}$) at (x = 0.75L, z = h/2), Transverse Displacement ($\bar{w}$) at (x = 0.75L, z = 0.0), Axial Stress ($\bar{\sigma}_x$) at (x = 0.75L, z = h/2), Maximum Transverse Shear Stresses $\bar{\tau}_{zx}^{CR}$ and $\bar{\tau}_{zx}^{EE}$ at (x = 0.0, z = 0.0) of the Simply Supported Beam subjected to varying load for Aspect Ratio 10. (Example 1)

Source	Model	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
Present	HPSDT	85.7800	0.5981	33.0076	2.5005	2.4984
Ghugal	TSDT	84.0500	0.5981	34.0105	2.5801	2.5803
Krishnamurthy	HSDT	85.7798	0.5981	33.0075	2.4995	2.5796
Timoshenko	FSDT	92.7734	0.5981	32.8125	2	2.5000
Bernoulli	ETB	85.4818	0.5811	32.8125	-	5

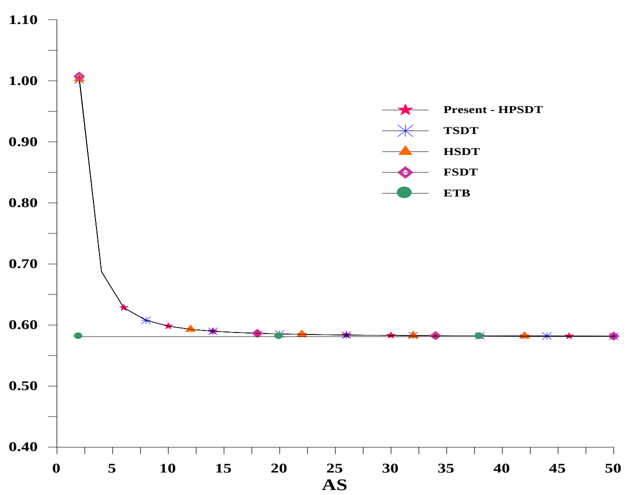

Figure-2: Variation of Transverse Displacement ($\bar{w}$)

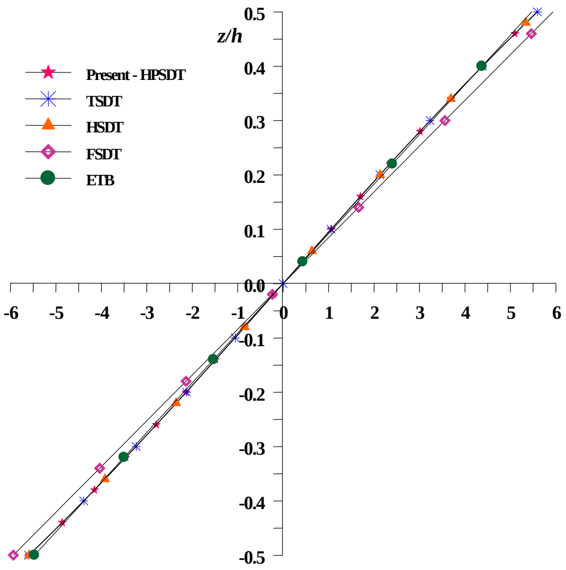

Figure-3: Variation of Maximum Axial displacement $\bar{u}$ for AS 04

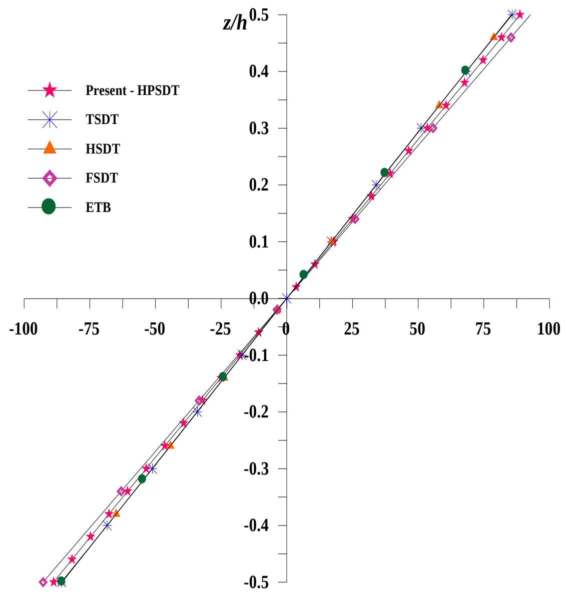

Figure-4: Variation of Maximum Axial displacement $\bar{u}$ for AS 10

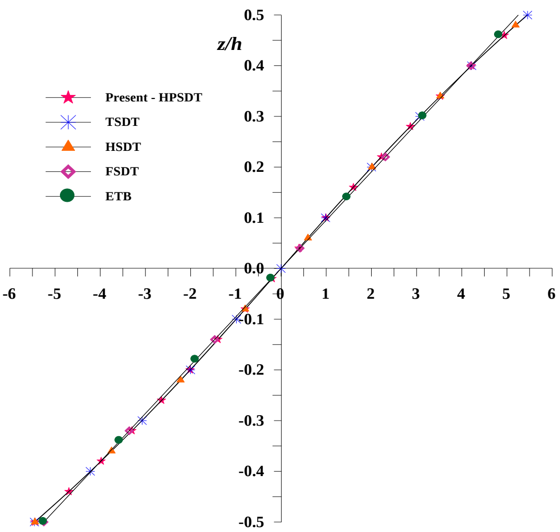

Figure-5: Variation of Maximum Axial stress $\overline{\sigma}_x$ for AS 04

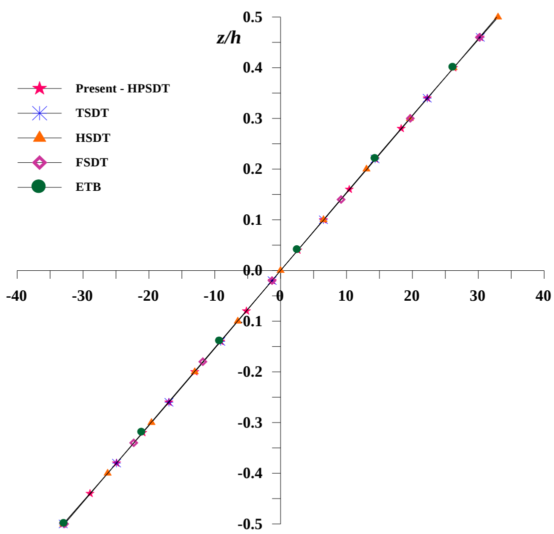

Figure-6: Variation of Maximum Axial stress $\overline{\sigma}_x$ for AS 10

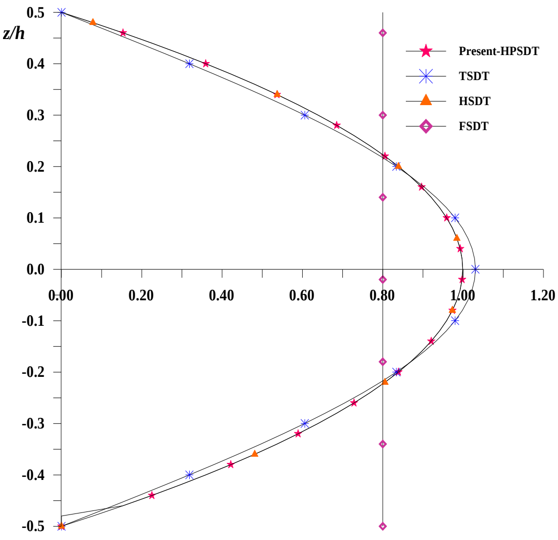

Figure-7: Variation of Transverse shear stress $\bar{\tau}_{zx}^{CR}$ for AS 04

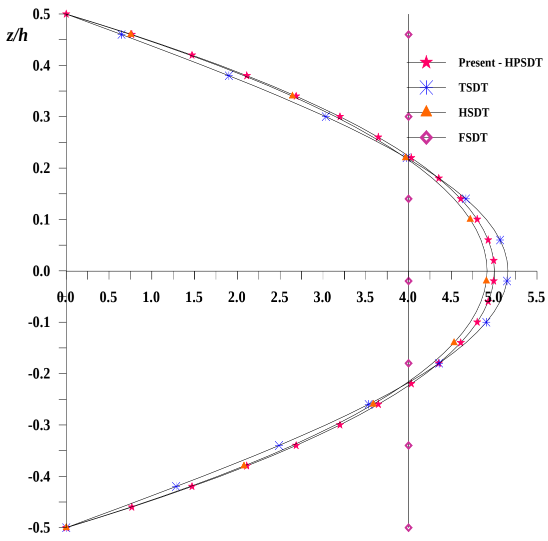

Figure-8: Variation of Transverse shear stress $\bar{\tau}_{zx}^{CR}$ for AS 10

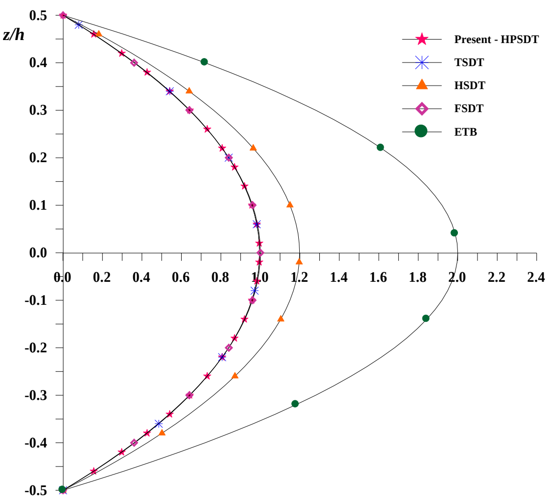

Figure-9: Variation of Transverse shear stress $\bar{\tau}_{zx}^{EE}$ for AS 04

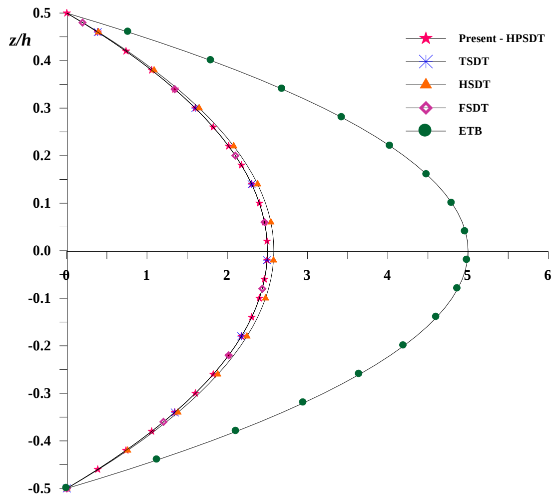

Figure-9: Variation of Transverse shear stress $\bar{\tau}_{zx}^{EE}$ for AS 04

5. CONCLUDING REMARK

From the static flexural analysis of simply supported beam following conclusion are drawn:

1. The result of maximum transverse displacement $\bar{w}$ obtained by present theory is in excellent agreement with those of other equivalent refined and higher order theories. The variation of for AS 4 and 10 are presented as shown in Fig-2.
2. From Fig-3 and Fig-4, it can be observed that, the result of axial displacement $\bar{u}$ for beam subjected touniform load varies linearly through the thickness of beam for AS 4 and 10 respectively.
3. The maximum Non-dimensional axial stresses $\bar{\sigma}_x$ for AS 4 and 10 varies linearly through the thickness of beam as shown in Figure 5 and Figure 6 respectively.
4. The transverse shear stresses $\bar{\tau}_{zx}^{EE}$ and $\bar{\tau}_{zx}^{CR}$ are obtained directly by constitutive relation. Fig-7, 8, 9 and Fig-10 shows the through thickness variation of transverse shear stress for thick isotropic beam for AS 4 and 10. From this figures it can be observed that, the transverse shear stresssatisfies the zero condition at top (z = +h/2) and at bottom (z = -h/2) surface of the beam.

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