

## **NEW METHOD FOR SOLVING SINGLE MACHINE SCHEDULING PROBLEM WITH FUZZY PROCESSING TIME AND DISTINCT DUE DATE**

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**Abstract:** This paper was focused on the development of a new method base on moving average to solve single machine scheduling problem with fuzzy processing time and distinct due date. The concept of single machine with fuzzy processing time has been presented and a new method base on moving average to sequence the jobs on the single machine, so that the total penalty cost to be minimum has been proposed. This cost is composed of the total earliness and the total tardiness cost. The performance of the proposed the new method has been measured on numerical examples. This is the first time a moving average was use to resolve the problem and the results are very encouraging for further investigation.

**Key Words:** Single Machine, Fuzzy processing, Earliness and Tardiness, Distinct Due Date, Moving Average

### **I. INTRODUCTION**

The study of earliness and tardiness penalty in scheduling is a relatively recent area of inquiry. For many years scheduling research focused on single performance measures .most of the literature deals with regular measure such as mean flow time, mean lateness percentage of jobs tardy mean tardiness and in deterministic time. But the environment in modern society is neither fixed nor probabilistic .So here we are considering fuzzy environment i.e. .the processing time of each job is in un deterministic environment. The time considered here is in three situations (a, b ,c )where a-in favorable (high)condition ,b-normal (medium)condition and c-worse (bad )conditions . Baker and Scudder (1990) Studied sequences with earliness and tardiness penalties in a JIT scheduling environment, jobs that complete early must be held in finished in good inventory until their due date. while jobs that complete after their due date may cause a customer to shut down operation .therefore an optimal schedule ,is one in which all jobs finish on their assigned due dates . This can be translated to scheduling objective in several ways .The most obvious objective is to minimize the deviation of job completion time around these due dates in non-deterministic time.

The concept of penalizing both earliness and tardiness has spawned anew and rapidly developing line of research in the scheduling field. Because the use of both earliness and tardiness penalties give rise to a non-regular performance

measure, it has led to new methodological issues in the design of solution procedure.

In this paper we present special case of Early / Tardy (E/T) single machine schedule problem in a fuzzy environment and having distinct due dates and the earliness and tardiness are penalized at the rate fixed by demand maker for the jobs. In the next Section we introduce the concepts of single machine and the fuzzy processing time of the jobs. .The moving average method and the scheduling of some small systems are determined in Section after. An algorithm based on these arguments is developed and it is justified by a numerical examples.

### **II. SINGLE MACHINE**

Now a day, in competitive and flexible market installing of machines is very expensive, as the technology changes very frequently and the out dated machines can not satisfy the demands of the modern market. Installing of more than one machines of the same type can speed up the work but needs more maintenance and supervision Also installation of machines demands for more space to install, which also increases the idle cost of the project. So reduce the expenditure, contractor wishes to process the work on a single machine using an intelligent scheduling system and for the small systems single machine maximizes the profit of whole the project.

### **III. FUZZY PROCESSING TIME**

The processing time of a job can vary in many ways, may be due to environmental factor or due to the different work places .We find that when a contractor takes the work from a department, he calculates total expenditure at the time of allotment. But due to many factors like non-available of labor ,weather not favorable ,or sometimes abnormal condition cost may vary .Hence due to these reasons work can be completed late and creates due date problem i.e. order cannot be delivered on time , on the other hand if the work completes before the due date time it arises the inventory problem. So to overcome these factors, the processing time of a job considered here is in three situations favorable (high), Normal (Medium) and worse (Bad) conditions. In this paper, a new concept of different processing time of each job is considered which helps the contractor to estimate the cost of the work at the time of allotment. Here also different due dates for each of the job be considered which meets the demand maker with more satisfaction level.

### **IV. ASSUMPTION AND NOTATION**

The machine scheduling problem studied in this paper requires  $n$  independent jobs,  $j_i$  ( $i= 1, 2, 3, \dots, n$ ) to be processed on a single machine with the following assumptions:

(i) All jobs are available for processing at time zero

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- (ii) The single machine can process at most one job at a time
  - (iii) No pre-emption is allowed
- Let S Schedule for the n jobs

sequence of schedule is 4-2-1-5-3 with total cost equal 237.67

- < a, b, c > Processing time of job i on the machine in fuzzy environment.
- $M_i$  Moving average of the processing time
- $(a, b, c) = \{a+2b+c\}/4$
- $d_i$  Due date for the job i
- $c_i$  Completion time of job i
- $T_i$  Max.  $(0, c_i - d_i)$
- $E_i$  Max  $(0, d_i - c_i)$
- $Sl_i$  Slack time of job i
- $\alpha_i$  Penalty per unit tim for the earliness of job i
- $\beta_i$  Penalty per unit time for the tardiness of job i

In this example we use the same data and solve it by our algorithm so as to compare between two schedule solutions.

**Table (1)**

Job i	$P_i$	<MA>	$d_i$	$Sl_i$	$\alpha_i$	$\beta_i$
1	5,7,8	6.75	9	2.25	2	3
2	3,5,7	5	7	2	2	3
3	11,15,17	14.5	18	3.5	2	3
4	6,8,11	8.25	10	1.75	2	3
5	7,8,10	8.25	10	1.75	2	3

An important special case in the family of earliness and tardiness problems involves minimizing the sum of absolute deviation of job completion time from a distinct due date having processing time in fuzzy environment. In particular, the objective function can be written as follows:

$$f(s) = \sum \alpha_i E_i + \beta_i T_i$$

When we write the objective function in the above form, it is clear that the earliness and tardiness are penalized at the rate  $\alpha_i$  and  $\beta_i$  for all jobs. In this paper, processing time of the jobs considered are in triangular fuzzy that means at < a, b, c >

The Five jobs of Table .1, having fuzzy processing time (a, b, c) are converted first to moving average by using  $\langle MA \rangle = M_i = |a + 2b + c|/4$ . After that we use the algorithm of Section (V). Finally we found that sequence of the jobs are 4 - 5 - 2 - 1 - 3, with total cost equal 198.5 units, which is better than their cost. **Table .2** shows the sequence schedule of the jobs, the total flow time of the system and the total optimized penalty cost due to earliness/tardiness of the jobs.

**V. MOVING AVERAGE ALGORITHM**

**Table .2**

Step1: Find Moving Average (MA) of the fuzzy processing time (a, b, c) of all The jobs.

$$M_i = |a + 2b + c|/4$$

Seq.	$p_i$	$c_i$	$d_i$	$c_i-d_i$	$\alpha_i$	$\beta_i$	cost
4	8.25	8.25	10	1.75	2	3	$1.75*2=3.5$
5	8.25	16.5	10	6.5	2	3	$6.5*3=19.5$
2	5	21.5	7	14.5	2	3	$14.5*3=43.5$
1	6.75	28.25	9	19.5	2	3	$19.*3=58.5$
3	14.5	42.75	18	24.5	2	3	$24.5*3=73.5$

Step2: Find the slack time of all the jobs

$$Sl_i = |M_i - d_i|$$

Step3: Arrange the jobs in increasing order of their slack time.  
If two jobs have the same Slack time then considers the jobs of the lowest processing time at the earlier position.

Step4: Using the sequence obtained in step 3 find the total penalty of all the jobs using earliness  $\alpha_i$  and  $\beta_i$  penalty cost .

**EXAMPLE .2**

**VI. NUMERICAL EXAMPLES**

**EXAMPLE. 1**

**Table .3** below shows a 10 jobs having fuzzy processing time, single machine, distinct due date and the Penalty cost  $\alpha_i$  for earliness and  $\beta_i$  tardiness is also given. This example was already solved by E. Janaki and R. Vigithra (2015). By Average High Ranking (AHR) and they found that the total cost is equal 1219.66, with jobs sequence 1, 9, 4,3, 2, 10, 6, 8, 5, 7. We need to solve the same example by our algorithm so as to compare between two Schedule solutions.

**Table (1)** shows a 5-jobs having fuzzy processing time, single machine and distinct due dates. Penalty cost  $\alpha_i$  for earliness and  $\beta_i$  tardiness is also given. This example already solved by S. Gupta and M. Rambha (2011). They used average high ranking method and found that the

**VII. CONCLUSION**

The single machine with fuzzy processing time and distinct due dates has been studied. The objective was to find an optimal scheduling that minimizes a total cost function containing earliness and tardiness costs with penalties. We developed a moving average algorithm to solve this problem. It was found that our method of moving average gives an optimal solution. Further investigation can be carried out for multi machine system

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**Table. 3**

Job <sub>i</sub>	P <sub>i</sub>	<MA>	d <sub>i</sub>	Sl <sub>i</sub>	α <sub>i</sub>	β <sub>i</sub>
1	3,4,5	4	5	1	2	3
2	8,10,12	10	7	3	2	3
3	5,6,8	6.25	10	3.6	2	3
4	8,9,11	9.25	12	2.6	2	3
5	11,12,15	12.5	8	4.5	2	3
6	7,9,11	9	15	6	2	3
7	5,7,9	7	17	10	2	3
8	12,14,15	13.75	10	3.6	2	3
9	5,7,8	6.75	6	0.6	2	3
10	6,8,10	8	14	6	2	3

The10 jobs having fuzzy processing time (a , b , c ) are converted into moving average method by using,  
 $M_i = |a + 2b + c|/4$  , and as per algorithm mentioned in Section V we found that the optimal sequence is 9-1-4-2-3-8-5-10-6-

**Table 4.**

Seq.	p <sub>i</sub>	C <sub>i</sub>	d <sub>i</sub>	c <sub>i</sub> - d <sub>i</sub>	α <sub>i</sub>	β <sub>i</sub>	cost
9	6.25	6.75	6	0.75	2	3	2.25
1	4	10.75	5	5.75	2	3	17.25
4	9.25	20	12	8	2	3	24
2	10	30	7	23	2	3	69
3	6.25	36.25	10	26.25	2	3	78.75
8	13.75	50	10	40	2	3	120
5	12.5	62.5	8	54.5	2	3	163.5
10	8	70.5	14	56.5	2	3	169.5
6	9	79.5	15	64.5	2	3	193.5
7	7	86.5	17	69.5	2	3	208.25

**Table 4** shows the total flow time of the system and the total optimized penalty cost of the jobs, which equal **1046.25**. This is better than their cost

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