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MARKOV CHAIN APPLICATION IN LITHOFACIES MODELING: A NIGER DELTA FIELD.

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ABSTRACT

Six lithofacies were grouped on a gross lithological unit and given individual codes: cross-bedded coarse grained sandstone (SCC), cross-bedded medium grained sandstones (SCM), planar/parallel laminated sandstone (SPL), wavy rippled sandy heterolith(SWH), parallel laminated mudstone (MPL), and wavy rippled muddy heterolith(MWH). The analytical procedure of markov chain method was applied in this order: Transition frequency matrix, Upward transition probability matrix, Downward transition probability matrix, Independent trial matrix, Difference matrix, Expected frequency matrix and then the Chi square test. The calculated Chi square value was higher than the limiting value at 5% level of significance, and this implied that the null hypotheisis rejected. This thus explains a markovian and presence of cyclic distribution arrangement of facies within the field of the study area.

Keywords: Markov chain, Transition frequency matrix, Chi square test, and cyclic distribution.

INTRODUCTION

Walther (1893) law of correlation states that,facies that occur in conformable vertical successions of strata also occur in laterally adjacent environment, proving that a relatively gradual transition from one facies to another indicates that the two facies were once laterally adjacent to each other, this is due to processes of transgressions and regressions. Cyclicityas applied in sedimentary facies, defines a succession of lithologic unit in a repeated pattern. This refers to a series of connected order of depositional facies that is linked to a depositional environment.

The Markov chain method is a stochastic process that uses mathematical model in a probabilistic approach. It aided in this study to statistically evaluate the cyclic character of the lithostratigraphic sequence of the field, evaluate the degree of ordering of the facies deposition and understand better the broad depositional environment of the field.

There are two types of types of cyclicity in Markov approach (type 1-Embedded Markov matrix- Krumbein and Darcey 1969; based on following an order of sequence only, and type 2- Regular Markov matrix- Read, 1969, based on certain order of repetition along a vertical scale of sedimentary succession) , and are applied according to the geological issue to be addressed. In this research work, we aimed at examining, if present, the cyclicity within a lithofacies model using the individual bed as a unit, and thus type 1- Embedded Markov Matrix method was applied, which has little or no consideration to individual thickness of lithofacies as it examines cyclicity of sequence.

AIM OF THE STUDY

To apply the principle of markov chain to analyse the lithofacies model within the study area, and evaluate the statistical cyclic character.

GEOLOGIC SETTING AND STRATIGRAPHY OF STUDY AREA

The study area is located within the central swamp depobelt, onshore region of the Niger delta (fig. 1). The Niger delta is located between latitude 3°N to 6°N and longitude 5°E and 8°E in the gulf of guinea, west Africa.

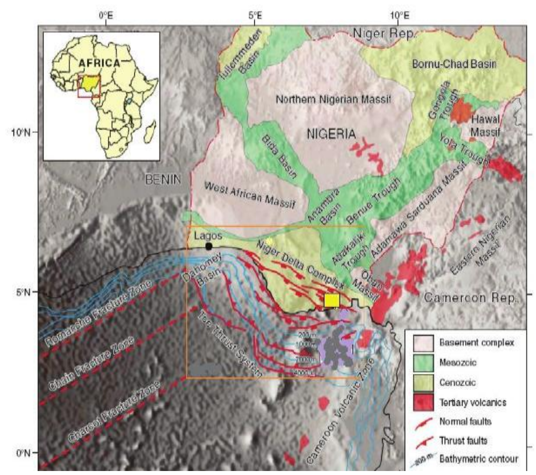

Fig. 1: Onshore location of study area (after Corredor et. al. 2005).

Etu-Efeotor J.O. (1997), described the tertiary Niger Delta as formed from a complex regressive offlap sequence of clastic sediments, ranging in thickness from 900 – 1200 meters. Short and Stauble (1967),studied the origin of the Niger Delta anddescribed it as an accurate, wave and tide dominated prograding deltaic system and the sediments range from Eocene to Quaternary.Thedeltaic complex is divided into three (3) major facie units based on the dominant environmental inference. These lithofaciesare locally designated from the bottom as Akata,Agbadaand Benin formation respectively (Fig.2). The Agbada formation constitutes the main reservoirs of hydrocarbon in the Niger Delta province (Doust and Omatsola, 1990).

Akata (Marine Shales) Formation

The lithofacies consists of shales, clays and silts situated at the base of the delta sequence. There is a few streaks of sand,

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possibly of turbiditic origin, and were deposited in delta front to deeper marine (holomarine) environments. The marine shale sequence is typically overpressured (Doust et. al. 1989).

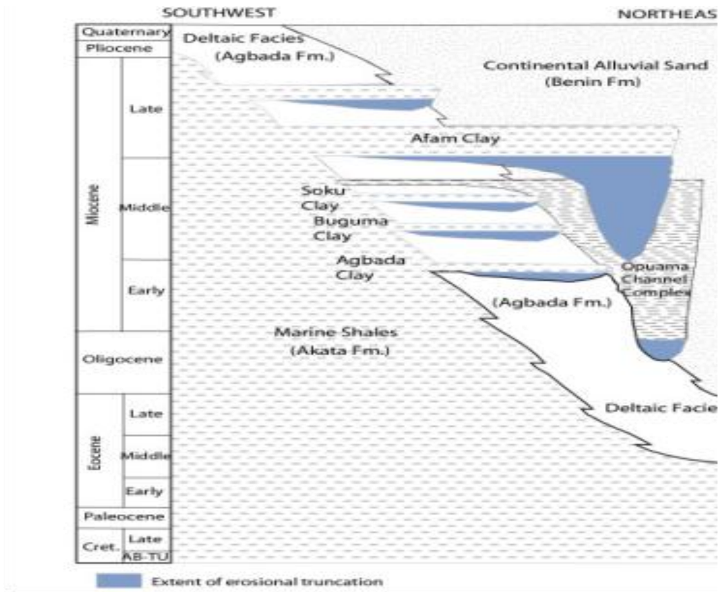

Fig.2.: Stratigraphic column showing the three formations of the Niger Delta.
(Adapted from Doust and Omatsola 1990).

The thickness of this sequence is about 7000m in the central part of the delta. The marine shales ranging from Paleocene to Holocene in age form the base of the sequence in each depobelt. The outcrops are seen in northeastern part of the delta, known as Imo shale.

Agbada (ParallicClastic) Formation
This has an alternation of sands, silts, and clays in various proportions and thicknesses, and this represents cyclic sequence of offlap units. They were deposited in a number of delta front, delta topset, and fluvio-deltaic environments. This paralic sequence is present in all depobelt and ranges in age from Eocene to Pleistocene. The thickness of this sequence is more than 3000m, and most exploration wells in the Niger Delta are bottomed in this lithofacie (Doust et. al., 1989). Its outcrop equivalent is the Ogwashi-Asaba of the Anambra Basin.

Benin (Continental Sands) Formation
The Benin formation is entirely a non-marine sand, and is the shallowest part of the sequence. This formation was deposited on upper coastal plain environment due to southward shift of deltaic deposition into a new depobelt. They lack fauna and so impossible to directly date, however the oldest continental sands are probable Oligocene.

LITERATURE REVIEW

A Markov process is defined as a succession of states or events where the occurrence of a given state depends on the present and not the past. A Markovian process is one in which the probability of the process being in a given state at a particular time may be deduced from the knowledge of the immediate preceding state (Harbaugh and Bonham-Carter, 1970).

The Markov chain method provides an objective approach to modeling of discrete variables such as lithologies or facies (Krumbein and Dacey, 1969; Read, 1969; Selley, 1970b; Harbaugh and Bonham – Carter, 1970; Miall,1973;Casshyap,1975; Allen, 1982). The Markov Chain analysis, in relation to stratigraphic studies defines that the probability of one lithology overlying another in a section is a function of their frequency (i.e. the number of times they occur) and the 'memory' of the depositional process (Selley, 1970; Miall, 1973; Cant and Walker 1976; Walker R.G, 1979 and 2006).

Several researchers (Carr, 1982; Billinton, 1992; Hota and Pandya, 2002; Hotaet. al., 2003; Hota and Maejima et. al., 2004;Purkis et. al., 2005; Alpaydın, 2010) have used the Markov chain analysis of geostatistics method to provide an ideal sedimentary sequence.It is one of the statistical methods that can be used to study the possibility of occurrence and repeat of different rock units at the time of deposition and based on proposed and interpreted depositional model. Rankey(2002), stated that this is achieved by characterizing the complexity of transition probabilities between sub-facies and testing whether they are non-random.

METHODOLOGY

The major analytical processes of markov chain involves defining transition count matrix (Fij), Observed transition probability matrix (Pij), Independent trial probability matrix (Random probability) (Rij) and difference probability matix (Dij).

The transition count matrix (Fij) involved recording observed coded facies in a tallied upward transition. From the transition count matrix, the other three matrices were then calculated; Observed transition probability matrix (Pij), Independent trial probability matrix (Random probability) (Rij) and difference probability matix (Dij). The work flow was adapted from Walker 1979 (fig. 3)

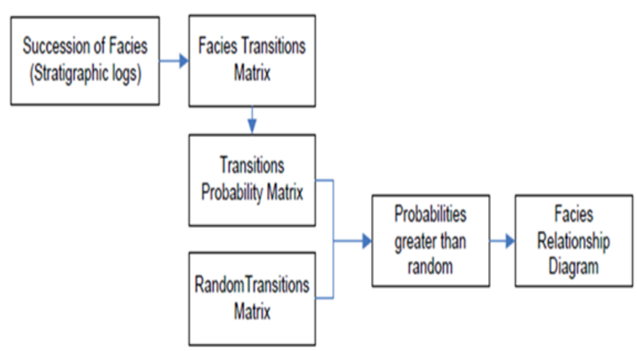

Fig. 3: Markov Flow chart (Walker 1979).

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The Observed transition probability matrix (Pij), calculates probability of lithofacies in an upward transition succession**.** The Independent trial probability matrix (Random probability) (Rij), records probability of assumed facies in a random sequence while the Difference probability matrix (Dij), records the difference between the observed transition probability (Pij) and the Independent trial probability matrix (Rij).

The following formulars were applicable:
Pij = Fij/SRi
Rij=Nj/N – Ni
Dij = Pij – Rij

Where;

- **Pij**is the observed Upward transition probability of**i** being followed by **j. Fij** is the number of transitions of **i** to **j,SRi** is the row total (Mial, 1973).
- **Rij** is the random probability of transition from facies**i** to facies **j**, **ni** and **nj**are the number of occurrences of facies**i** and **j** respectively, and **N** is the total number of occurrences of all facies (Walker, 1979)
- **Dij** is the difference between **observed probability (Pij)** and the **random probability (Rij)** (Mial, 1973; Cant and Walker, 1976; Walker, 1979).

The probability range which is between 0 to 1 determines the certainty and cyclicity order of the facies. With a probability scale of $0 \leq P \leq 1$. Where the probability of occurrence of an event (P) is 0 or lesser, the probability of occurrence is false, and where P is 1 or greater, the probability of occurrence is true.

RESULTS AND DISCUSSION

Two interpreted wells, Well X1 and Well X4 were used in this study to determine the vertical and areal distribution of the lithofacies within 'X' field. Six interpreted lithofacies based on sedimentology, biofacies report, primary sedimentary structures and paleocurrent patterns were grouped on a gross lithological unit and given individual codes. These six coded lithofaciesare presented in table 1.

Lithofacies	Codes
Crossbedded Coarse Grained Sandstone	SCC
Crossbedded Medium Grained sandstone	SCM
Planar/parallel laminated Sandstone	SPL
Wavy Rippled Sandy Heterolith	SWH
Parallel Laminated mudstone	MPL
Wavy Rippled Muddy Heterolith	MWH

Table 1: Coded lithofacies within the study area

The lithofacies for well X1 and well X4were grouped in a sequence each, and the total count of each facies recorded (Tables 2, 3 and 4).

Well X1

Depth(m)	Facie Co	Bed-Thickness	Column Thicknes
6750 -6754	SCC	4	
6800 - 6804	SCC	4	
6850 - 6854	SCC	4	
6910 - 6914	SCC	4	16
7308 - 7312	SCM	4	
7515 - 7518	SCM	3	7
7810 - 7815	SPL	5	
7900 -7904	SPL	4	
8050 -8055	SPL	5	14
8100 - 8105	SWH	5	5
8112-8118	SPL	6	
8202 -8206	SPL	4	10
8305 -8308	SWH	3	3
8410-8414	SPL	4	4
8515 -8520	MWH	5	5
8800 -8804	MPL	4	
9100-9104	MPL	4	8
9218 - 9222	MWH	4	4
9450 -9455	MPL	5	
9660-9664	MPL	4	
10250-10255	MPL	5	14
10330-10334	MWH	4	
10415 -10418	MWH	3	
10512 -10515	MWH	3	10
10812 -10818	SCM	6	6
10910 -10914	SCC	4	4
10948 -10952	SCM	4	4
11210 - 11215	SCC	5	5
11220 -11224	SCM	4	4

Table 2: Sequence facies arrangement for well X1

Well X4

Depth	Facie Code	Bed Thickness	Column Thickness
6500 - 6504	SCM	4	
6600 - 6605	SCM	5	9
6715 - 6718	SPL	3	
6912 - 6915	SPL	3	
7050 - 7058	SPL	8	14
7212 -7215	SWH	3	
7418 - 7422	SWH	4	
7544 - 7548	SWH	4	
7640 -7645	SWH	5	16
7800 - 7804	MPL	4	
8010 - 8015	MPL	5	
8100 - 8104	MPL	4	13
8215 - 8218	MWH	3	
8415 - 8419	MWH	4	
8715 - 8718	MWH	3	
8800 - 8805	MWH	5	15
8910 - 8915	SWH	5	
9110- 9113	SWH	3	8
9200 -9205	SPL	5	
9280 - 9283	SPL	3	8
9290 -9301	SWH	11	
9350-9355	SWH	5	
9410 -9414	SWH	4	20
9480 - 9483	SPL	3	
9510 - 9515	SPL	5	8
9618 - 9622	SWH	4	
9810 -9815	SWH	5	9
9900 - 9904	SPL	4	4
10108 -10115	MWH	7	7

Table 3: Sequence faciesarrangement for well X4

Individual Occurrences(**ni**)	Facies States
3	SCC
5	SCM
7	SPL
6	SWH
3	MPL
5	MWH
29	

Table 4: Composite transition frequency matrix for well X1 and X4.

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The analytical procedures of markov chain were applied in this order:

- Transition Frequency Matrix(F)
- Upward Transition Probability Matrix (P)
- Downward Transition Probability Matrix (Q)
- Independent Trial Matrix (R)
- Difference Matrix (D)
- Expected Frequency Matrix (E)
- Chi Square Test (X^2)

Transition Frequency Matrix: A frequency count matrix was structured, having **'i'** as row count while **'j'** as the column count. This involved counting transition from one facies to another. The diagonal elements are all zero because the embedded markov chain method considers transition occurrances as one lithology to another and not their thicknesses. The transition frequency matrix is presented in table 5.

(Fij) TALLY COUNT MATRIX

FROM	TO						Row Total
	SCC	SCM	SPL	SWH	MPL	MWH	
SCC	0	3	0	0	0	0	3
SCM	2	0	2	0	0	0	4
SPL	0	0	0	5	0	2	7
SWH	0	0	5	0	1	0	6
MPL	0	0	0	0	0	3	3
MWH	0	1	0	1	2	0	4
Column TOTAL	2	4	7	6	3	5	27

Table 5: Transition frequency tally count matrix

Upward Transition Probability Matrix (P): This describes the upward probability ordering of lithofacies in a succession and is given as:

$$P_{ij} = F_{ij}/SR_i$$

Where SR_irepresents the corresponding row total of each lithofacies.
This was achieved by dividing the transition frequency matrix count (F_{ij}) by the corresponding row total (SR_i). This is presented in table 6a.

(Pij)UPWARD PROBABILITY MATRIX (Well X1)

FROM	TO						Row Total
	SCC	SCM	SPL	SWH	MPL	MWH	
SCC	0	1	0	0	0	0	1
SCM	0.5	0	0.5	0	0	0	1
SPL	0	0	0	0.714286	0	0.28571	1
SWH	0	0	0.8333	0	0.166667	0	1
MPL	0	0	0	0	0	1	1
MWH	0	0.25	0	0.25	0.5	0	1
							6

Table 6a: Upward transition probability matrix

Downward Transition Probability Matrix (Q): This describes the downward probability ordering of lithofacies in a succession. This was determined by dividing the transition frequency matrix counts (F_{ij}) by the corresponding column total (SC_j) , and is given as:

$$Q_{ji} = F_{ij}/SC_j$$

whereSC_j is the column total. This is presented in table 6b.

DOWNWARD PROBABILITY MATRIX(Qij)

FROM	TO						Row Total
	SCC	SCM	SPL	SWH	MPL	MWH	
SCC	0	0.75	0	0	0	0	0.75
SCM	1	0	0	0	0	0	1
SPL	0	0	0	0.833333333	0	0.4	1.23333
SWH	0	0	0.7143	0	0.333333333	0	1.04762
MPL	0	0	0	0	0	0.6	0.6
MWH	0	0	0	0.166666667	0.666666667	0	0.83333
							5.46429

Table 6b: Downward transition probability matrix

The upward and downward transition probability matrix were both analysed to understand the probabilities of each lithofacies occurring across each other in an upward and downward order. (Fig. 4a and 4b).

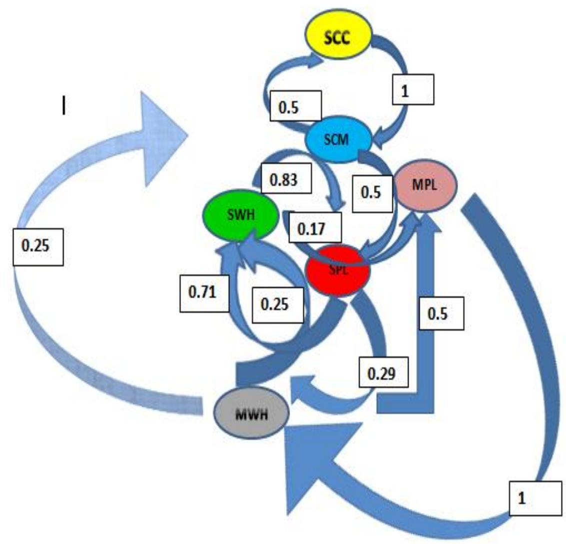

Fig. 4a: View of Upward transitionprobability matrix

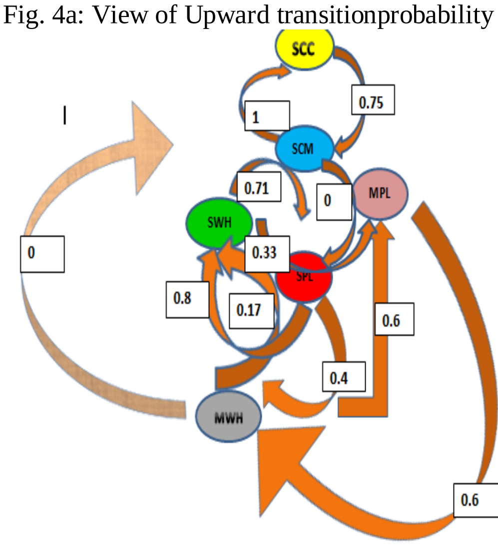

Fig. 4b: View of Downward transition probability matrix

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Independent Trial Matrix (R): This expresses the probability of randomness within the transition. It is expressed as:

$R_{ij}=N_j/N - N_i$ or $R_{ij} = S_{cj}/ST\text{-}SR_{i,}$

which was achieved by dividing facie states column total of the frequency matrix count (S_{cj}) by sum of total number of facies (S_T) and corresponding row total (SR_i) of facies states in the frequency matrix count. This is presented in table 7.

(Rij)Independent Trial Probability Matrix

FROM	TO						Row Total
	SCC	SCM	SPL	SWH	MPL	MWH	
SCC	**0**	0.166666667	0	0	0	0	0.1666667
SCM	0.087	**0**	0.3043	0	0	0	0.3913043
SPL	0	0	**0**	0.3	0	0.25	0.55
SWH	0	0	0.3333	**0**	0.125	0	0.4583333
MPL	0	0	0	0	**0**	0.20833	0.2083333
MWH	0	0.173913043	0	0.285714	0.130435	**0**	0.5900621
							2.3647

Table 7: Independent trial matrix

Difference Matrix (D): This expressed the transitions that have higher probability of occurrence if sequence were in random order.

This is given as:

$$D_{ij}= P_{ij}- R_{ij}$$

In this case, a preferred upward path of facies transition wasestablished which was interpreted in accordance with the works of Read 1969 and Miall 1973, relating it to depositional processes that had resulted to the arrangement of the facies. Where negative values are present, it indicates less frequency in occurrence while positive values are interpreted as having high frequency of occurrence. This is presented in table 8.

(Dij)DIFFERENCE PROBABILITY MATRIX

FROM	TO						Row Total
	SCC	SCM	SPL	SWH	MPL	MWH	
SCC	**0**	0.83		0	0	0	0.8333333
SCM	0.41	**0**	0.1667	0	0	0	0.5797101
SPL	0	0	**0**	0.41	0	0.03571	0.45
SWH	0	0	0.50	**0**	0.041667	**0**	0.5416667
MPL	0	0	0	0	**0**	0.79	0.7916667
MWH	0	0.076086957	0.00	-0.03571	0.37	**0**	0.4099379
							3.60631

Table 8: Difference probability matrix

Expected Frequency matrix (E) : This is calculated as the expected number of transitions from one facie to another. It is expressed as:

$$E_{ij}= R_{ij} \text{ X } SR_i$$

This is presented in table 9. The expected frequency matrix (E), is an essential element in the test for significance using Chi's square.

EXPECTED FREQUENCY MATRIX (Eij)

FROM	TO						Row Total
	SCC	SCM	SPL	SWH	MPL	MWH	
SCC	**0**	0.50	0	0	0	0	0.5
SCM	0.35	**0**	1.217391304	0	0	0	1.57
SPL	0	0	**0**	2.10	0	1.75	3.85
SWH	0	0	2.00	**0**	0.75	0	2.75
MPL	0	0	0		**0**	0.625	0.625
MWH	0	0.6957	0.00	1.142857143	0.52	**0**	2.3602484
							11.650466

Table 9: Expected frequency matrix

Chi- Square Test (X^2): This is a test to determine the significance difference between the expected frequencies and the observed frequency (as in table 4.7). In this work, the computed Chi-square value is higher than the limiting value at 5% level of significance, implying that the null hypothesis is rejected. This explains a markovian and cyclic distribution arrangement of facies within the field X. This is presented in table 10.

Fij	Eij	(Fij –Eij)^2/Eij
3	0.50	12.5
2	0.35	7.778571
2	1.22	0.503771
5	2.10	4.004762
2	1.75	0.035714
5	2.00	4.5
1	0.75	0.083333
3	0.65	8.496154
1	0.695	0.133849
1	1.142	0.017657
2	0.52	4.212308
		42.26612

Chi Square test

Test for Significance

Computed Value	5% significance level (95% confidence)	Degree of freedom (n^2-2n)
42.27	**36.42**	**24**

Table 10: Chi-square test

CONCLUSION

This research revealed the presence of a markovian relationship within the lithofacies as proved by the probability transition counts. An extended application of entropy concept in this work would aid in depositional environment setting interpretation of the study area.

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