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A Study of Neural Network Based Function Approximation

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***Abstract*— This paper provides a comprehensive study of neural network based function approximation. Different dimensional and different variable functions are used. Different standard test function for optimization are also used. Neural network architecture is used to approximate the functions.**

***Index Terms*— Function; Neural Network; Approximation**

1) Introduction

Neural network are used for function approximation and it has been widely used in different applications. Neural networks are being successfully applied across an extraordinary range of problem domains, in fields as diverse as computer science, finance, medicine, engineering, physics, etc. The main reason for such popularity is their ability to approximate arbitrary functions. For the last 30 years a number of results have been published showing that the artificial neural network called a feed forward network with one hidden layer can approximate arbitrarily well any continuous function of several real variables. These results play an important role in determining boundaries of efficacy of the considered networks. But the proofs are usually do not state how many neurons should be used in the hidden layer. The purpose of this paper is to prove constructively that a neural network having only one neuron in its single hidden layer can approximate arbitrarily well all continuous functions defined on any compact subset of the real axis.

2) Artificial Neural Network

Artificial neural network (ANN) takes its name from the network of nerve cells in the brain. Recently, ANN has proved to be an important technique for classification and optimization problems. McCulloch and Pitts have developed the neural networks for different computing machines. There are extensive applications of various types of ANN in the field of communication, control, instrumentation and forecasting. The ANN is capable of performing nonlinear mapping between the input and output space due to its large parallel interconnection between different layers and the nonlinear processing characteristics. An artificial neuron basically consists of a computing element that performs the weighted sum of the input signal and the connecting weight. The sum is added with the bias or threshold and the resultant signal is then passed through a nonlinear function of sigmoid or hyperbolic tangent type. Each neuron is associated with three parameters whose learning can be adjusted; these are the connecting weights, the bias and the slope of the nonlinear function. For the structural point of view, a neural network (NN) may be single layer or it may be multilayer. In multilayer structure, there is one or many artificial neurons in each layer and for a practical case there may be a number of layers. Each neuron of the one layer is connected to each neuron of the next layer. The functional-link ANN is another type of single layer NN. In this type of network the input data is allowed to pass through a functional expansion block where the input data are nonlinearly mapped to more number of points. This is achieved by using trigonometric functions, tensor products or power terms of the input. The output of the functional expansion is then passed through a single neuron.

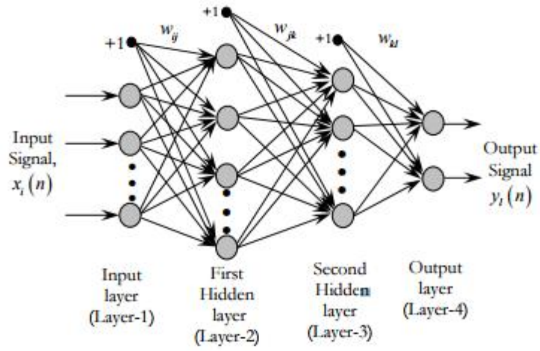

Fig. 1. Multi layer perceptron (MLP)

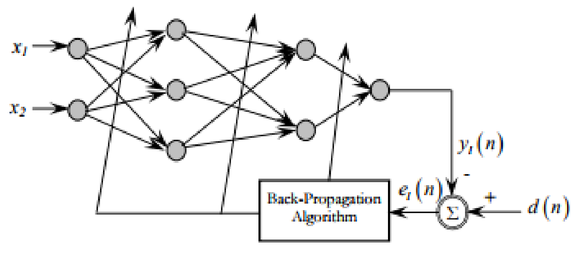

Fig. 2. Backpropagation algorithm for training of MLP

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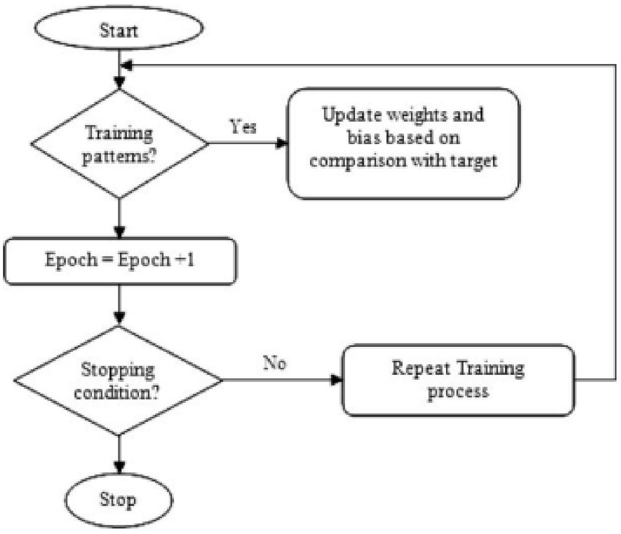

Fig. 3. Block Diagram of Supervised Training Algorithm

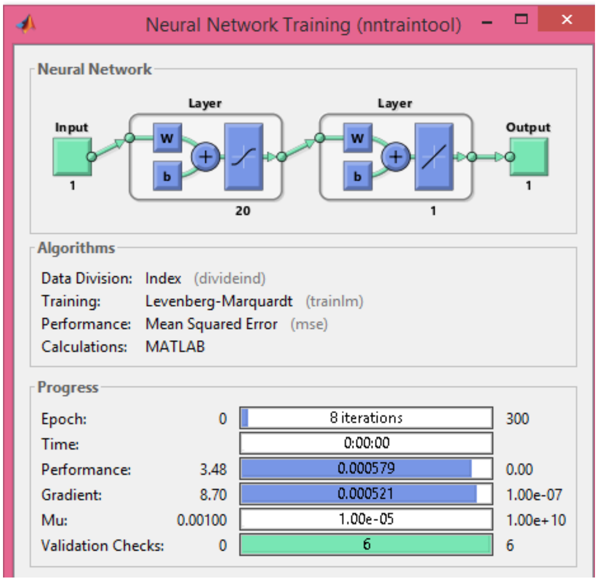

Fig. 6. Neural network training using LM method

3) SIMULATION RESULTS

This section provides simulation results of function approximation using ANN. Figure 4 and Figure 5 shows the function approximation of sine wave using MLP. Figure 6 shows the setting of neural network training algorithm.

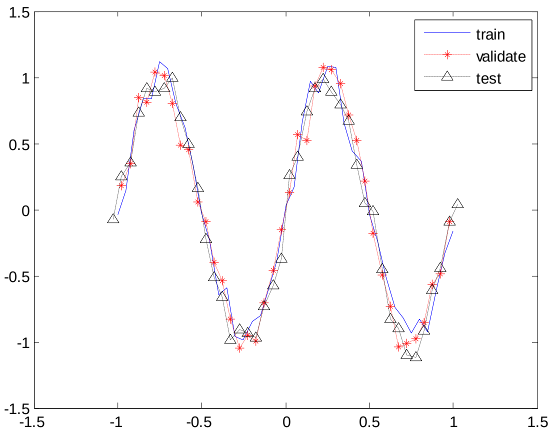

Fig. 4. Approximation of sine wave using MLP

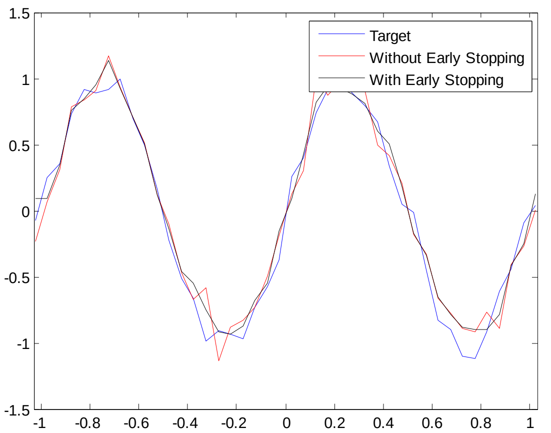

Fig. 5. Approximation of sine wave with noise using MLP

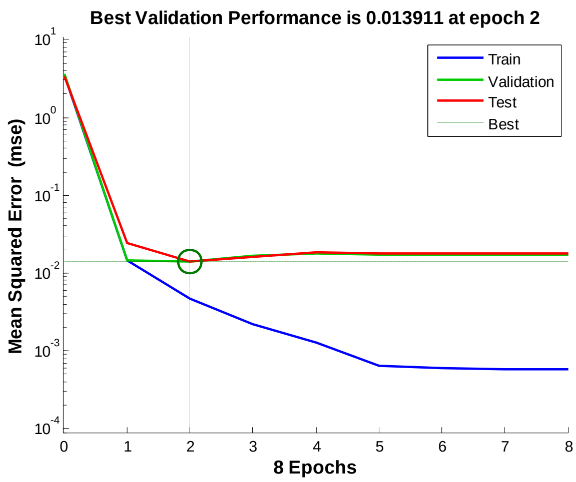

Fig. 7. Plot for MSE vs Epoch for sine wave approximation

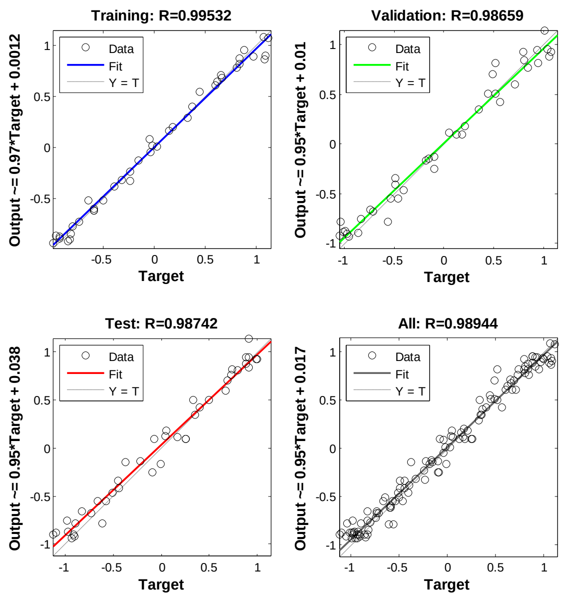

Fig. 8. Regression analysis

Figure 7 shows the plot for MSE and epoch for sine wave approximation and Figure 8 shows the regression analysis of back propagation neural network. Figure 9 and Figure 10 shows the function approximation of addition of sine wave. Figure 11 shows the graph between error and epoch. Figure 12 shows the setting of back propagation algorithm. Figure 13 shows the regression analysis. Table I shows some of the standard test functions.

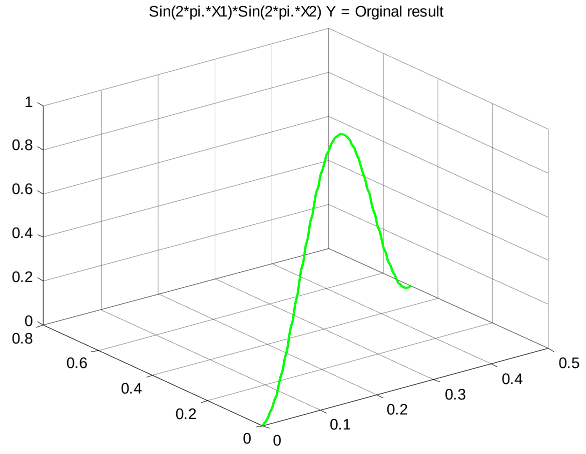

Fig. 9. Function approximation of addition of sine wave in 3D view

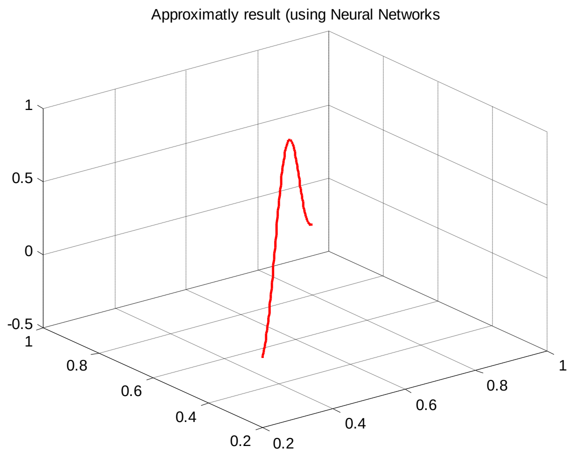

Fig. 10. Approximation results

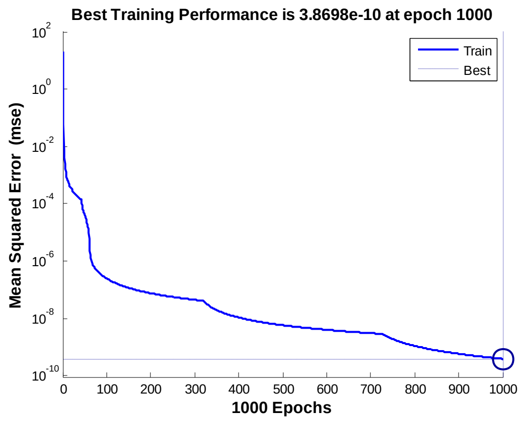

Fig. 11. Graph for epoch and error

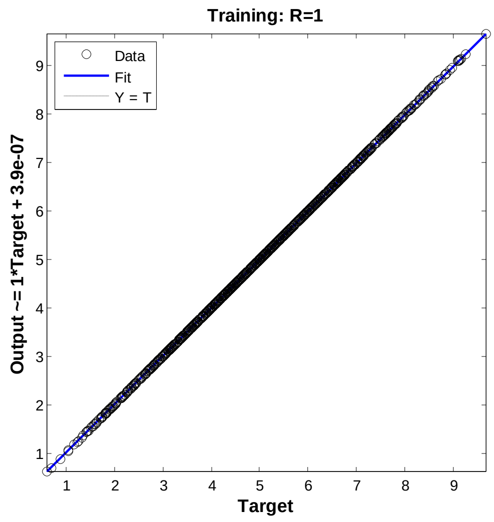

Fig. 12. Regression analysis

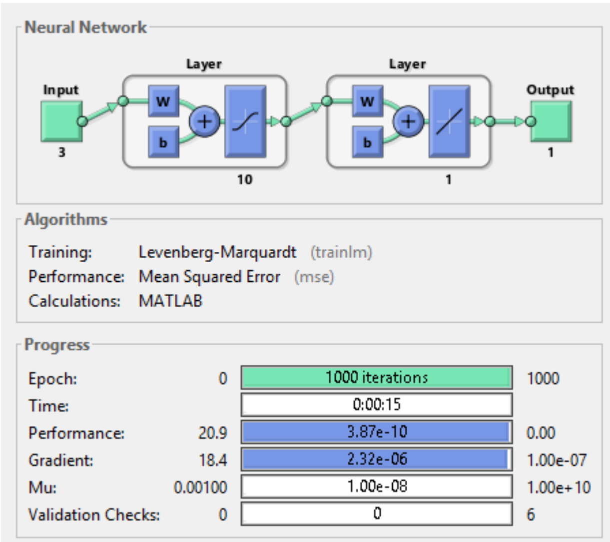

Fig. 13. Setting of neural network

Table I: Test Functions for optimization

Function	Formula	Search Domain
Rastrigin	$f(x) = An + \sum_{i=1}^{n} \left[x_i^2 - A\cos(2\pi x_i) \right]$	$A = 10$ $x_i \in [-5.12, 5.12]$
Ackley	$f(x,y) = -20e^{\left[-0.2\sqrt{0.5(x^2+y^2)}\right]} - e^{\left[0.5(\cos 2\pi x + \cos 2\pi y)\right]} + e + 20$	$-5 \le x, y \le 5$
Sphere	$f(x) = \sum_{i=1}^{n} x_i^2$	$-\infty \le x_i \le \infty$ $1 \le i \le n$
Rosenbrock	$f(x) = \sum_{i=1}^{n-1} \left[100\left(x_{i+1} - x_i^2\right)^2 + \left(x_i - 1\right)^2 \right]$	
Booth	$f(x,y) = (x + 2y - 7)^2 + (2x + y - 5)^2$	$-10 \le x, y \le 10$
Schaffer N2	$f(x,y) = 0.5 + \frac{\sin^2\left(x^2 - y^2\right) - 0.5}{\left[1 + 0.001\left(x^2 + y^2\right)\right]^2}$	$-100 \le x, y \le 100$
Schaffer N4	$f(x,y) = 0.5 + \frac{\cos^2\left[\sin\left(\left\|x^2 - y^2\right\|\right)\right] - 0.5}{\left[1 + 0.001\left(x^2 + y^2\right)\right]^2}$	

4) CONCLUSION

This paper provides a comprehensive study of different function approximation using artificial neural network. Different single and multi-variable functions are considered and are approximated using MLP and back propagation algorithm. Different test functions for optimization has also been used.

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