

NON DARCIAN APPROACH TO MHD FLOW OF NEWTONIAN FLUID THROUGH HIGHLY POROUS MEDIUM BETWEEN TWO SEMI-INFINITE PARALLEL PLATES

Dr. Shiva Shanker K.

Assistant Professor, Dept of Mathematics
Kakatiya Institute of Technology & Science,
Warangal, Telangana, India.

Abstract—The aim of this paper is to investigate the flow of incompressible viscous fluid between two semi-infinite parallel plates. The space between the parallel plates is filled with highly porous medium. Brinkmann equation is applied to study the fluid flow. Transverse magnetic field is applied perpendicular to the length of the plates. The velocity and flow rate of the fluid are obtained in elegant forms. The effect of magnetic parameter on velocity and flow rate of the fluid is investigated. The results are graphically represented.

Index Terms— Incompressible viscous fluid, Porous medium, magnetic field, permeability parameter.

1) INTRODUCTION

The study of flow through porous medium assumed importance because of the interesting applications in the diverse fields of science, Engineering and Technology. The practical applications are in the percolation of water through soil, extraction and filtration of oils from wells, the drainage of water, irrigation and sanitary engineering and also in the inter disciplinary fields such as biomedical engineering etc. The lung alveolar is an example that finds applications in an animal body. The classical Darcy's law Muskat [1] states that the pressure gradient pushes the fluid against the body forces exerted by the medium which can be expressed as

$$\vec{V} = -\left(\frac{k}{\mu}\right)\nabla P$$

The law gives good results in the situations when the flow is uni-directional or the flow is at low speed. In general, the specific discharge in the medium need not be always low. As the specific discharge increases, the convective forces get developed and the internal stress generates in the fluid due to its viscous nature and produces distortions in the velocity field. In the case of highly porous medium such as fiber glass, paper of dandelion the flow occurs even in the absence of the pressure gradient.

Modifications for the classical Darcy's law were considered by the Beavers and Joseph [2], Saffman [3] and others. A generalized Darcy's law proposed by Brinkmann [4] is given by

$$O = -\nabla P - \left(\frac{\mu}{K}\right)\vec{v} + \mu \nabla^2 \vec{v}$$

Dr.K.Shiva shanker, Department of Mathematics, Kakatiya Institute of Technology & Science, Warangal, Telangana, Warangal, India, 9704989229.

where μ and K are coefficients of viscosity of the fluid and permeability of the porous medium.

The applications of flows through porous medium bears wide spread interest in Geophysics, biology and medicine. In many of these areas the flow consists of more than one phase, such type of flows find applications in the inter disciplinary fields such as bio-medical engineering etc., the flow of blood is one such application. The blood may be represented as Newtonian fluid and the flow of the blood is in two layered. Lightfoot [5], Shukla *et al.* [6] and Chaturani [7]. Bird *et al.* [8] found an exact solution for the laminar flow of two immiscible fluids between two parallel plates. Bhattacharya [9] discussed the flow of immiscible fluids between rigid plates with a time dependent pressure gradient. Vajravelu *et al.* [10] have discussed the effect of magnetic field on unsteady flow of two immiscible conducting fluids between two permeable beds. Transient coquette flow in a rotating non-Darcian porous medium parallel plate configuration is studied by Anwarbeg *et al.* [11] Kandryzakaria *et al.* [12] discussed magneto hydrodynamics instability of interfacial waves between two immiscible cylindrical fluids. Earlier Narasimhacharyulu *et al.* [13] studied the problem of two phase fluid flow between parallel plates with porous lining and Narasimhacharyulu *et al.* [14] examined the flow of micropolar fluid between parallel plates coated with porous lining. Narasimhacharyulu *et al.* [15] studied two phase flow between two semi infinite parallel plates under transverse magnetic field.

In this present paper we are considering the fluid flow between two parallel plates, the space between the plates is filled with highly porous medium. Transverse magnetic field is applied perpendicular to the length of the plates. The results are graphically represented.

2) MATHEMATICAL FORMULATION OF THE PROBLEM

The flow of an incompressible viscous liquid is considered between two semi infinite parallel plates given by $y = \pm h$. The space between the plates is filled with porous region. The coordinate system is taken such that x-axis lies parallel to the length of the plates and y-axis perpendicular to the length of the plates. The fluid flows under a constant pressure gradient.

$$G = -\frac{\partial p}{\partial x}$$

A transverse magnetic field is applied perpendicular to the flow of the fluid. The induced magnetic effect is negligible in comparison with the transverse magnetic field due to low magnetic Reynold's number, as a result of slightly conducting fluid [15]. Further the electric force E given by ohm's law $J = (E + V \times B)$ when $B = (H_0, 0, 0)$ and the electrical conductivity is assumed to be a null vector for the simplicity of the problem.

The velocity of the fluid $\vec{V} = (u, 0, 0)$ satisfies the equation of continuity, the physical quantities depend on y only. The equation of motion is given by

$$\frac{d^2u}{dy^2} - \frac{u}{k} - \frac{\sigma\beta_0^2}{\mu}u = -\frac{G}{\nu} \quad \dots(2.1)$$

Where $-h < y < h$

and $G = -\frac{\partial p}{\partial x}$ is a constant pressure gradient, in the x direction, ν is coefficient of viscosity of the fluid, k is permeability of the porous medium. using the following non-dimensional quantities

$$u^* = \frac{uh}{\nu}, y^* = \frac{y}{h}, G^* = \frac{Gh^3}{\nu}, \beta^2 = \frac{h^2}{K}, M^2 = \frac{\sigma\beta_0^2 h^2}{\mu} \quad \dots (2.2)$$

After removing *, the non-dimensional form of equation of motion is

$$\frac{d^2u}{dy^2} - \alpha^2 u = -\frac{G}{\nu}; \quad -1 < y < 1 \quad \dots(2.3)$$

where

$$\alpha^2 = \beta^2 + M^2, \quad \beta^2 = \frac{h^2}{K}, \quad M^2 = \frac{\sigma\beta_0^2 h^2}{\mu}$$

The boundary conditions are given by

$$u = 0 \quad \text{at} \quad y = \pm 1 \quad \dots (2.4)$$

3) SOLUTION OF THE PROBLEM

Solving the equations (2.3) employing boundary conditions (2.4) we get

$$u = \frac{G}{\nu M^2} \left(1 - \frac{\cosh My}{\cosh M} \right) \quad \dots(2.5)$$

$$\text{Flow rate } Q = \int_{-1}^1 u dy$$

$$\text{Flow rate } Q = \frac{2G}{\nu M^2} \left(1 - \frac{\text{Tanh}M}{M} \right) \quad \dots(2.6)$$

where

$$\alpha^2 = \beta^2 + M^2, \quad \beta^2 = \frac{h^2}{K}, \quad M^2 = \frac{\sigma\beta_0^2 h^2}{\mu}$$

4) RESULT AND CONCLUSION

MHD flow of incompressible viscous liquid is studied between two semi-infinite parallel plates filled with highly porous medium.

Fig. 1 shows the velocity in the porous medium decreasing with increasing magnetic field values.

From Fig. 2 it is observed that increasing values of magnetic parameter the values of flow rate are decreasing and also thickness of the porous medium is increasing, the flow rate decreasing.

As viscosity of the fluid is increasing the velocity of the fluid is decreasing. Further it is also observed that as viscosity of the fluid is increasing, the flow rate of the fluid is decreasing.

The results of the problem have great importance to the petroleum engineer concerned with the movement of oil, gas and water through reservoir of an oil or gas field. Beyond this, the results of the present problem are widely applicable in soil mechanics, water purification and power metallurgy.

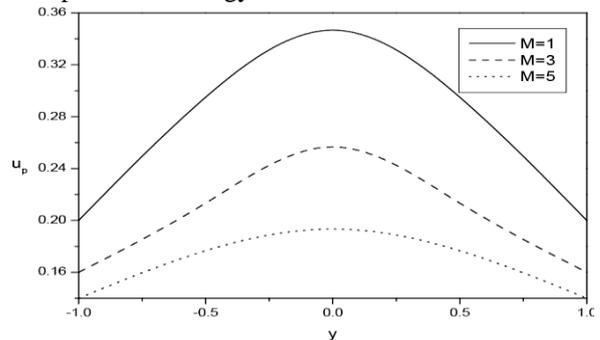


Fig. 1 : Variation of u with different values of M

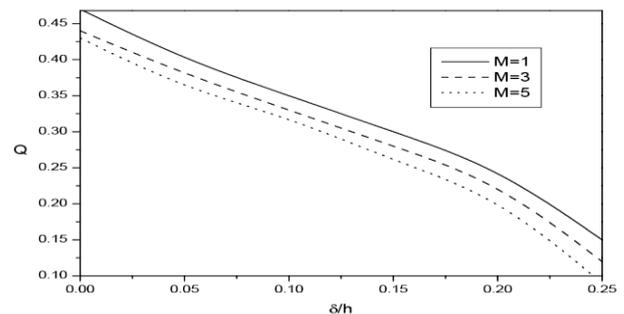


Fig. 2 : Flow rate for different values of magnetic parameter M

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Dr. K. Shiva shanker received Ph.D. from Kaktiya University, Warangal in 2014, received M.Sc.(Applied Mathematics) from NIT, Warangal in 1998. Presently working as senior Asst.Professor in KITS, Warangal and research area of interest is Fluid Mechanics, Numerical and Advanced optimization techniques.