

Design of Phase Lead and Phase Lag Compensator for Pitch Controller of a BOEING Aircraft using Bode diagram

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Abstract—This paper describes design of phase lead and phase lag compensator for pitch controller of aircraft. Firstly, the mathematical model of a pitch controller is calculated. Then all of design parameters of compensator and design steps are implemented using MATLAB without manual calculation except the gain corresponding to get desired steady state error. Depending on the calculated mathematical model, required specifications for Pitch Controller, phase lead and phase lag compensator can be designed. Finally, the result of phase lead and phase lag compensator is presented by using MATLAB-GUI. The proposed phase lead and phase lag compensator design can be used for any system that the mathematical model have been known to meet desired specifications.

Keywords— mathematical model, phase lead and phase lag compensator, desired specifications, MATLAB-GUI

1) INTRODUCTION

The phase-lead and phase-lag controllers are often preferred in the design of control systems. Although the phase-lead and phase-lag controllers are similar to each other in terms of general structure, they are different in terms of placement of the zeros and poles. The purpose of phase lead and phase lag compensator design in the frequency domain generally is to satisfy specifications on steady-state accuracy and phase margin. A phase margin specification can represent a requirement on relative stability due to desired transient response characteristics that have been translated from the time domain into the frequency domain. If the compensation is to be performed by a single-stage compensator. The gain crossover frequency and closed-loop bandwidth for the lag-compensated system will be lower than for the uncompensated plant (after the steady-state error specification has been satisfied), so the compensated system will respond more slowly in the time domain. Lead and lag compensators using the Bode plot method are designed to improve the phase margin and satisfy steady state tracking specs. The Bode plot analysis will give us a different perspective on why the pole and zero of the lag compensator are usually chosen to be very small.

Each component of the control system is defined in terms of its transfer function. The simplest form of the control system is an open-loop system in which the control action is independent of the output. In order to obtain a more accurate control system, the connection between the output and input

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signal must be established, forming a closed-loop feedback control system composed of a comparator, a controller and a control object. The controller design process called controller synthesis must assure dynamical stability and wanted dynamical behaviour of a system expressed through response characteristics of a system.

The equations governing the motion of an aircraft are a very complicated set of six nonlinear coupled differential equations. However, under certain assumptions, they can be decoupled and linearized into longitudinal dynamics. In this paper it will be designed an autopilot that controls the pitch of an aircraft. The lead and lag compensator technique is one of the design analysis tools used in this paper for controller synthesis of a BOEING aircraft.

This paper consists of mathematical model, design specifications, design procedures of lead and lag compensator, stability, results and conclusions.

2) A MATHEMATICAL MODEL OF AIRCRAFT

The first step in control system analysis is defining a mathematical model which contains all the essential information about the physical properties of a given system, so that a physical system can be carried out on its mathematical model. The mathematical model of a physical system is described by differential equations.

The basic coordinate axes and forces acting on an aircraft are shown in the figure given below.

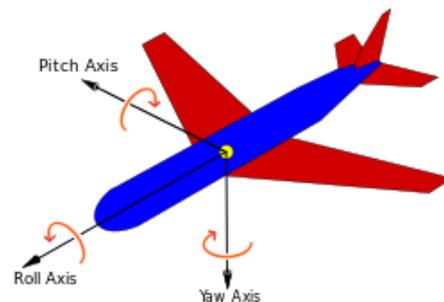


Fig. 1. Three axes definition of aircraft [9]

The pitch axis (also called transverse or lateral axis) has its origin at the centre of gravity and is directed to the right, parallel to a line drawn from wingtip to wingtip. Motion about this axis is called pitch. A positive pitching motion raises the nose of the aircraft and lowers the tail. The elevators are the primary control of pitch.

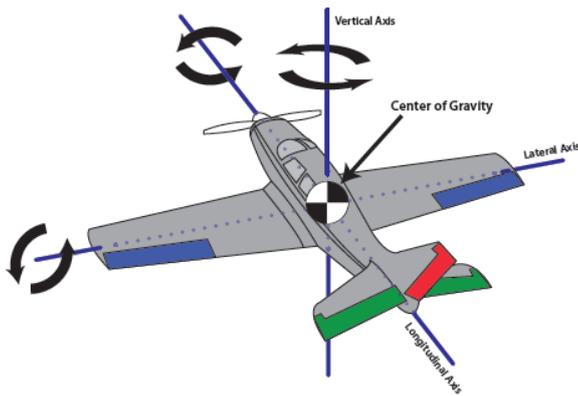


Fig. 2. Definition of the centre of gravity of aircraft[9]

Regardless of the type of aircraft, there are three axes upon which it can move: Left and Right, Forwards and Backwards, Up and Down. In aviation though, their technical names are the lateral axis, longitudinal axis and vertical axis. The lateral axis runs from wing tip to wing tip. The aircraft pitches around this axis (Fig. 3).

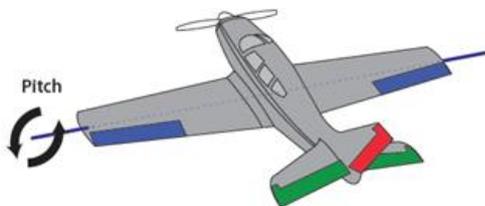


Fig. 3 The lateral axis[9]

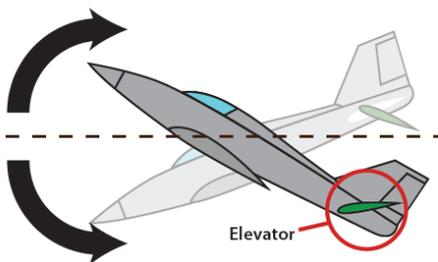


Fig. 4 Elevator and pitch movement[9]

In order for an aircraft to reach its destination, the forces of flight have to be precisely manipulated. To do this, the aircraft has control surfaces Fig. 4 which can direct airflow in very specific ways.

Elevator is usually located on the tail of the aircraft and serves two purposes. The first is to provide stability by producing a downward force on the tail. Airplanes are traditionally nose-heavy and this downward force is required to compensate for that. The second is to direct the nose of the aircraft either upwards or downwards, known as pitch, in order to make the airplane climb and descend.

The aircraft is in steady-cruise at constant altitude and velocity; thus, the thrust, drag, weight and lift forces balance each other in the x - and y -directions. It is assumed that a change in pitch angle will not change the speed of the aircraft under any circumstance (unrealistic but simplifies the problem a bit). Under these assumptions, the longitudinal equations of motion for the aircraft can be written as modelling equation in following form [2].

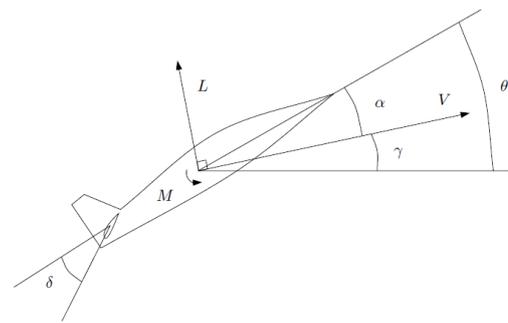


Fig.5 Variables parameters of aircraft

$$\dot{\alpha} = \mu\Omega\sigma[-(C_L + C_D)\alpha + \frac{1}{(\mu - C_L)}q - (C_W \sin \gamma)\theta + C_L] \quad (1)$$

$$\dot{q} = \frac{\mu\Omega}{2I_{yy}}[[C_M - \eta(C_L + C_D)]\alpha + [C_M + \sigma C_M(1 - \mu C_L)]q + (\eta C_W \sin \gamma)\delta] \quad (2)$$

$$\dot{\theta} = \Omega q \quad (3)$$

Laplace transform of the above modelling equations are shown below,

$$sA(s) = -0.313A(s) + 56.7Q(s) + 0.232\Delta(s) \quad (4)$$

$$sQ(s) = -0.0139A(s) + 0.0203\Delta(s) \quad (5)$$

$$s\Theta(s) = 56.7Q(s) \quad (6)$$

where

α = Angle of attack.

q = Pitch rate.

θ = Pitch angle.

δ = Elevator deflection angle.

These values are taken from the data from one of Boeing's commercial aircraft [2].

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.313 & 56.7 & 0 \\ -0.0139 & -0.426 & 0 \\ 0 & 56.7 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} 0.232 \\ 0.0203 \\ 0 \end{bmatrix} [\delta] \quad (7)$$

The system output is pitch angle θ and the input is the elevator deflection angle δ .

Matlab code for transfer function ($\theta(s)/\Delta(s)$)

$$A = [-0.313 \ 56.7 \ 0; -0.0139 \ -0.426 \ 0; 0 \ 56.7 \ 0];$$

$$B = [0.232; 0.0203; 0];$$

$$C = [0 \ 0 \ 1];$$

$$D = [0];$$

$$[\text{num}, \text{den}] = \text{ss2tf}(A, B, C, D);$$

$$p_pitch = \text{tf}(\text{num}, \text{den})$$

$$p_pitch =$$

$$\frac{1.151 s + 0.1774}{s^3 + 0.739 s^2 + 0.9215 s}$$

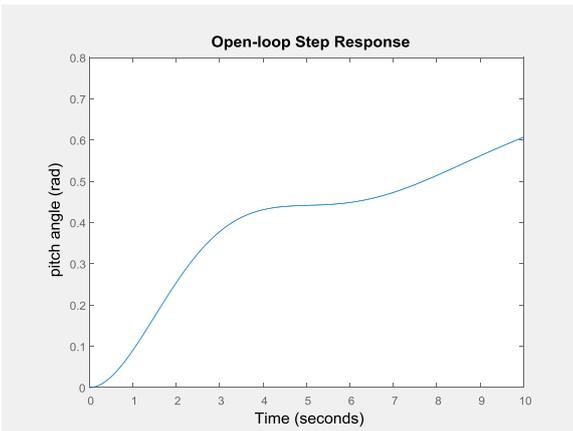


Fig. 6 The response of open loop transfer of pitch controller

Fig.6 show the response of open loop transfer of pitch controller.

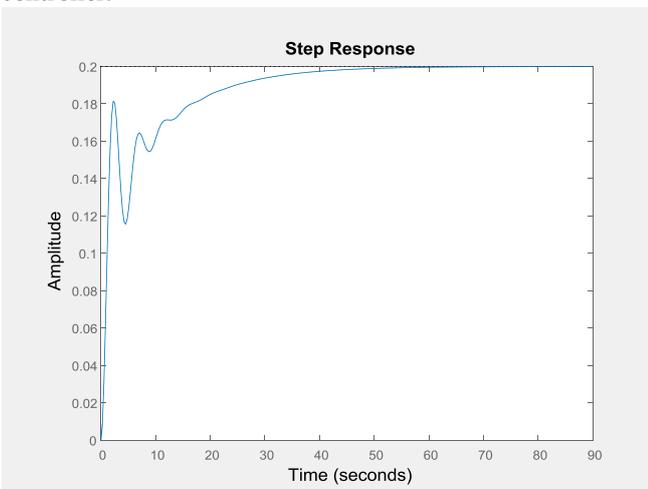


Fig. 7 The response of closed loop transfer function

Fig. 7 show the response of closed loop transfer function . From examining Fig.6 and Fig. 7, it can be seen that dynamical behaviour of an aircraft is not acceptable considering overshoot, rise time, settling time and steady-state error values, and must be designed using phase lead and phase lag compensator with Bode diagram.

3) DESIGN SPECIFICATIONS

The next step is to choose some design criteria. The design specifications are the following.

For a step reference of 0.2 radians, the design specifications are the following.

- 1) Overshoot less than 10% ,Phase margin > 60°
- 2) Rise time less than 2 seconds
- 3) Settling time less than 10 seconds
- 4) Steady-state error less than 2%

4) DESIGN PROCEDURE

1) The design procedure a for a phase -lead network on the Bode diagram is as follows:[8]

- Evaluate the uncompensated system phase margin when the error constant are satisfied.
- Allowing for a small amount of safety ,determine the necessary additional phase lead Φ_m .
- Evaluate α from

$$\alpha = \frac{1 + \sin \Phi_m}{1 - \sin \Phi_m} \quad (8)$$

- Evaluate $10 \log \alpha$ and determine the frequency where the uncompensated magnitude curve is equal to $-10 \log \alpha$ dB.
- Calculate the pole $p = \omega_m \sqrt{\alpha}$ and $z = p/\alpha$.
- Draw the compensated frequency response, check the resulting phase margin , and repeat the steps if necessary. Finally, for an acceptable design, raise the gain of the amplifier in order to account for attenuation $(1/\alpha)$.

2) The design procedure for a phase- lag network on the Bode diagram is as follows:

- Obtain the Bode diagram of the uncompensated system with the gain adjusted for the desired error constant.
- Determine the phase margin of the uncompensated system and, if it is insufficient ,proceed with the following steps.
- Determine the frequency where the phase margin requirement would be satisfied if the magnitude curve crossed the 0-dB line at this frequency ω_c' . (Allow for 5° phase lag from the phase-lag network when determining the new crossover frequency.)
- Place the zero of the compensator one decade below the new crossover frequency ω_c' , and thus ensure only 5° of additional phase lag at ω_c' due to the lag network.
- Measure the necessary attenuation at ω_c' to ensure that the magnitude curve crosses at this frequency.
- Calculate α by noting that the attenuation introduced by the phase-lag network is $-20 \log \alpha$ at ω_c' .

All of the design procedures are implement using matlab codes without manual calculation to satisfy phase margin and steady state error specifications.

The pitch controller to be controlled is described by the transfer function

$$p_pitch =$$

$$\frac{1.151 s + 0.1774}{s^3 + 0.739 s^2 + 0.9215 s}$$

This is a type 1 system ,so the closed loop system will zero steady state error for a step input and a nonzero finite steady state error for a ramp input .

To satisfy steady-state error 2%,the gain k was calculated as follow

$$e_{ss} = \frac{A}{K_V}, K_V = 10$$

$$K_V = \lim_{s \rightarrow 0} s k G(s) = k \frac{0.1774}{0.921} = 10$$

$$k = 52$$

5) RESULT

The simulation program is implemented with two GUI file. The first GUI show the information of this paper. If the user run the cover window.fig ,MATLAB GUI generate Fig.8 as follow.



Fig.8. Main GUI design of the system

The user can simulate the phase lead or phase lag program by clicking these two buttons displayed in Fig.8. It consists of two axes ,one is for Bode diagram with gain margin and phase margin and another one is for the response of the system with information to display.

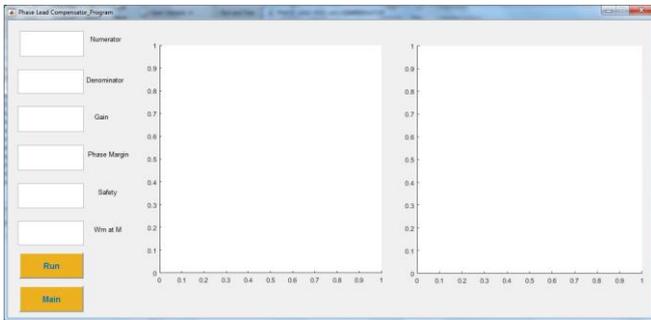


Fig.9. Simulation of Phase Lead Compensator

The above Fig.9 depicted for phase lead compensator. The user enters the numerator and denominator of transfer function and gain k. Then click on Run button the uncompensated system with GM and PM will appear as shown in Fig.10.

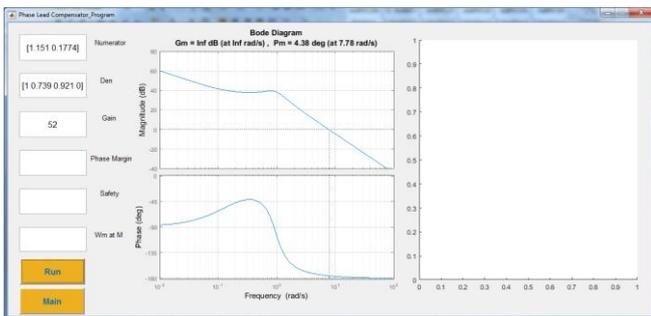


Fig.10. Uncompensated results by numerator, denominator and Gain

If the desired PM does not meet ,desired PM and safety enter the display will show as shown in Fig.11.

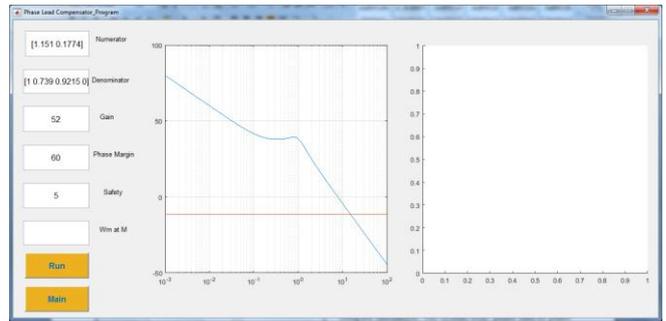


Fig.11 Value of ω_{cm} at intersection point

Finally if the user enter ω_{cm} that intersection pt between the red line and curve final result will be generated Fig.12 as follow.

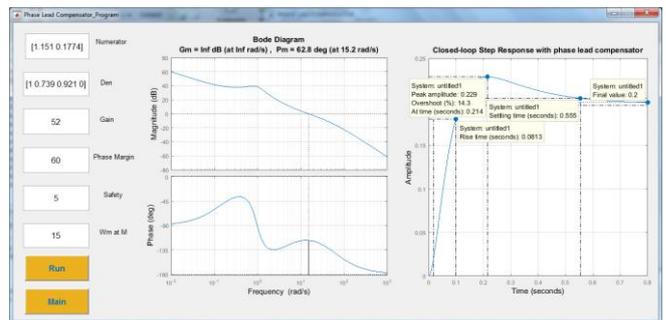


Fig.12. Final result of phase lead compensated design.

$$G_c = \frac{s + 3.932}{s + 57.23}$$

RiseTime: 0.0813
SettlingTime: 0.5549
SettlingMin: 0.1831
SettlingMax: 0.2287
Overshoot: 14.3294
Undershoot: 0
Peak: 0.2287
PeakTime: 0.2138

When main button is clicked cover window will be displayed. If the phase lead or phase lag program is clicked the procedure as shown Fig.13 will be done.

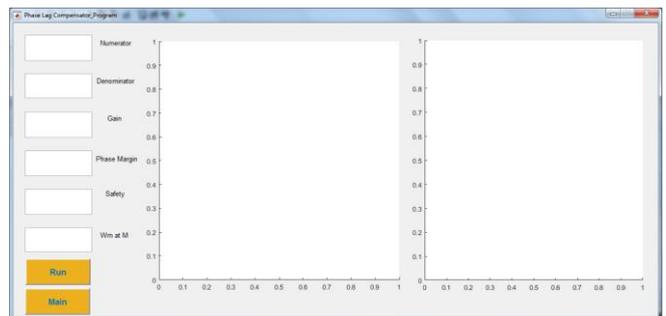


Fig.13. Simulation of Phase Lag Compensator

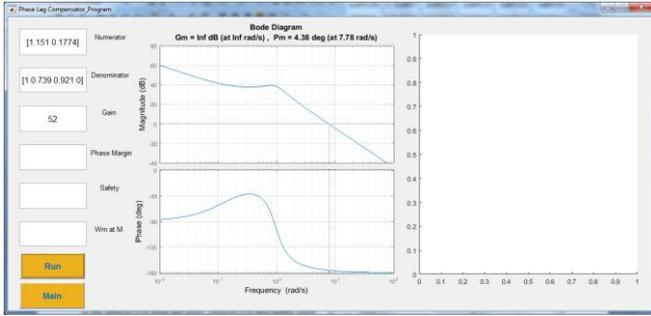


Fig.14. Uncompensated results by numerator, denominator and Gain

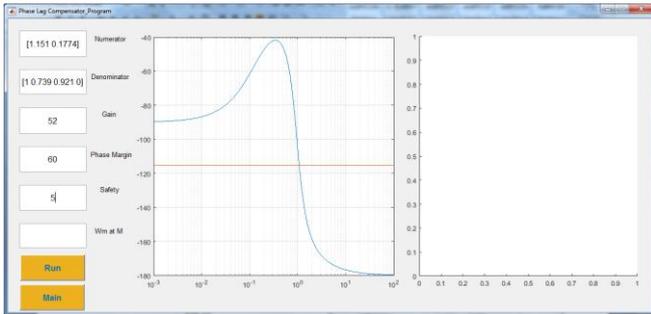


Fig.15. Value of ω_{cm} at intersection pt

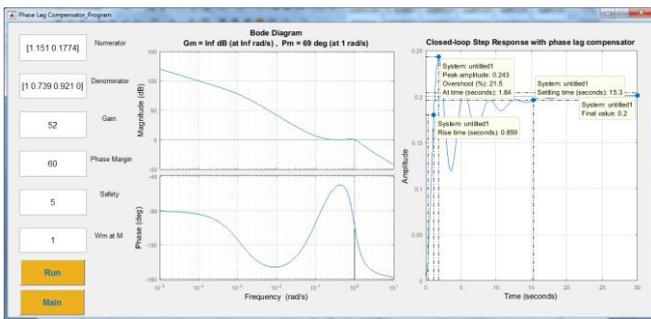


Fig.16. Final result of phase lag compensated design.

Above Fig.13 , Fig.14, Fig.15 and Fig.16 are for phase lag compensator.

$G_c =$

$$\frac{s + 0.1}{s + 0.001227}$$

when $\alpha = \omega_p/\omega_z$ is reduced, result is as follow:
ans =

- RiseTime: 0.8589
- SettlingTime: 15.2691
- SettlingMin: 0.1191
- SettlingMax: 0.2430
- Overshoot: 21.5054
- Undershoot: 0
- Peak: 0.2430
- PeakTime: 1.8428

6) CONCLUSION

The open loop and closed loop of pitch controller does not meet the desired specification. In order to increase the accuracy and efficiency ,phase lead or phase lag compensator must be used. It can also reduced the design time and manual complex calculation. The system with phase lead or phase lag compensator completely satisfied desired PM > 60° and steady- state error. Phase lead compensator increase system bandwidth and phase lag compensator decrease system bandwidth. The following tables provide a comparison between phase lead and phase lag compensator in this paper. For phase lead compensator ,the gain must be adjusted to get more accurate in time domain specifications. For phase lag compensator, if the ratio of ω_p/ω_z is $10 < \omega_p/\omega_z < 100$,two stages compensator design must be considered and adjustment by gain is not sufficient for these systems. By using this two stages of compensation design, system will be more accurate in time domain specifications .

For frequency response (desired PM >60 deg)

Table.1 Frequency response

	pitch controller	lead compensated	Lag compensated
PM	4.38 deg	60.5deg	69 deg
GM	∞	∞	∞
Kv	10	10	10

Table.2 Time domain

controller	G_{c_lead}	G_{c_lag}	G_{c_lag}	G_{c_lag}
Over shoot	13.43%	8.8%	41.4 %	21.55
Settle time	0.527 sec	42 sec	28.63sec	15.26
RiseTime	0.0813	1.17sec	0.632 sec	0.859
Error constant	10	20	50	10

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