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Parametric method for spectral estimation of clutter based on autoregressive model

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Abstract— **The purpose of this paper is to solve the problem of spectral estimation of clutter by parametric method based on the modified Autoregressive (AR) model. If limited by the order of the AR model while estimating the spectrum of clutter, the proposed method has increased the accuracy by up to 4 times compared with parametric method based on the conventional AR model.**

Index Terms— **autoregressive model, clutter, parametric method, spectral estimation.**

1) INTRODUCTION

While in operation, radio systems (e.g., targeting radars, navigational radars, radio measurement devices, and so on) are required to ensure optimal performance in adherence to technical limitations and they are always affected by interference under practical conditions. Among the interference, clutter has an unexpected impact on the working efficiency of radio systems, which reduces the probability of detection, misleads the parameters of a target and makes errors in navigation indicators [1], [4]. Therefore, improving the efficiency of clutter reduction in the current radio systems is always considered as a top-priority task. For coping with interferences in general and clutter in particular, the phase of estimating the parameters, the power and the power spectral density of interference plays a particularly important role, which determines to eliminate interference effects of the next phase. One of the approaches is to apply classical methods such as Periodogram and Correlogram [2], [5], [6], or nonparametric methods such as MUSIC and EV [6] to estimate the power spectral density. Although these methods have obtained certain results, they still have many drawbacks such as the largely remained errors of estimation with respect to the limited number of observation patterns, the cost of memory usage, and the optimization of processing time. Currently, modern methods for estimating the power spectral density of clutter such as parametric methods: Yula-Walker, Berg, Covariance, Modified covariance, Levinson, etc [2], [6], [8] have overcome a number of limitations of classical methods and nonparametric methods. The paper proposed a more accurate method of estimating the spectrum of clutter in the case of the low-order AR model, i.e., it limits the correlation coefficient of clutter corresponding to the conventional AR model.

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2) SETTING OUT THE PROBLEM

In fact, clutter impacting on radio systems commonly has a narrow spectrum [1], [4], so it is appropriate to use the AR model to describe them.

The AR model is constructed in Figure 1 [2], where x_n is the nth input sample, y_n is the nth output sample, z^{-1} denotes the linear delay in a T cycle**,** × denotes the multiplier and Σ denotes the adder.

The parameters of the AR model are found by solving a set of Yule-Walker linear equations [2], based on the calculation of the inverse values of the clutter correlation matrix R sized as $((q+1)\times(q+1))$. The Yule-Walker equations are written as

$$\mathbf{R}\begin{bmatrix}1\\ \cdots \\ \mathbf{a}\end{bmatrix}=\begin{bmatrix}\sigma^2\\ \cdots\cdots \\ \mathbf{0}\end{bmatrix}, \quad (1)$$

where **a** is the vector of the model parameters, σ^2 is the variance of white noise, 0 is the column vector of $(q\times1)$ elements, whose values equal 0. From (1), it can be expanded as

$$\begin{bmatrix} R_1 & R_0 & R_1^* & \cdots & R_{q-1}^* \\ \vdots & R_1 & R_0 & \ddots & \vdots \\ R_{q-1} & \vdots & \ddots & \ddots & R_1^* \\ R_q & R_{q-1} & \cdots & R_1 & R_0 \end{bmatrix}\begin{bmatrix}1\\ \mathbf{a}\end{bmatrix}=\mathbf{0}, \quad (2)$$

where R_k is the correlation coefficient of the clutter, for k = 0,1, 2, ..., q. The * denotes the conjugate of a complex number.

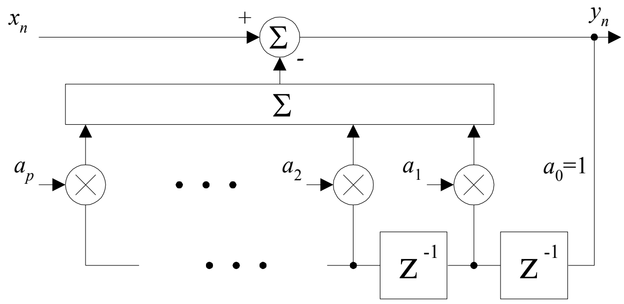

Figure 1. The structure of the AR model

In order to calculate the coefficients of the vector **a** for the AR model, the expression (2) is developed as

$$\begin{bmatrix} R_1 \\ \vdots \\ R_{q-1} \\ R_q \end{bmatrix} + \begin{bmatrix} R_0 & R_1^* & \cdots & R_{q-1}^* \\ R_1 & R_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & R_1^* \\ R_{q-1} & \cdots & R_1 & R_0 \end{bmatrix} \mathbf{a} = \mathbf{0}. \quad (3)$$

Alternatively, the equation (3) can be rewritten as

$$\mathbf{Ra} = -\mathbf{r}, \quad (4)$$

where $\mathbf{r}^T = [R_1; R_2; \ldots; R_q]$ is the vector of clutter autocorrelation coefficients and "T" denotes matrix transposition operator. Thus, from (4) we can calculate the vector of the coefficient a of the AR model as follow:

$$\mathbf{a} = -\mathbf{R}^{-1}\mathbf{r}. \quad (5)$$

The problem is that when the order of the AR model is low (q is small), the coefficient of the corresponding vector **a** is not large enough. Then, it leads to decreasing the accuracy of spectral estimation of the clutter because the clutter autocorrelation coefficients $R_{j,\,k}$ with their index $|j\text{-}k| > q$ will not affect the process of creating spectrum of the clutter, making the assessment of the clutter correlation coefficients decrease accuracy.

3) SOLUTION TO THE PROBLEM

In order to increase the accuracy of the vector elements of the AR model of clutter in (5), the proposed method takes into account the correlation coefficients $R_{j,\,k}$ with the index $|j\text{-}k| > q$ while still maintaining the same order of AR model. It means finding the optimal vector that satisfies the least squares standard E of the error vector ε, described as follow:

$$\mathrm{E} = \boldsymbol{\varepsilon}^H \boldsymbol{\varepsilon}, \text{ with } \boldsymbol{\varepsilon} = \tilde{\mathbf{r}} - \tilde{\mathbf{R}}\tilde{\mathbf{a}}, \quad (6)$$

where $\tilde{\mathbf{a}}$ is the coefficient vector of the transformed AR model, $\tilde{\mathbf{R}}$ is the correlation matrix sized $[(c+q)\times q]$ with c is the number of the added correlation coefficients, $\tilde{\mathbf{r}}$ is the column vector of length (q + c) and the symbol "H" denotes the operator of transposition and complex conjugate.

When E obtains the minimum value, i.e., the expression (6) can be written as

$$\mathrm{E} = (\tilde{\mathbf{r}} - \tilde{\mathbf{R}}\tilde{\mathbf{a}})^H (\tilde{\mathbf{r}} - \tilde{\mathbf{R}}\tilde{\mathbf{a}}) \to \min_{\tilde{\mathbf{a}} \in C^q}, \quad (7)$$

where C^q is the q-dimensional complex coordinate space. In order to find the value of the vector that satisfies (7), we compute the second derivative of E with respect to $\tilde{\mathbf{a}}$ [7], that is,

$$\begin{aligned} d\mathrm{E}/d\tilde{\mathbf{a}} &= d[(\tilde{\mathbf{r}} - \tilde{\mathbf{R}}\tilde{\mathbf{a}})^H (\tilde{\mathbf{r}} - \tilde{\mathbf{R}}\tilde{\mathbf{a}})]/d\tilde{\mathbf{a}} \\ &= -2\tilde{\mathbf{R}}^H \tilde{\mathbf{r}} + 2\tilde{\mathbf{R}}^H \tilde{\mathbf{R}}\tilde{\mathbf{a}}. \end{aligned} \quad (8)$$

Thus, $\tilde{\mathbf{a}}$ is the vector satisfying the equation when assuming the right side of (8) to equal 0, then the vector value is obtained as

$$\tilde{\mathbf{a}} = -(\tilde{\mathbf{R}}^H \tilde{\mathbf{R}})^{-1} \tilde{\mathbf{R}}^H \tilde{\mathbf{r}}. \quad (9)$$

In order to confirm that the vector $\tilde{\mathbf{a}}$ calculated in (9) satisfies the request of (7), the second derivative of E must be not negative [7]. It is obvious because it can be easily seen that

$$d^2\mathrm{E}/d\tilde{\mathbf{a}}^2 = 2\tilde{\mathbf{R}}^H \tilde{\mathbf{R}} > 0 .$$

4) ANALYSIS AND SIMULATION

Figure 2 shows the results of the spectral estimation of the clutter using the parameter method in the presence of the relatively uncorrelated noise power $P_n = 10^{-6}$. The curve 1 describes the power spectral density of the clutter in the ideal case, i.e., the order of the AR model is large (q = 100) and the number of observed samples is unlimited. This spectrum is used for the researching methods to refer to. The curve 2 describes the power spectral density of the clutter by the AR model (its order q = 6) satisfying (5). The curve 3 is the proposed method for estimating the power spectral density of the clutter on the basis of the AR model at the order q = 6 and the correlation coefficient c = 20 satisfying (9). The clutters having the spectrum as shown in Figure 2 are normally encountered in the radar systems for detecting and tracking targets and the clutters are returned from meteorology and hydrology elements [1], [4].

It is clear from Figure 2 that the spectral estimation by the proposed method is better than that by the known parameter methods. In order to further clarify the advantage of the proposed algorithm, let us take the squared modulus of the error vectors $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\varepsilon}_2$ for the methods based on the conventional AR model and the proposed method corresponding to (10) and (11), that is,

$$\boldsymbol{\varepsilon}_1 = \mathbf{c}_{opt} - \mathbf{s}_{AR}, \ \mathrm{E}_1 = \boldsymbol{\varepsilon}_1^T \boldsymbol{\varepsilon}_1 / \mathrm{L}, \quad (10)$$

$$\boldsymbol{\varepsilon}_2 = \mathbf{c}_{opt} - \mathbf{s}_{MAR}, \ \mathrm{E}_2 = \boldsymbol{\varepsilon}_2^T \boldsymbol{\varepsilon}_2 / L, \quad (11)$$

where $\mathbf{c}_{opt}$ is the vector of the power spectral density of the clutter in the real case, when the AR order is large (q=100) and the observed sample number is infinitive, L is the sample number of the spectrum, $\mathbf{s}_{AR}$ and $\mathbf{s}_{MAR}$ are the vectors of the power spectral density of the clutter by the known method and the proposed method, respectively.

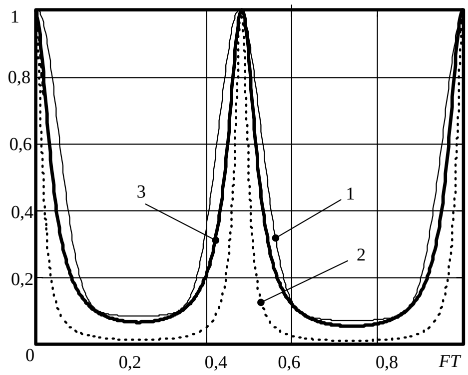

Figure 2. The power spectral density of the clutter

Table I. The error values by the two methods

Method / Error	known (q=6, c=0)	proposed (q=6, c=15)
E	0,076	0,019

From Table 1, it can be seen that the accuracy of the proposed method is 4 times better than the known method.

5) Conclusion

The previously used method has advanced the effectiveness of the spectral estimation of the clutter, which is a very important task, having significant meanings in the term of signal processing for the current radio systems, especially in the radar systems and the electronic warfare systems. Although the proposed method has remarkable advantages over the conventional AR model-based approach (Figure 2, Table I), there are certain limitations, for example, the proposed method has the larger number of calculations than the known method. It is also the author's the next study to solve that important problem.

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